Applicability of hydrodynamics in small and large systems

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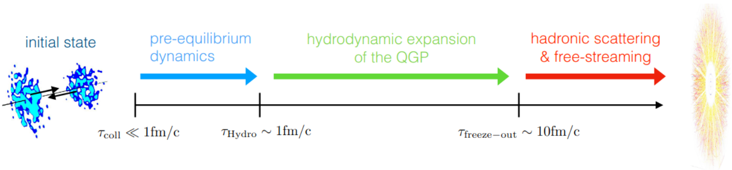
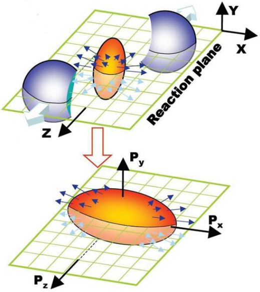
PRD **107** (2023) 094013 (arXiv:2211.14379)
PRL **130** (2023) 152301 (arXiv:2211.14356)

Universitatea de Vest din Timişoara

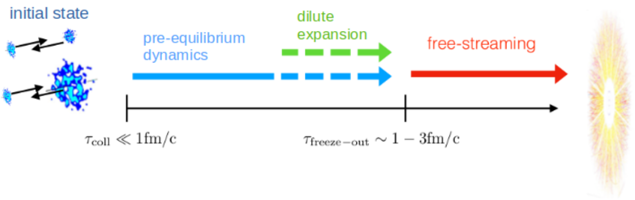
[1]Based on slides prepared by C. Werthmann.



- Shortly after the collision, the system is in a far-from-equilibrium stage.
- Pre-equilibrium dynamics require a non-equilibrium description.
- Large systems $(A + A)$ equilibrate quickly and hydrodynamics becomes applicable.
- Strongly-interacting QGP leaves imprints of thermalization and collectivity in final-state observables:
  $\mathbf{v_n}$, $\langle p_T \rangle$, particle yields, ...



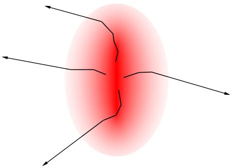
Hiroshi Masui (2008)



Very dilute, hydrodynamics not necessarily applicable

- still collective behaviour is observed!

  Nagle, Zajc Ann.Rev.Nucl.Part. 68 (2018) 211



Collectivity can also be explained in kinetic theory, a mesoscopic description which does not rely on equilibration.

- KT interpolates between free streaming at small opacities and hydrodynamics at large opacities!

Aim

Benchmarking of hydro for transverse flow observables w.r.t. kinetic theory for a simplified (conformal) fluid on full range from small to large system sizes.

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- Mesoscopic description in terms of averaged on-shell phase-space distribution of massless bosons:

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\rm eff}} \frac{{\rm d}N}{{\rm d}^3x\, {\rm d}^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y),$$

  with $\nu_{\rm eff} = 2(N_c^2 - 1) + \frac{7}{8} \times 4N_cN_f = 42.25$ for $N_f = 2.5$ flavours of massless quarks.
- Time evolution is described via the Boltzmann eq. in conformal RTA

$$p^\mu \partial_\mu f = C_{\rm RTA}[f] = -\frac{p^\mu u_\mu}{\tau_R}(f - f_{\rm eq}), \quad f_{\rm eq} = \frac{1}{e^{p^\mu u_\mu/T} - 1}, \quad \tau_R \frac{5\eta}{sT}.$$

- We assume boost invariance $\Rightarrow$ f depends only on $y - \eta$.
- At τ_0, we assume $f(\tau_0)$ depends only on $|\mathbf{p}_\perp|$ (no transverse anisotropies).
- $T^{\mu\nu} = \int_{\mathbf{p}} p^\mu p^\nu f$ is initialized as

$$T_0^{\mu\nu} = \epsilon_0(\mathbf{x}_\perp) \times {\rm diag}(1, 1/2, 1/2, 0),$$

  i.e. the longitudinal pressure vanishes, $P_L(\tau_0) = 0$.
- $\Rightarrow$ system evolution depends only on $\epsilon_0(\mathbf{x}_\perp)$ and **opacity** $\hat{\gamma}$.

