Quantum constitutive equations for finite temperature Dirac fermions under rotation

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Abstract

The experimental confirmation of the polarization of the Lambda hyperons observed in relativistic heavy ion collisions experiments [1] has renewed the interest in anomalous transport of fermions due to the spin-orbit coupling (e.g., through the chiral vortical effect [2]). Using a non-perturbative technique [3], exact expressions are derived for the thermal expectation values of the stress-energy tensor (SET), vector charge current (VCC) and axial charge current (ACC) for the case of massless rigidly-rotating fermions at finite chemical potential. Compared to the relativistic kinetic theory analogue, the quantum corrections are expressed as constitutive equations in terms of local kinematic quantities, such as the vorticity and acceleration.

Classical approach: Relativistic kinetic theory

- In rigid rotation ($\vec{\Omega} = \Omega \vec{k}$), global equilibrium entails [3]:

$$u = \Gamma(\partial_t + \Omega\partial_\varphi), \quad \Gamma = (1 - \rho^2\Omega^2)^{-1/2}, \quad T = \Gamma T_0,$$

where ρ is the distance to the rotation axis ($T_0 = \text{const}$).

- The acceleration $a = \nabla_u u$, vorticity $\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu \nabla_\alpha u_\beta$ and the fourth vector, $\tau^\mu = \varepsilon^{\mu\nu\alpha\beta}\omega_\nu a_\alpha u_\beta$, are:

$$a = -\rho\Omega^2\Gamma^2\partial_\rho, \quad \omega = \Omega\Gamma^2\partial_z, \quad \tau = -\Omega^3\Gamma^5(\rho^2\Omega\partial_t + \partial_\varphi).$$

- In rigid rotation, these vectors are mutually orthogonal.
- We propose the following distributions for chiral fermions:

$$f^\chi_{q/\bar{q}} = \frac{1}{(2\pi)^3}\left[e^{(p\cdot u \mp \mu_V - \chi\mu_A)/T} + 1\right]^{-1},$$

where $\mu_{V/A} = \Gamma\mu_{V/A;0}$ are the vector/axial chemical potentials, p is the microscopic momentum and $\chi = \pm 1$ is the chirality.

- The vector charge current (VCC) J^μ_V, axial charge current (ACC) J^μ_A and stress-energy tensor (SET) $T^{\mu\nu}$ are defined through:

$$\begin{pmatrix} J^\mu_V \\ J^\mu_A \\ T^{\mu\nu} \end{pmatrix} = \sum_\chi \int \frac{d^3p}{p_0} \begin{pmatrix} p^\mu(f^\chi_q - f^\chi_{\bar{q}}) \\ \chi p^\mu (f^\chi_q + f^\chi_{\bar{q}}) \\ p^\mu p^\nu (f^\chi_q + f^\chi_{\bar{q}}) \end{pmatrix}.$$

- It can be shown that $J^\mu_{V/A} = Q^{\rm RKT}_{V/A} u^\mu$ and $T^{\mu\nu} = P_{\rm RKT}(4u^\mu u^\nu - g^{\mu\nu})$, where

$$Q^{\rm RKT}_{V/A} = \frac{\mu_{V/A}T^2}{3} + \frac{\mu_{V/A}}{3\pi^2}(\mu^2_{V/A} + 3\mu^2_{A/V}),$$

$$P_{\rm RKT} = \frac{7\pi^2 T^4}{180} + \frac{T^2}{6}(\mu_V^2 + \mu_A^2) + \frac{\mu_V^4 + 6\mu_V^2\mu_A^2 + \mu_A^4}{12\pi^2}.$$

QFT approach: Finite temperature field theory

- The VCC, ACC and SET operators for the Dirac field are

$$\widehat{J}^\mu_V = \frac{1}{2}[\widehat{\overline{\Psi}}, \gamma^\mu \widehat{\Psi}], \qquad \widehat{J}^\mu_A = \frac{1}{2}[\widehat{\overline{\Psi}}, \gamma^\mu\gamma^5\widehat{\Psi}],$$

$$\widehat{T}_{\mu\nu} = \frac{i}{4}[\widehat{\overline{\Psi}}, \gamma_{(\mu}\overleftrightarrow{\partial}_{\nu)}\widehat{\Psi}].$$

- The thermal expectation value (t.e.v.) of a quantum operator $\widehat{A}$ can be computed by tracing over Fock space:

$$\langle : \widehat{A} : \rangle = \mathcal{Z}^{-1}{\rm tr}(\hat{\rho} : \widehat{A} :), \qquad \mathcal{Z} = {\rm tr}(\hat{\rho}),$$

where $: \widehat{A} := \widehat{A} - \langle 0|\widehat{A}|0\rangle$ denotes normal ordering.

