# Rigidly-rotating Maxwell-Jüttner states

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## Abstract

We consider rigidly-rotating Maxwell-Jüttner thermal states on space-times with central symmetry. We discuss the topology of the Killing horizons associated with rigidly-rotating observers for the maximally-symmetric spaces (Minkowski, de Sitter and anti-de Sitter), as well as for the static black hole space-times (Schwarzschild and Reissner-Nordström black holes).
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## Global thermodynamic equilibrium in GR

- The Boltzmann equation in general relativity reads:

$$p^\mu \partial_\mu f - \Gamma^i{}_{\mu\nu} p^\mu p^\nu \partial_{p^i} f = J[f].$$

- The collision term $J[f]$ drives $f$ towards the Maxwell-Jüttner equilibrium distribution:

$$f^{(\rm eq)}_{\rm M-J} = Z \exp(\beta \mu_E + \beta p^\lambda u_\lambda),$$

where
  - $Z$ is the number of degrees of freedom;
  - $\mu_E$ is the chemical potential (set to $0$ hereafter);
  - $\beta$ is the inverse temperature;
  - $u = u^\mu \partial_\mu$ is the four-velocity of the flow;
  - $p = p^\mu \partial_\mu$ is the four-momentum of the particle.
- Since $J[f^{(\rm eq)}_{\rm M-J}] = 0$, $f^{(\rm eq)}_{\rm M-J}$ satisfies (1) if:

$$\nabla_\lambda \beta \mu_E = 0, \qquad \underbrace{(\beta u_\lambda)_{;\nu} + (\beta u_\nu)_{;\lambda} = 0}_{\text{Killing equation}}.$$

## Rotating flows on central charts

- The metric for a space-time with spherical symmetry can be written as:

$$ds^2 = w^2 \left(-dt^2 + \frac{dr^2}{u^2} + \frac{r^2}{v^2} d\Omega^2\right),$$

where $w$, $v$ and $u$ depend only on $r$.
- The Killing vector field corresponding to a rigidly-rotating flow is:

$$\beta u = \beta_0 (\partial_t + \Omega \partial_\varphi),$$

- where the inverse temperature $\beta$ is ($u^2 = -1$):

$$\beta = w \gamma^{-1} \beta_0, \qquad \gamma^{-1} = \sqrt{1 - \left(\frac{\rho \Omega}{v}\right)^2}.$$

## Killing horizons

- In co-rotating coordinates, the line element becomes:

$$ds^2 = w^2 \left[-\frac{dt^2}{\gamma^2} + \frac{2\rho^2 \Omega}{v^2} dt d\varphi + \frac{dr^2}{u^2} + \frac{r^2}{v^2} d\Omega^2\right].$$

- The surfaces where $g_{00} = 0$ represent Killing horizons:

$$-g_{00} = w^2 \left(1 - \frac{\rho^2 \Omega^2}{v^2}\right) = \frac{\beta^2}{\beta_0^2} = 0.$$

