

# Rotating fermions on anti-de Sitter space

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VEA, EW, PLB **749** (2015) 597; CQG **34** (2017) 145010; Symmetry **2021**, 13, 2019.



## 1 Introduction

## 2 Rigid rotation: kinematics

## 3 Finite-temperature field theory

## 4 Results

## 5 Conclusion

Vortical effects relevant for:

- ▶ Rotating black holes (frame dragging)

M. Casals *et al.*, PRD **87** (2013) 064027

- ▶ Neutron stars (anomalous transport)

M. Kaminski *et al.*, PLB **760** (2016) 170

- ▶ QGP (spin polarisation)

STAR Collaboration, Nature **548** (2017) 62

VEA, M.N. Chernodub, EPJC **82** (2022) 61

Why AdS?

- ▶ AdS/CFT duality

Aharony, Gubser, Maldacena, Ooguri, Oz, Phys. Rep. **323** (2000) 183

Son, Starinets, JHEP **03** (2006) 052.

- ▶ AdS has timelike boundary  $\rightarrow$  no SLS for “mild”  $\Omega$ .

R. Panerai, PRD **93** (2016) 104021

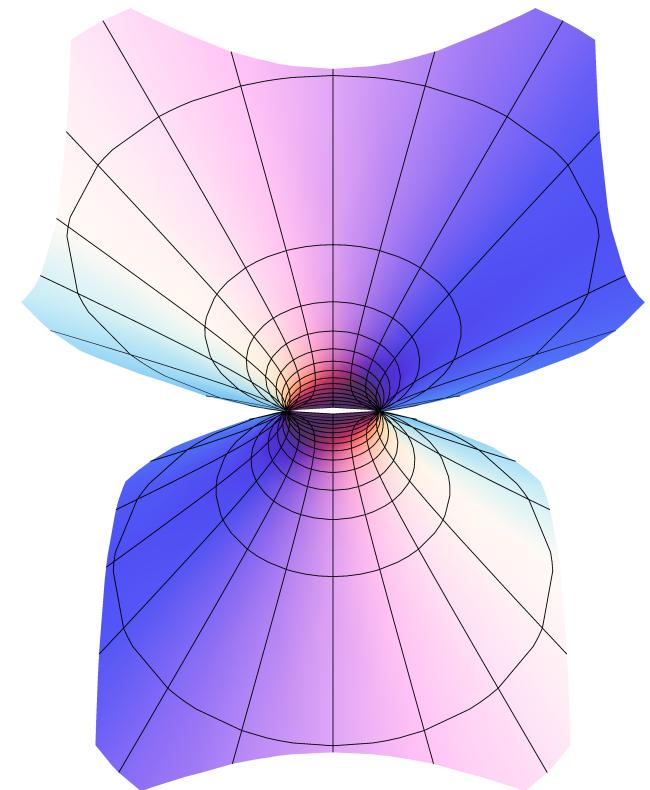
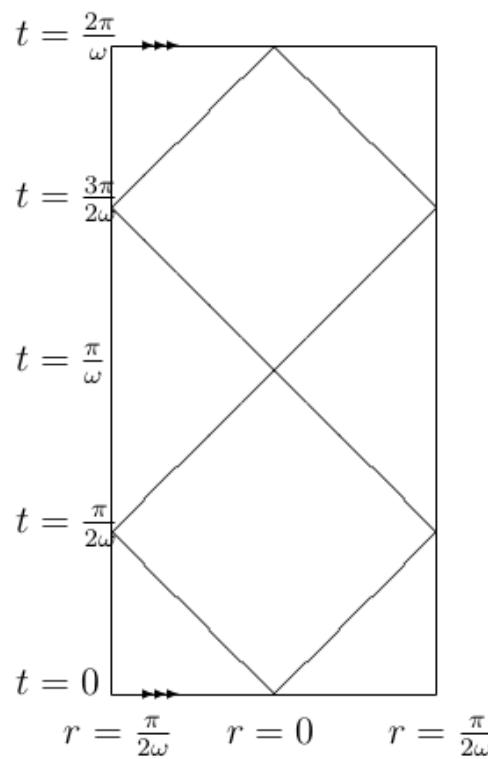
- ▶ Maximal symmetry  $\Rightarrow$  analytically tractable.