System opacity $\hat{\gamma}$

- The system evolution depends only on the opacity $\sim$ "total interaction rate"

  Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

$$\hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{1}{a\pi} R \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\right)^{1/4}, \qquad a = \frac{\pi^2}{30}\nu_{\rm eff}.$$

  - $\hat{\gamma}$ encodes dependencies on viscosity , transverse size and energy scale, with

$$\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta} = \int_{\mathbf{x}_\perp} \tau_0\epsilon_0, \qquad\qquad R^2\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta} = \int_{\mathbf{x}_\perp} \tau_0\epsilon_0\mathbf{x}_\perp^2.$$

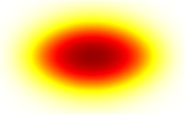
- We take as initial condition the centrality class-average of Pb+Pb at $5.02\ \mathrm{TeV} \Rightarrow R \simeq 2.78\ \mathrm{fm}$ and $\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta = 1280\ \mathrm{GeV}$

  Borghini, Borrell, Feld, Roch, Schlichting, Werthmann PRC 107 (2023), 034905

  - Since we fix the initial profile, $\hat{\gamma}$ is varied via η/s:

$$\hat{\gamma} \approx \frac{11}{4\pi\eta/s}.$$

30-40%



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- In hydro, the system is described directly by the energy-momentum tensor,

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}, \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu.$$

- Energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$ entails

$$\dot{\epsilon} + (\epsilon + P)\theta - \pi^{\mu\nu}\sigma_{\mu\nu} = 0,$$
$$(\epsilon + P)\dot{u}^\mu - \nabla^\mu P + \Delta^\mu{}_\lambda \partial_\nu \pi^{\lambda\nu} = 0,$$

  where $\theta = \partial_\mu u^\mu$ and $\sigma_{\mu\nu} = \nabla_{\langle\mu} u_{\nu\rangle}$,[2] with $\nabla_\mu \equiv \Delta^\alpha_\mu \partial_\alpha$.
- In ideal hydro, $\pi^{\mu\nu} = 0$.
- In MIS viscous hydro, $\pi^{\mu\nu}$ evolves according to

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi^{\lambda\langle\mu}\sigma_\lambda^{\nu\rangle} + \phi_7 \pi_\alpha^{\langle\mu}\pi^{\nu\rangle\alpha},$$

  where $\omega_{\mu\nu} = \frac{1}{2}[\nabla_\mu u_\nu - \nabla_\nu u_\mu]$ is the vorticity tensor.
- The transport coefficients are chosen for compatibility with RTA:

[Ambruș, Molnár, Rischke, PRD **106** (2022) 076005]

$$\eta = \frac{4}{5}\tau_\pi P, \quad \delta_{\pi\pi} = \frac{4\tau_\pi}{3}, \quad \tau_{\pi\pi} = \frac{10\tau_\pi}{7}, \quad \phi_7 = 0, \quad \tau_\pi = \tau_R.$$

- Numerical solution obtained using **vHLLE**. [Karpenko, Huovinen, Bleicher, CPC 185 (2014) 3016]

[2] $A^{\langle\mu\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta} A^{\alpha\beta}$, $\Delta^{\mu\nu}_{\alpha\beta} = \frac{1}{2}(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\alpha \Delta^\nu_\beta) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$.

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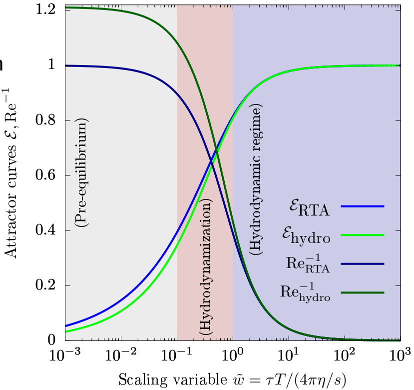
- KT and hydro disagree far from eq.
- For $\tau \ll R$, long. exp. dominate $\Rightarrow$ system behaves as a set of $0+1$-D Bjorken flows.
- In Bjorken flow, $T^{\mu\nu}$ is diagonal:

$$T^{\mu\nu} = \mathrm{diag}(\epsilon, P_T, P_T, \tau^{-2} P_L)$$
$$P_L = P + \pi_d, \quad P_T = P - \pi_d/2,$$

with $\pi_d = \tau^2 \pi^{\eta\eta}$.