- For rigid rotation, the following density operator is used:

$$\hat{\rho} = \exp\left[-\beta_0(\widehat{H} - \Omega\widehat{J}^z - \mu_{V;0}\widehat{Q}_V - \mu_{A;0}\widehat{Q}_A)\right],$$

where $\widehat{J}^z$ and $\widehat{Q}_{V/A}$ are the angular momentum and vector/axial charge operators ($\beta_0 = T_0^{-1}$).

- The field operator is expanded as $\widehat{\Psi} = \sum_j (U_j \hat{b}_j + V_j \hat{d}_j^\dagger)$, where $\widetilde{E}_j = E_j - \Omega m_j > 0$ is enforced when defining the rotating vacuum state [4].
- The building blocks for the t.e.v.s are

$$\langle b^\dagger_j b_{j'}\rangle = \delta(j,j')[e^{\beta_0(\widetilde{E}_j - \mu_{V;0} - \chi_j\mu_{A;0})} + 1]^{-1},$$

$$\langle d^\dagger_j d_{j'}\rangle = \delta(j,j')[e^{\beta_0(\widetilde{E}_j + \mu_{V;0} - \chi_j\mu_{A;0})} + 1]^{-1}.$$

Charge currents

- In the β (thermometer) frame, the t.e.v.s of the CC can be decomposed w.r.t. $u = \Gamma(\partial_t + \Omega\partial_\varphi)$, as follows:

$$\langle : \widehat{J}^\mu_{V/A} : \rangle = Q^{\rm QFT}_{V/A}u^\mu + \sigma^\tau_{V/A}\tau^\mu + \sigma^\omega_{V/A}\omega^\mu,$$

$$Q^{\rm QFT}_{V/A} = Q^{\rm RKT}_{V/A} + \Delta Q_{V/A}, \qquad \Delta Q_{V/A} = \frac{\mu_{V/A}}{4\pi^2}(\vec{\omega}^2 + \vec{a}^2),$$

$$\sigma^\tau_{V/A} = \frac{\mu_{V/A}}{6\pi^2}, \quad \sigma^\omega_V = \frac{\mu_V\mu_A}{\pi^2}, \quad \sigma^\omega_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}.$$

- $\Delta Q_{V/A}$ represent quantum corrections to the charge densities.
- The conductivities $\sigma^\tau_{V/A}$ and $\sigma^\omega_{V/A}$ drive anomalous transport [5].

Stress-energy tensor

- The SET can be decomposed as:

$$\langle : \widehat{T}^{\mu\nu} : \rangle = P_{\rm QFT}(4u^\mu u^\nu - g^{\mu\nu}) + \Pi^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu,$$

where $P_{\rm QFT} = P_{\rm RKT} + \Delta P$ and

$$\Delta P = \frac{3\vec{\omega}^2 + \vec{a}^2}{24}\left(\frac{T^2}{3} + \frac{\mu_V^2 + \mu_A^2}{\pi^2}\right)$$

represents a quantum correction.

- The anisotropic stress $\Pi^{\mu\nu}$ can be written as

$$\Pi^{\mu\nu} = \Pi_1\left(\tau^\mu\tau^\nu - \frac{\vec{\omega}^2}{2}a^\mu a^\nu - \frac{\vec{a}^2}{2}\omega^\mu\omega^\nu\right) + \Pi_2(\tau^\mu\omega^\nu + \tau^\nu\omega^\mu),$$

where $\Pi_2 = -\mu_A/3\pi^2$, while $\Pi_1 = 0$ [5].

- The heat flux can be decomposed as $W^\mu = \kappa^\tau\tau^\mu + \kappa^\omega\omega^\mu$, where the azimuthal and vortical heat conductivities are [5]

$$\kappa^\tau = -\frac{1}{6}\left(\frac{T^2}{3} + \frac{\mu_V^2 + \mu_A^2}{\pi^2}\right),$$

$$\kappa^\omega = \frac{\mu_A}{3}\left[T^2 + \frac{\mu_A^2 + 3\mu_V^2}{\pi^2} + \frac{1}{4\pi^2}(\vec{\omega}^2 + \vec{a}^2)\right].$$

- The results agree with those reported in Ref. [6].

Conclusion

- An effective RKT can capture the equilibrium effects of μ_A.
- The charge and energy densities receive quantum corrections which are $\sim \Omega^2$.
- Anomalous transport is driven by the azimuthal and vortical conductivities.
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