# Anti-de Sitter space

- ▶ Maximal symmetry:  
 $R_{\mu\nu} = \frac{1}{4}Rg_{\mu\nu}$ ,  
 $R = 4\Lambda = -12\ell^{-2}$ .
- ▶ Boundary at  $r = \pi\ell/2$   
reached in a finite time  
only by null geodesics.
- ▶ Closed timelike loops  
 $\Rightarrow$  CAdS ( $-\infty < t < \infty$ ).
- ▶ Line element:

$$ds^2 = \frac{\ell^2}{\cos^2 \bar{r}} \left[ -d\bar{t}^2 + d\bar{r}^2 + \sin^2 \bar{r} (d\theta^2 + \sin^2 \theta d\varphi^2) \right],$$

where  $\bar{t} = t/\ell$ ,  $\bar{r} = r/\ell$ .



- ▶ For the maximally symmetric vacuum state,  $S_F$  can be written as:

[W. Mück, J. Phys. A 33 \(2000\) 2000 3021](#)

$$iS_{\text{vac}}^F(x, x') = [\mathcal{A}(s) + \mathcal{B}(s)\not{p}] \Lambda(x, x').$$

- ▶ The geodesic interval  $s = \ell \bar{s}$  can be given through:

$$\cos \bar{s} = \frac{\cos \Delta \bar{t}}{\cos \bar{r} \cos \omega \bar{r}'} - \cos \gamma \tan \bar{r} \tan \bar{r}',$$

where  $\cos \gamma = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \Delta \varphi$ .

- ▶  $n_\mu = \nabla_\mu s(x, x')$  is the normalised tangent to the geodesic at  $x$ .
- ▶  $\mathcal{A}$  and  $\mathcal{B}$  depend only on  $\bar{s}$  and satisfy:

$$i \frac{d}{d\bar{s}} \begin{pmatrix} \mathcal{A} \\ \mathcal{B} \end{pmatrix} + \frac{3i}{2} \begin{pmatrix} -\mathcal{A} \tan(\bar{s}/2) \\ \mathcal{B} \cot(\bar{s}/2) \end{pmatrix} - \ell M \begin{pmatrix} \mathcal{B} \\ \mathcal{A} \end{pmatrix} = \begin{pmatrix} 0 \\ i(-g)^{-1/2} \delta(x, x') \end{pmatrix}.$$

- ▶ The equations can be solved exactly:

$$\mathcal{A}_F = \frac{\Gamma_k}{16\pi^2\ell^3} \frac{\cos(\bar{s}/2)}{[-\sin^2(\bar{s}/2)]^{2+k}} {}_2F_1 \left( 1+k, 2+k; 1+2k; \operatorname{cosec}^2 \left( \frac{\bar{s}}{2} \right) \right),$$

$$\mathcal{B}_F = \frac{i\Gamma_k}{16\pi^2\ell^3} \frac{\sin(\bar{s}/2)}{[-\sin^2(\bar{s}/2)]^{2+k}} {}_2F_1 \left( k, 2+k; 1+2k; \operatorname{cosec}^2 \left( \frac{\bar{s}}{2} \right) \right),$$

where  $k = \ell m$ , while the normalisation constant  $\Gamma_k$  is given by

$$\Gamma_k = \frac{\Gamma(2+k)\Gamma\left(\frac{1}{2}\right)}{4^k\Gamma\left(\frac{1}{2}+k\right)}.$$