- Noneq. effects can be measured using

$$\mathrm{Re}^{-1} = \sqrt{\frac{6\pi^{\mu\nu}\pi_{\mu\nu}}{\epsilon^2}},$$



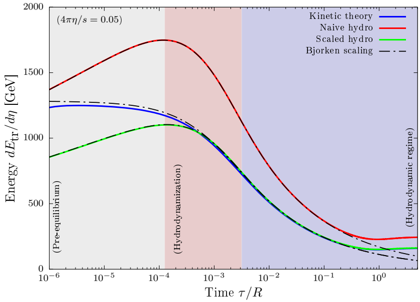
which depends only on the conformal variable $\tilde{w} = \tau T/(4\pi\eta/s)$.

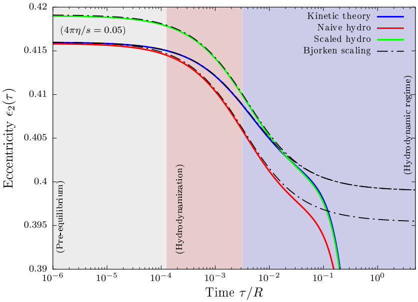
- The energy density admits a universal scaling function, $\tau^{4/3}\epsilon = (\tau^{4/3}\epsilon)_\infty \mathcal{E}(\tilde{w})$, with

$$\mathcal{E}(\tilde{w}) \simeq \begin{cases} C_\infty^{-1}\tilde{w}^\gamma, & \tilde{w} \ll 1, \\ 1 - \frac{2}{3\pi\tilde{w}}, & \tilde{w} \gg 1, \end{cases} \qquad (\tau^{4/3}\epsilon)_\infty = \text{const. dep. on } \tau_0,\ \epsilon_0,\ \gamma \text{ and } C_\infty,$$

while $\gamma = 4/9$ (0.526) and $C_\infty \simeq 0.88$ (0.80) for KT (hydro).

- Hydro and KT agree when $\tilde{w} \gtrsim 1 \Leftrightarrow \mathrm{Re}^{-1} \lesssim 0.4$.



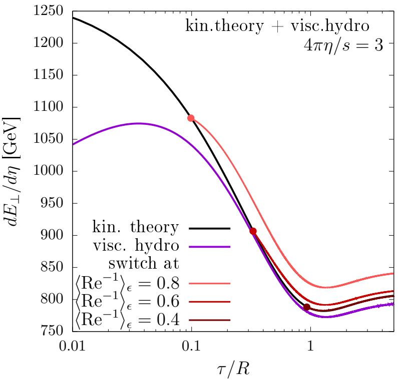


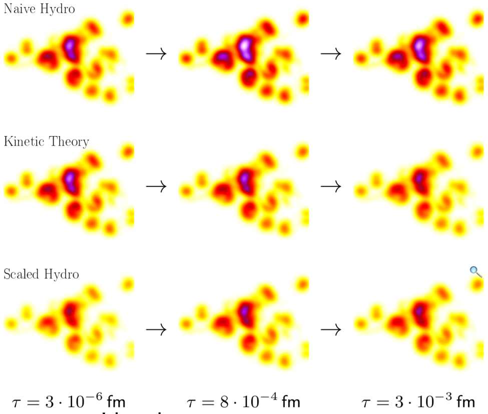
- Less work done during preeq. in hydro: $\dfrac{{\rm d}E_{\rm tr}}{{\rm d}\eta} \simeq \left(\dfrac{\tau_0}{\tau}\right)^\alpha \dfrac{{\rm d}E_\perp^{(0)}}{{\rm d}\eta}$, $\alpha = \begin{cases} 0 & \text{in KT}, \\ -0.07 & \text{in hydro}. \end{cases}$
- Inhomogeneous cooling affects shape (eccentricities) of equilibrated profile:

$$\epsilon_n \simeq \left[\int_{\mathbf{x}_\perp} x_\perp^n \epsilon_0^{1-\gamma/4}\right]^{-1} \times \int_{\mathbf{x}_\perp} x_\perp^n \epsilon_0^{1-\gamma/4} \cos(n\phi).$$