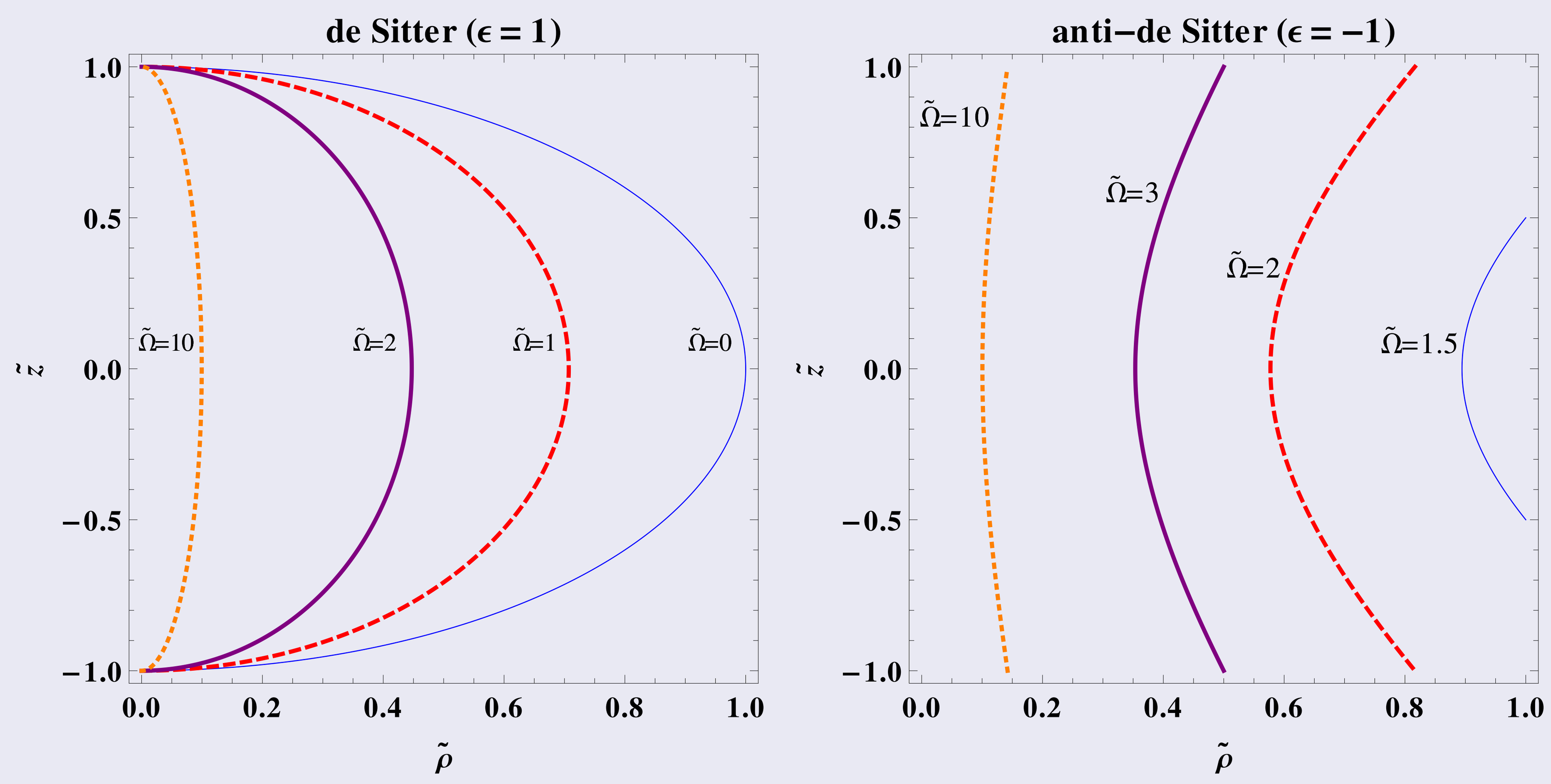
- The horizons are:
  - Rotation horizons (SOLs): $1 - \rho^2 \Omega^2 / v^2 \to 0$;
  - Cosmological (event) horizons (EHs): due to $w$ and $v$.
- The horizon structure is non-trivial, revealing an interplay between the properties of the space-time and the rigid rotation.

## Maximally-symmetric (M-S) space-times

$$ds^2 = -(1 - \epsilon \omega^2 r^2) dt^2 + \frac{dr^2}{1 - \epsilon \omega^2 r^2} + r^2 d\Omega^2,$$

$$\epsilon = 0 \ (\text{Minkowski}), \qquad \epsilon = 1 \ (\text{dS}), \qquad \epsilon = -1 \ (\text{adS}).$$