- ▶ In the limit  $k \rightarrow 0$ , we have  $\Gamma_k \rightarrow 1$  and

$$\lim_{k \rightarrow 0} \mathcal{A}_F = \frac{1}{16\pi^2\ell^3} \left( \cos \frac{\bar{s}}{2} \right)^{-3}, \quad \lim_{k \rightarrow 0} \mathcal{B}_F = \frac{i}{16\pi^2\ell^3} \left( \sin \frac{\bar{s}}{2} \right)^{-3}.$$

- The bi-spinor of parallel transport  $\Lambda(x, x')$  satisfies:

W. Mück, J. Phys. A 33 (2000) 3021

$$D_\mu \Lambda(x, x') = -i\omega S_{\mu\nu} n^\nu \Lambda(x, x') \tan\left(\frac{\bar{s}}{2}\right).$$

- Employing the Cartesian gauge for the tetrad:

I. I. Cotăescu, Rom. J. Phys. 52 (2007) 895

$$e_{\hat{t}} = \ell^{-1} \cos \bar{r} \partial_{\bar{t}}, \quad e_{\hat{i}} = \cos \bar{r} \left[ \frac{\bar{r}}{\sin \bar{r}} \left( \delta_{ij} - \frac{x^i x^j}{r^2} \right) + \frac{x^i x^j}{r^2} \right] \partial_j, \quad (1)$$

allows  $\Lambda(x, x')$  to be expressed as:

VEA, EW, CQG 34 (2017) 145010.

$$\begin{aligned} \Lambda(x, x') = & \frac{\sec(\bar{s}/2)}{\sqrt{\cos \bar{r} \cos \bar{r}'}} \left[ \cos \frac{\Delta \bar{t}}{2} \left( \cos \frac{\bar{r}}{2} \cos \frac{\bar{r}'}{2} + \sin \frac{\bar{r}}{2} \sin \frac{\bar{r}'}{2} \frac{\mathbf{x} \cdot \boldsymbol{\gamma}}{r} \frac{\mathbf{x}' \cdot \boldsymbol{\gamma}}{r'} \right) \right. \\ & \left. + \sin \frac{\Delta \bar{t}}{2} \left( \sin \frac{\bar{r}}{2} \cos \frac{\bar{r}'}{2} \frac{\mathbf{x} \cdot \boldsymbol{\gamma}}{r} \boldsymbol{\gamma}^{\hat{t}} + \sin \frac{\bar{r}'}{2} \cos \frac{\bar{r}}{2} \frac{\mathbf{x}' \cdot \boldsymbol{\gamma}}{r'} \boldsymbol{\gamma}^{\hat{t}} \right) \right]. \quad (2) \end{aligned}$$

- ▶ Using the ansatz  $S_F = (\alpha + \beta \not{h})\Lambda$ , the v.e.v. of  $T_{\mu\nu}$  can be computed as

$$\langle 0 | T_{\mu\nu} | 0 \rangle = 8i \lim_{x' \rightarrow x} \left[ g_{\mu\nu} \frac{\omega}{2} \cot \frac{\omega s}{2} \beta - n_\mu n_\nu \left( \frac{\partial}{\partial s} - \frac{\omega}{2} \cot \frac{\omega s}{2} \right) \beta \right].$$

- ▶ Due to maximal symmetry,  $T_{\mu\nu} \sim g_{\mu\nu}$ .

$$\langle 0 | T_{\mu\nu} | 0 \rangle = \frac{i}{2} \lim_{x' \rightarrow x} \text{tr} \left\{ \gamma_{(\nu}(x) D_{\mu)}(x) S_F(x, x') - S_F(x, x') \overleftarrow{D}_{(\mu}(x') \gamma_{\nu)}(x') \right\}.$$

- ▶ The renormalisation of  $T_{\mu\nu}$  is performed by subtracting renormalisation counter-terms:

[S. M. Christensen, Phys. Rev. D 17, 946–963 \(1978\)](#)

$$T_{\text{ren;SdW}} = -\frac{\omega^4}{4\pi^2} \left\{ \frac{11}{60} + k - \frac{k^2}{6} - k^3 + 2k^2(k^2 - 1) \left[ \ln \frac{\nu}{\omega} - \psi(k) + \ln 2 \right] \right\}.$$

- ▶ The result agrees with the Pauli–Villars regularisation method.