- Preeq. discrepancies can be accounted for by scaling $\epsilon_0(\mathbf{x}_\perp)$ in hydro:

$$\epsilon_0^{\rm hydro} = \left[\left(\frac{4\pi\eta/s}{\tau_0} a^{1/4}\right)^{\frac{1}{2} - \frac{9\gamma}{8}} \left(\frac{C_\infty^{\rm RTA}}{C_\infty^{\rm hydro}}\right)^{9/8} \epsilon_0^{\rm RTA}\right]^{\frac{8/9}{1-\gamma/4}}.$$

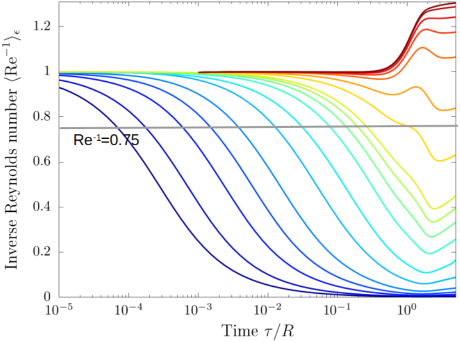


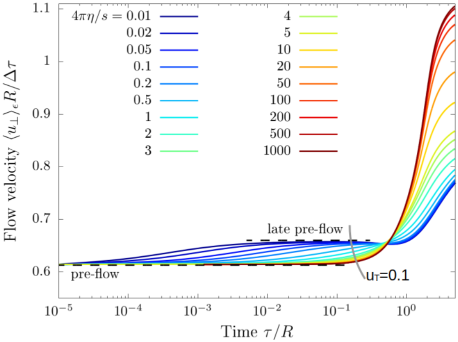


To counteract preequilibrium discrepancies, we considered:

- Scaled hydro, using modified, locally-scaled initial profile $\epsilon_0^{\rm hydro}(\mathbf{x}_\perp)$.
  - Fails if eq. time $\tau_{\rm eq} \sim \hat{\gamma}^{-4/3}$ is comparable to R and eq. is interrupted by transv. exp.
- Hybrid simulations, switching from KT to hydro at $\tau_{\rm sw} > \tau_0$.
  - When $\mathrm{Re}^{-1}(\tau_{\rm sw}) \gtrsim 0.4$, part of the system is still in preeq. $\Rightarrow$ discrepancies will appear at late times $\Rightarrow$ $\mathrm{Re}^{-1}(\tau_{\rm sw})$-based criterion!
  - For small $\hat{\gamma}$, $\mathrm{Re}^{-1}(\tau_{\rm eq})$ is still large $\Rightarrow$ Re^{-1}-based switching criterion is never reached!