$$\beta = \beta_0 \sqrt{1 - (\epsilon \omega^2 + \Omega^2 \sin^2 \theta) r^2},$$

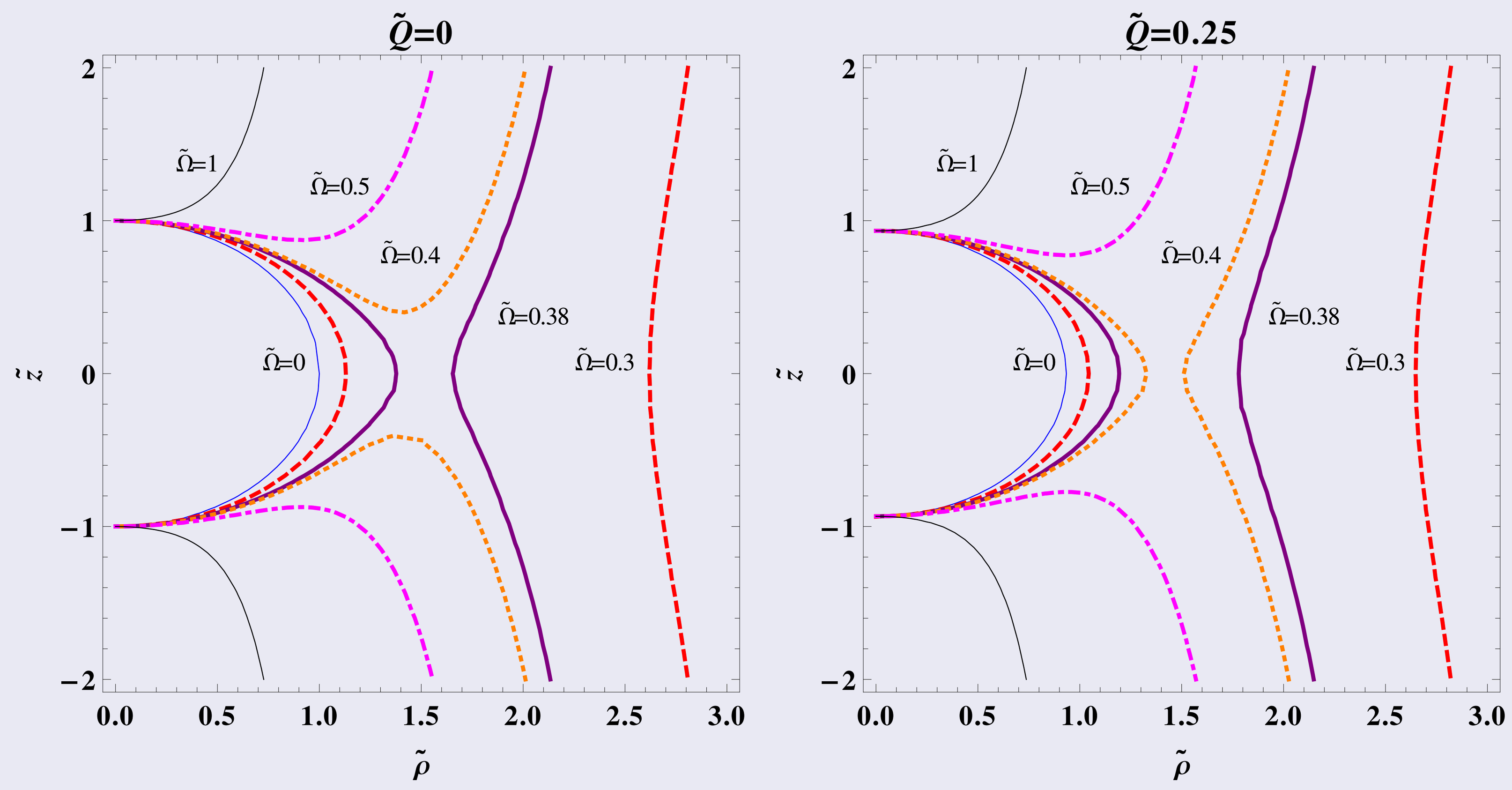



SOL at $r_{\rm SOL} = (\epsilon \omega^2 + \Omega^2 \sin^2 \theta)^{-1/2}$.

## Spherically-symmetric black holes

$$w^2 = u = v = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

$$\beta = \beta_0 \sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \rho^2 \Omega^2}.$$




- Increasing $\widetilde{\Omega} = 2M\Omega$ brings the SOL closer to, and pushes the event horizon away from, the rotation axis.
- Increasing $\widetilde{Q} = Q/2M$ has an inverse effect.

## Conclusion

- On M-S spaces, the SOL forms closer (farther) to the rotation axis on dS (adS) compared to Minkowski.
- No SOL forms on adS if $\Omega < \omega$.
- On Reissner-Nordström, the rotation enhances the EH, which joins the SOL at large $\Omega$.
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