[R. Camporesi and A. Higuchi, Phys. Rev. D 45, 3591–3603 \(1992\)](#)

# Hadamard regularisation of $S_F$

- ▶ Hadamard regularisation uses the auxiliary function  $\mathcal{G}_F$ :

$$S_F(x, x') = (i\gamma^\lambda D_\lambda + \mu)\mathcal{G}_F(x, x').$$

- ▶ The Hadamard theorem states that:

$$\mathcal{G}_F(x, x') = \frac{1}{8\pi^2} \left( \underbrace{\frac{u}{\sigma} + v \ln |\nu^2 \sigma| + w}_{\text{Hadamard form } \mathcal{G}_H} \right) \Lambda(x, x'),$$

where  $\sigma = -\frac{1}{2}s^2$ ;  $u$ ,  $v$  and  $w$  are finite; and  $\nu$  is a renormalisation mass scale.

- ▶  $u$  and  $v$  can be determined from the Dirac equation:

$$u = \left( \frac{\omega s}{\sin \omega s} \right)^{\frac{3}{2}}, \quad v = \frac{\mu^2 - \omega^2}{2} \cos \frac{\omega s}{2} {}_2F_1 \left( 2 - k, 2 + k; 2; \sin^2 \frac{\omega s}{2} \right),$$

leading to

VEA, EW, PLB 749 (2015) 597

$$T_{\text{ren}} = -\frac{\omega^4}{4\pi^2} \left\{ \frac{11}{60} + k - \frac{7k^2}{6} - k^3 + \frac{3k^4}{2} + 2k^2(k^2 - 1) \left[ \ln \frac{\tilde{\nu}}{\omega} - \psi(k) \right] \right\},$$

where  $k = \mu/\omega$ ,  $\tilde{\nu} = \nu e^{-\gamma} \sqrt{2}$  and  $\gamma$  is Euler's constant.

# Rigid rotation in spherically-symmetric space-times

- ▶ SSST can be parametrised using central charts:

VEA, I. I. Cotăescu, PRD 94 (2016) 085022

$$ds^2 = W^2 \left[ -dt^2 + \frac{dr^2}{U^2} + \frac{r^2}{V^2} (d\theta^2 + \sin^2 \theta d\varphi^2) \right], \quad (3)$$

- ▶  $W = U = V = 1$  for Mink, while on adS,

$$W = \frac{1}{\cos \bar{r}}, \quad U = 1, \quad V = \frac{r}{\sin \bar{r}}. \quad (4)$$

- ▶ The relativistic Boltzmann equation,  $k^\mu \partial_\mu f = C[f]$ , is satisfied when

$$f = f^{(\text{eq})}(-\beta \cdot k - \alpha), \quad \nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0, \quad \nabla_\mu \alpha = 0, \quad (5)$$

where  $\beta^\mu = T^{-1} u^\mu$  and  $\alpha = \mu/T$  ( $= 0$  in this talk).

- ▶ Rigid rotation  $\beta = \beta_0(\partial_t + \Omega \partial_\varphi)$  is a solution of the Killing equation, with

$$u = \frac{\Gamma}{W(r)} (\partial_t + \Omega \partial_\varphi), \quad \begin{pmatrix} T \\ \mu \end{pmatrix} = \Gamma \begin{pmatrix} T_0 \\ \mu_0 \end{pmatrix}, \quad \Gamma = (1 - \rho^2 \Omega^2)^{-1/2}, \quad (6)$$

where  $\rho = r \sin \theta / V$  is  $r \sin \theta$  on Minkowski and  $\ell \sin \bar{r} \sin \theta$  on adS.