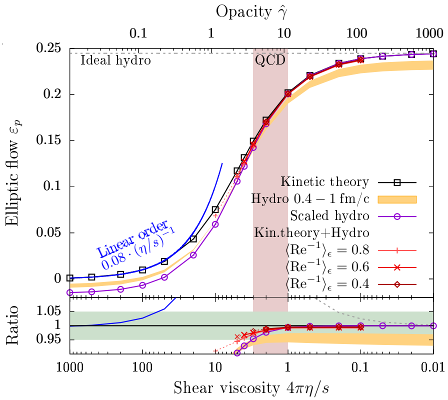
Transition between dynamical regimes

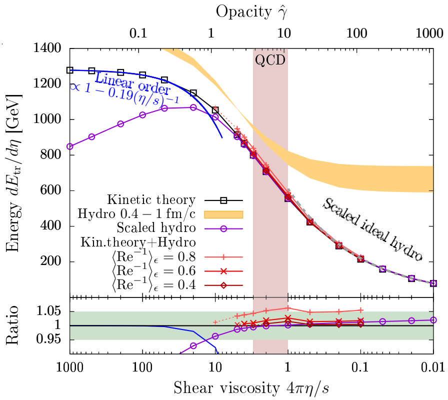




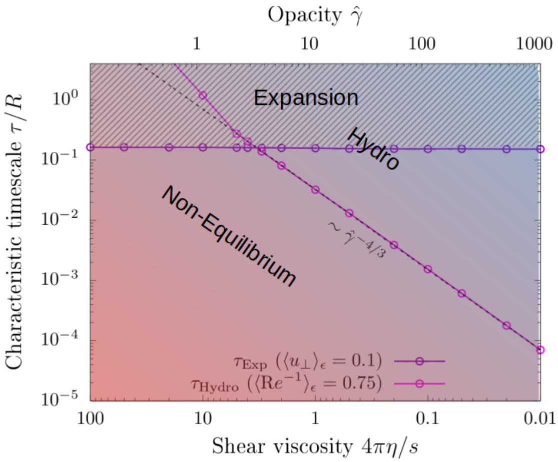
- Transverse expansion sets in when $\langle u_\perp \rangle_\epsilon \gtrsim 0.1$, for $\tau \simeq 0.2R$.
- Hydro is applicable when $\mathrm{Re}^{-1} \lesssim 0.75 \Rightarrow$ discrepancies can be expected for $4\pi\eta/s \gtrsim 3$.

Scaled and hybrid hydro vs. KT





- Naive hydro, initialized with same ϵ_0 as RKT at $\tau_0 = 0.4$–$1\ \mathrm{fm}/c$ underestimates ε_p and overestimates $dE_{\mathrm{tr}}/d\eta$.
- Scaled hydro is in perfect agreement at large $\hat{\gamma}$ but loses applicability as $\hat{\gamma} \lesssim 3$–4.
- Hybrid hydro can improve on scaled hydro, but only down to $\hat{\gamma} \simeq 1$.



- Transverse expansion sets in at $\tau_{\rm Exp} \sim 0.2R$, independent of opacity.
- Hydro applicable when $\text{Re}^{-1} \lesssim 0.75$.
- When $\hat{\gamma} \lesssim 3$, hydrodynamization is interrupted by transv. expansion.

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What does the criterion $\hat{\gamma} \gtrsim 3$ imply for the applicability of hydro to realistic collisions?

$$\mathrm{p+p}: \overset{\text{min.bias}}{\hat{\gamma}} \sim 0.7 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.12\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{7.1\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4}$$

far from hydrodynamic behaviour

$$\mathrm{p+Pb}: \overset{\text{min.bias}}{\hat{\gamma}} \sim 1.5 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.81\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{24\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4} \overset{\text{high mult.}}{\lesssim} 2.7$$

very high multiplicity events approach regime of applicability, but do not reach it

$$\mathrm{O+O}: \overset{30-40\%}{\hat{\gamma}} \sim 2.2 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{1.13\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{55\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4} \sim \overset{70-80\%}{1.4} - \overset{0-5\%}{3.1}$$

probes transition region to hydrodynamic behaviour

$$\mathrm{Pb+Pb}: \overset{30-40\%}{\hat{\gamma}} \sim 5.7 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{2.78\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{1280\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{42.25}\right)^{-1/4} \sim \overset{70-80\%}{2.7} - \overset{0-5\%}{9.0}$$

hydrodynamic behaviour in all but peripheral collisions

Summary

- We employed KT to explore transverse flow for a simplified, conformal fluid over the entire opacity range.
- Hydrodynamics is accurate at 5% level if Re^{-1} drops below ~ 0.75 before transv. exp. sets in.
- In small systems (p+p, p+Pb), transverse expansion interrupts equilibration $\Rightarrow$ hydro not applicable!
  - O+O covers transition regime to hydro behaviour

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Backup

How can hydro still describe small systems?

In theoretical descriptions:

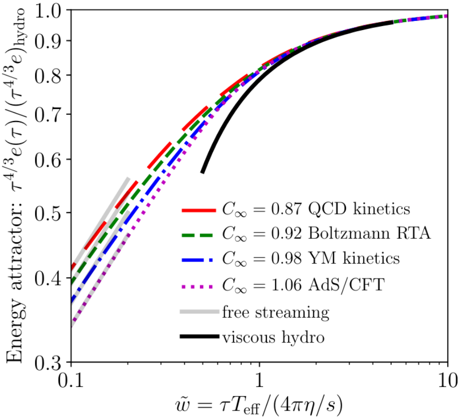
$$v_n = \kappa_{n,n} \cdot \epsilon_n$$

- Flow can be compared to experiment
- Response depends on the dynamical model
- Initial state geometry is poorly constrained in small systems

Varying initial condition in order to fit flow measurements will mask inaccuracies in the description of the dynamical response!

