Quantum corrections in rotating states

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Abstract

The thermal expectation value of the stress-energy tensor (SET) is calculated for a rigidly-rotating massless fermion field at finite temperature using kinetic theory and quantum field theory. The quantum corrections become dominant near the speed-of-light surface (SOL), where the SET diverges. In this poster, we compare the Eckart and Landau frames and highlight the presence of non-equilibrium terms which are not present in the classical result.

Kinetic theory result

- A flow in global thermodynamic equilibrium can be characterised using the Fermi-Dirac distribution:

$$f^{(\rm eq)} = \frac{Z}{\exp(-\beta p^\mu u_\mu) + 1}.$$

- The rigid rotation corresponds to:

$$\beta = \gamma^{-1}\beta_0, \ \mathbf{u} = \gamma(\partial_t + \Omega\partial_\varphi), \ \gamma = \frac{1}{\sqrt{1-\rho^2\Omega^2}}.$$

- The resulting SET is:

$$T^{\mu\nu} = \int \frac{d^3p}{p^{\hat 0}} f^{(\rm eq)}\, p^\mu p^\nu = E u^\mu u^\nu + P\Delta^{\mu\nu},$$

where $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$.

- Results in the massless case[a]:

$$E_{\rm Boltz} = \frac{7\pi^2\gamma^4}{60\beta_0^4}, \qquad P_{\rm Boltz} = \frac{1}{3}E_{\rm Boltz}.$$

[a] V. E. Ambruș, R. Blaga, Ann. WUT - Phys. 58 (2015) 89.

QFT results

- The Dirac field operator can be expanded as:

$$\psi(x) = \sum k (U_k b_k + V_k d_k^\dagger).$$

- The stress-energy tensor operator is:

$$T_{\mu\nu} = \frac{i}{4}\left\{[\overline{\psi}, \gamma_{(\mu} D_{\nu)}\psi] - [\overline{D_{(\mu}\psi}\gamma_{\nu)}, \psi]\right\}.$$

- Thermal expectation values can be computed[a] using the relations:

$$\langle b_k^\dagger b_{k'}\rangle_\beta = \langle d_k^\dagger d_{k'}\rangle_\beta = \frac{\delta_{kk'}}{e^{\beta\widetilde{E}_k} + 1}.$$

[a] V. E. Ambruș, E. Winstanley, Phys. Lett. B 734 (2014) 296.

Eckart frame

- The Eckart decomposition of the SET is:

$$T^{\mu\nu} = E u^\mu u^\nu + (P + \varpi)\Delta^{\mu\nu} + 2u^{(\mu}q^{\nu)} + \pi^{\mu\nu},$$

where

$$E_{\rm Eck} = \frac{7\pi^2\gamma^4}{60\beta^4} + \frac{\Omega^2}{8\beta^2}\left(\frac{4\gamma^6}{3} - \frac{\gamma^4}{3}\right),$$

$$q_{\rm Eck} = \frac{\rho\Omega^3\gamma^7}{18\beta^2}\left(\rho\Omega\partial_t + \rho^{-1}\partial_\varphi\right),$$

$$P_{\rm Eck} + \varpi_{\rm Eck} = \frac{1}{3}E_{\rm Eck}, \qquad \pi^{\mu\nu}_{\rm Eck} = 0.$$

Landau frame

- The Landau decomposition of the SET is:

$$T^{\mu\nu} = E_L u_L^\mu u_L^\nu + (P_L + \varpi_L)\Delta_L^{\mu\nu} + \pi_L^{\mu\nu},$$

where $\pi_L^{\mu\nu} \neq 0$ and u_L^μ and E_L can be found by solving the eigenvalue equation:

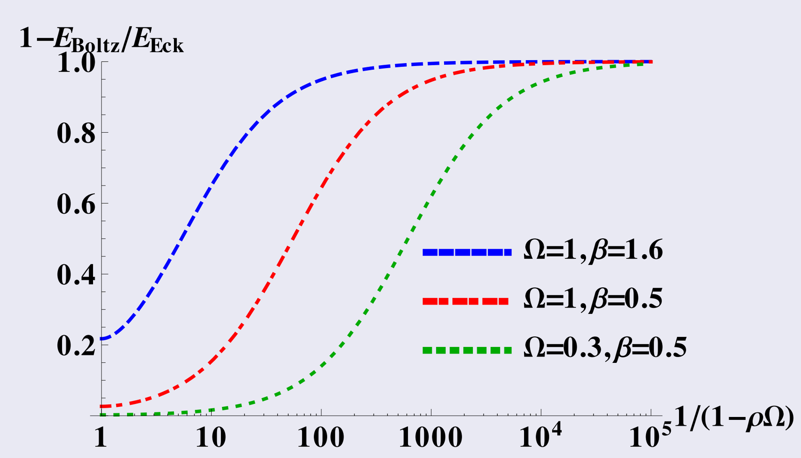
$$T^\mu{}_\nu u_L^\nu = -E_L u_L^\mu.$$

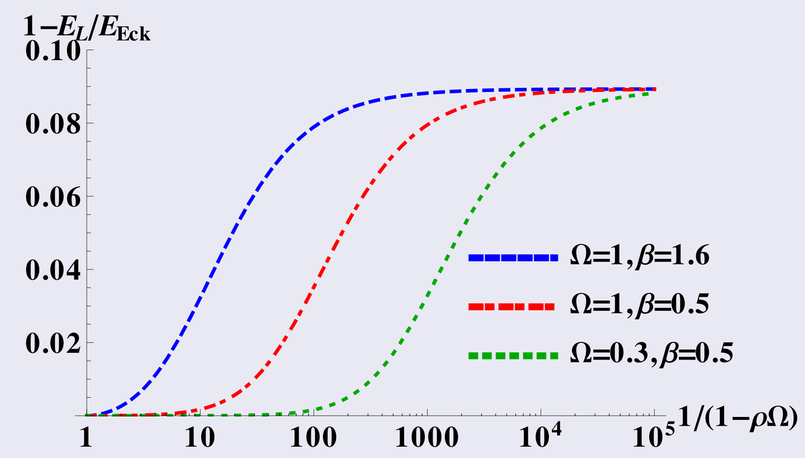
- With the above definitions, q^μ is concealed in u_L^μ:

$$E_L = \frac{E_{\rm Eck}}{3} + \sqrt{\frac{4E_{\rm Eck}^2}{9} - q_{\rm Eck}^2},$$

$$u_L^\mu = \sqrt{\frac{E_L + \frac{1}{3}E_{\rm Eck}}{2(E_L - \frac{1}{3}E_{\rm Eck})}}\left(u^\mu + \frac{q^\mu_{\rm Eck}}{E_L + \frac{1}{3}E_{\rm Eck}}\right).$$

Landau vs. Eckart vs. Boltzmann: energy



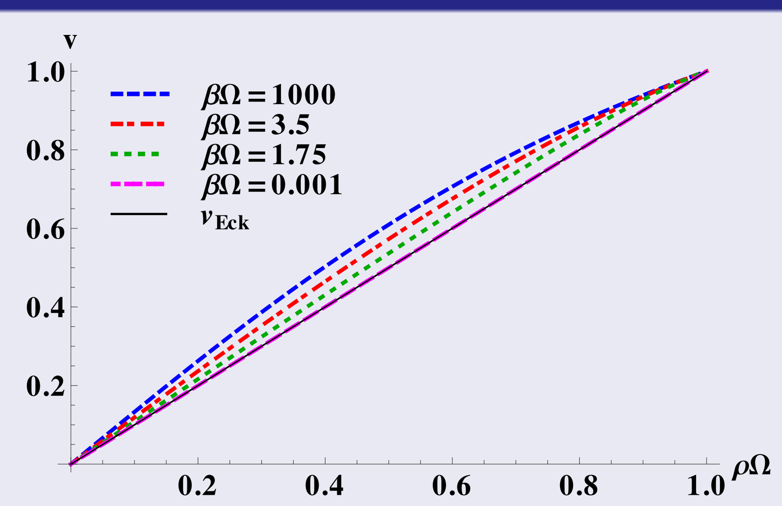


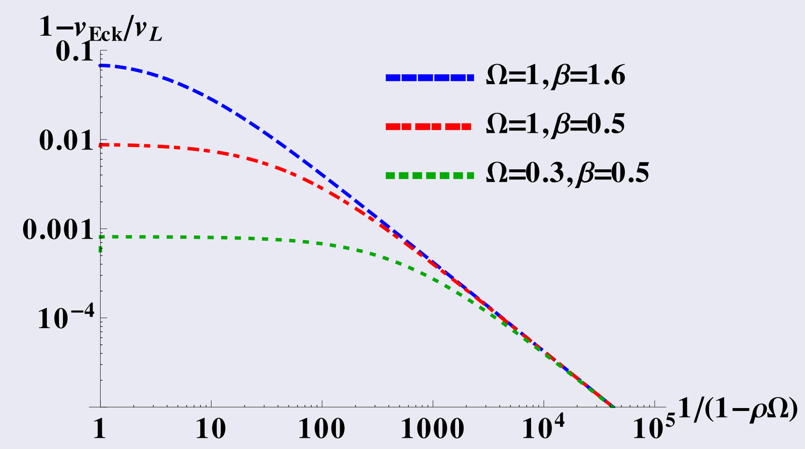
$E_{\rm Eck} \geq E_{\rm L} \geq E_{\rm Boltz}$.

$$\lim_{\rho\Omega\to 1}\frac{E_{\rm L}}{E_{\rm Eck}} = \frac{1}{3} + \frac{1}{\sqrt{3}}, \qquad \lim_{\rho\Omega\to 1}\frac{E_{\rm Boltz}}{E_{\rm Eck}} = 0,$$

while $E_{\rm Eck}\rfloor_{\rho=0} = E_{\rm L}\rfloor_{\rho=0}$.

Landau vs. Eckart vs. Boltzmann: velocity





- $v_{\rm L} \geq v_{\rm Eck}$ everywhere.
- $\lim_{\rho\Omega\to 1} v_{\rm L} = \lim_{\rho\Omega\to 1} v_{\rm Eck} = 1$.

Conclusion

- Quantum corrections become dominant at the SOL.
- At fixed ρ, the corrections increase with Ω and β.
- Landau and Eckart frames are distinct since $q \neq 0$.
- $E_{\rm L} \leq E_{\rm Eck}$ and $v_{\rm L} \geq v_{\rm Eck}$ everywhere.
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