# Kinematic tetrad

Becattini, Grossi, PRD **92** (2015) 045037



**Velocity :**

$$u = \Gamma(\partial_t + \Omega\partial_\varphi),$$

**Acceleration :**

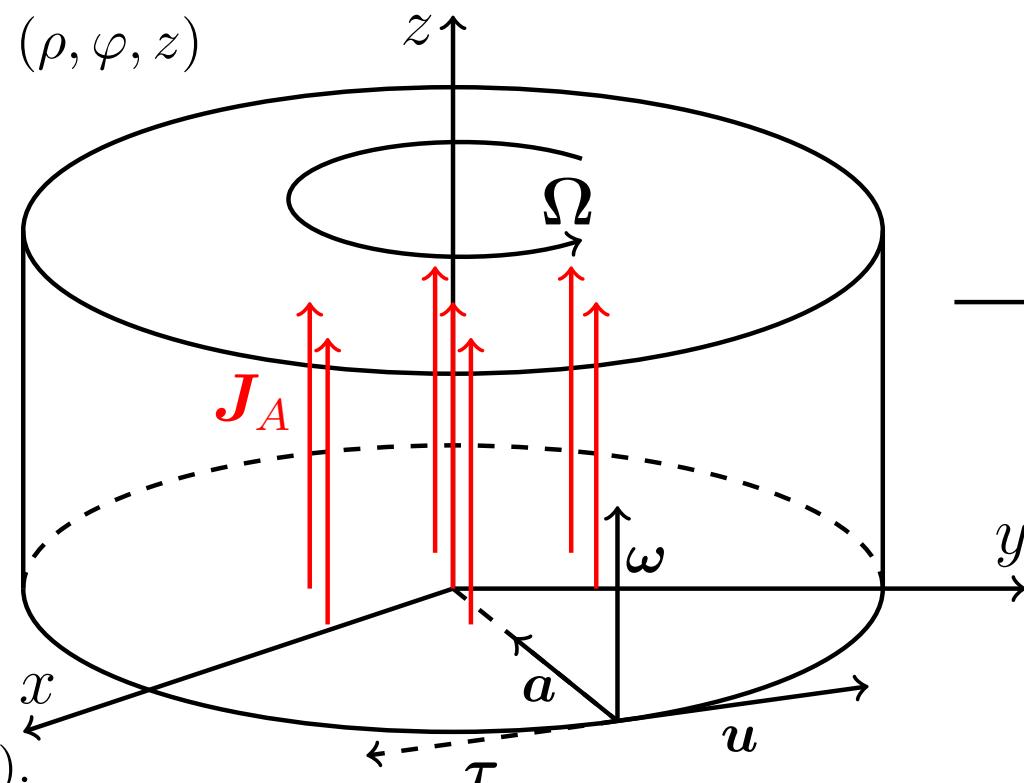
$$a = \nabla_u u = -\rho\Omega^2\Gamma^2\partial_\rho,$$

**Vorticity :**

$$\begin{aligned} \omega &= \frac{1}{2}\varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}}e_{\hat{\alpha}}u_{\hat{\beta}}(\nabla_{\hat{\gamma}}u_{\hat{\sigma}}) \\ &= \Gamma^2\Omega\partial_z, \end{aligned}$$

**Fourth vector :**

$$\begin{aligned} \tau &= -\varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}}e_{\hat{\alpha}}\omega_{\hat{\beta}}a_{\hat{\gamma}}u_{\hat{\sigma}} \\ &= -\Omega^3\Gamma^5(\rho^2\Omega\partial_t + \partial_\varphi). \end{aligned}$$



Axial vortical effect:

Kharzeev, Liao, Voloshin, Wang, PPNP **88** (2016) 1

$$J_A^\mu = \sigma_A^\omega \omega^\mu, \quad \sigma_A^\omega = \frac{T^2}{6} + \frac{1}{24\pi^2} (\omega^2 + 3a^2 - 6M^2) + O(M^4). \quad (7)$$