What might happen when going beyond RTA?

- more complex kernels will introduce further parameter dependence, but opacity dependence might still be "leading order approximation"
- in Bjorken flow, equilibration happens in very similar ways across different model descriptions:



Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

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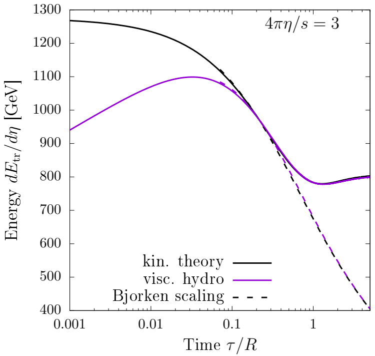
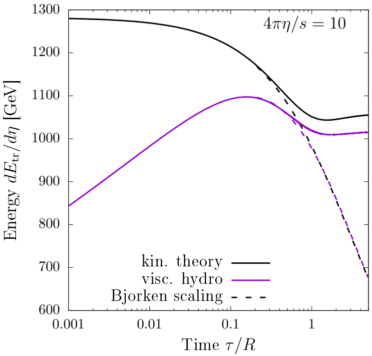
- accuracy depends on timescale separation of pre-equilibrium and transv. expansion

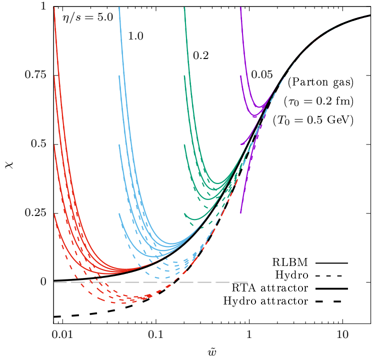




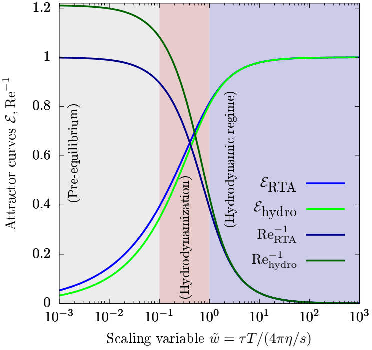
Bjorken flow attractor

- Longitudinal boost-invariant Bjorken flow exhibits universal behaviour.
- Time evolution curves converge to an attractor w.r.t. the scaling variable $\tilde{w} = \frac{\tau T}{4\pi\eta/s}$.
- The attractor can be described by universal scaling functions:
  $\chi(\tilde{w}) = p_L/p_T, \quad \mathcal{E}(\tilde{w}) \propto \tau^{4/3} e, \quad f_{E_\perp}(\tilde{w}) \propto \tau^{1/3} \mathrm{d}E_\perp/\mathrm{d}y, \ldots$

Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301

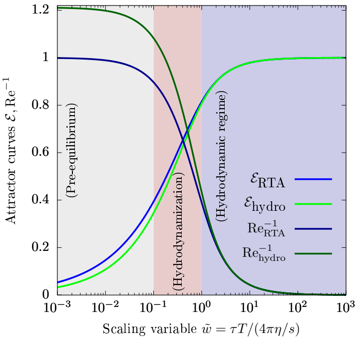


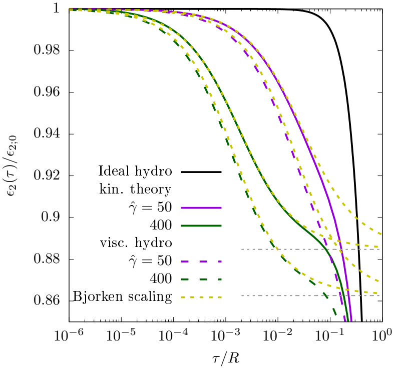
Ambruș, Bazzanini, Gabbana, Simeoni, Succi, Tripicione, Nature Comput. Sci. **2** (2022) 641.



Ambruș, Werthmann, Schlichting, Phys. Rev. D **107** (2023) 094013.

- $\tau \ll R$: no transverse expansion, system locally behaves like 0+1D Bjorken flow
  - universal attractor curve scaling in the variable $\tilde{w}(\tau, \mathbf{x}_\perp) = \frac{T(\tau, \mathbf{x}_\perp)\tau}{4\pi\eta/s}$
    Giacalone, Mazeliauskas, Schlichting, PRL 123 (2019) 262301
  - $\tilde{w} \gg 1$: $\tau^{4/3} e = \text{const.}$, $\tau^{1/3} \frac{dE_\perp}{dy} = \text{const.}$
  - $\tilde{w} \ll 1$: model dependent power law $\tau^{4/3} e \sim \tilde{w}^\gamma$





- Inhomogeneous cooling changes energy density profile

Early-time Bjorken scaling

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- The energy density in Bjorken flow is described by the universal attractor curve $\mathcal{E}(\tilde{w})$,

$$\tau^{4/3}\epsilon = (\tau^{4/3}\epsilon)_\infty \mathcal{E}(\tilde{w}), \quad (\tau^{4/3}\epsilon)_\infty = C_\infty \left(\frac{4\pi\eta}{s} a^{1/4}\right)^\gamma \left(\tau_0^{(\frac{4}{3}-\gamma)/(1-\gamma/4)} \epsilon_0\right)^{1-\gamma/4},$$

  where $\tilde{w} \to \tilde{w}(\tau, \mathbf{x}_\perp) = \tau T(\tau, \mathbf{x}_\perp)/(4\pi\eta/s)$.
- At early times, $\mathcal{E}(\tilde{w} \ll 1) = C_\infty^{-1} \tilde{w}^\gamma$ and

$$\epsilon(\tau) = \left(\frac{\tau_0}{\tau}\right)^{(\frac{4}{3}-\gamma)/(1-\frac{\gamma}{4})} \epsilon_0 = \begin{cases} \epsilon_0 & \text{in KT}, \\ (\tau/\tau_0)^{0.07} \epsilon_0 & \text{in hydro}. \end{cases}$$

- At late times, $\mathcal{E}(\tilde{w} \gg 1) = 1 - 2/(3\pi\tilde{w})$ for both KT and hydro.
- Since $(\tau^{4/3}\epsilon)_\infty$ depends on the theory, the late-time limit of KT and hydro is still different.
- Due to inhomogeneous cooling, the eccentricities of the equilibrated system are different from the early-time, free-streaming ones:

$$\epsilon_n = -\frac{\int_{\mathbf{x}_\perp} x_\perp^n \epsilon \cos(n\phi)}{\int_{\mathbf{x}_\perp} x_\perp^n \epsilon} \to -\frac{\int_{\mathbf{x}_\perp} x_\perp^n \epsilon_0^{1-\gamma/4} \cos(n\phi)}{\int_{\mathbf{x}_\perp} x_\perp^n \epsilon_0^{1-\gamma/4}}.$$

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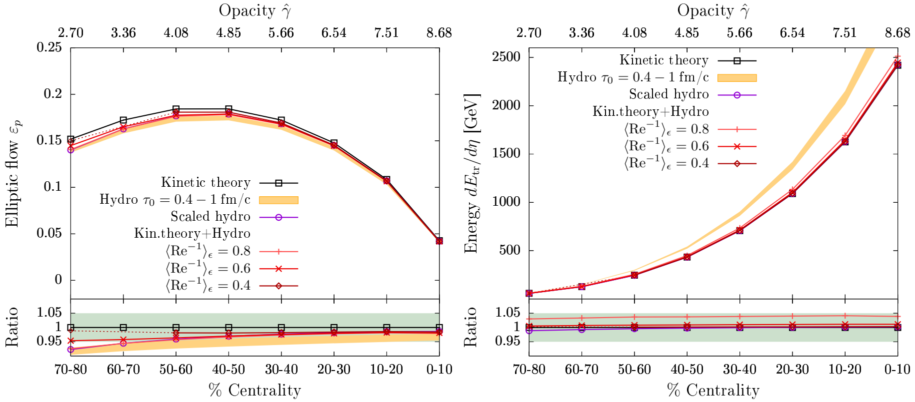
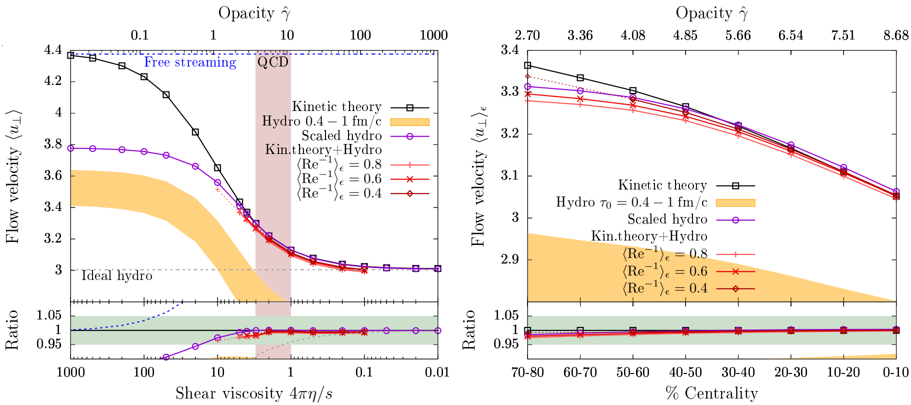
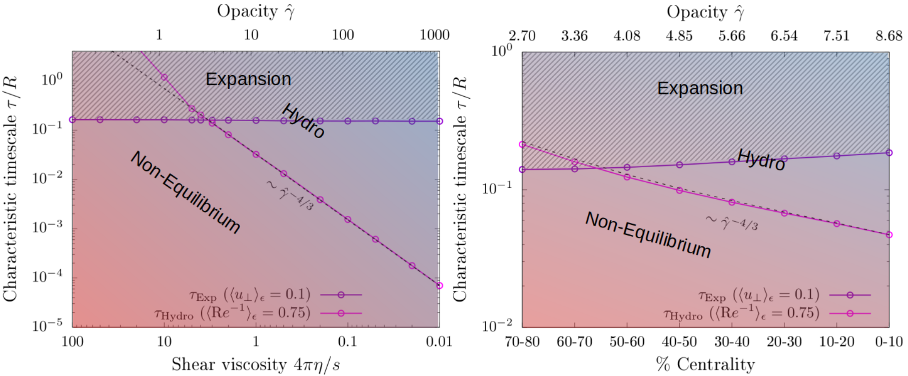

Opacity $\hat{\gamma}$
2.70 3.36 4.08 4.85 5.66 6.54 7.51 8.68
Elliptic flow ε_p
Kinetic theory
Hydro $\tau_0 = 0.4 - 1$ fm/c
Scaled hydro
Kin.theory+Hydro
$\langle \mathrm{Re}^{-1} \rangle_\epsilon = 0.8$
$\langle \mathrm{Re}^{-1} \rangle_\epsilon = 0.6$
$\langle \mathrm{Re}^{-1} \rangle_\epsilon = 0.4$
Ratio
70-80 60-70 50-60 40-50 30-40 20-30 10-20 0-10
% Centrality
Energy $dE_{\mathrm{tr}}/d\eta$ [GeV]

Transverse flow velocity



Hydrodynamization in viscosity and centrality dependence



- Transverse expansion sets in at $\tau_\perp \sim 0.2R$, independent of opacity
- Hydro applicable when $\text{Re}^{-1} < \text{Re}_c^{-1} \sim 0.75$ after timescale

$$\tau_{\rm Hydro}/R \approx 1.53\ \hat{\gamma}^{-4/3}\ \left[(\text{Re}_c^{-1})^{-3/2} - 1.21(\text{Re}_c^{-1})^{0.7}\right]$$

- Hydrodynamization sets in before transverse expansion when $\hat{\gamma} \gtrsim 3$.