# Kinematic tetrad for adS.

**Velocity** :  $u = \Gamma(e_{\hat{t}} + \rho\Omega e_{\hat{\varphi}})$ ,

**Acceleration** :  $a = a(0) + [a(\Omega) - a(0)]$ ,

$$a(0) = \frac{\sin \bar{r} \cos \bar{r}}{\ell^2} \partial_{\bar{r}},$$

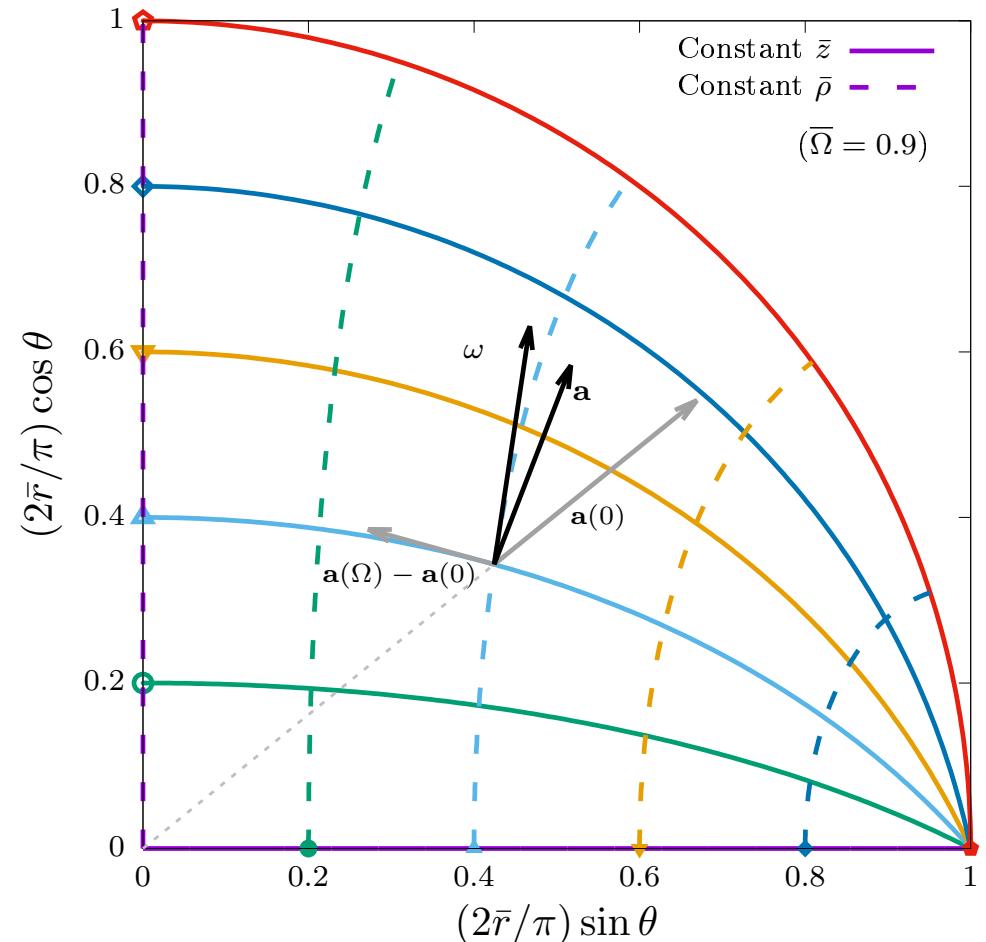
$$a(\Omega) - a(0) = -\rho\Omega^2\Gamma^2 \frac{1 - \bar{\rho}^2}{\sec^2 \bar{r}} \partial_{\rho},$$

**Vorticity** :  $\omega = \Omega\Gamma^2(1 - \bar{\rho}^2)\partial_z$ ,

**Fourth vector** :  $\tau = -\rho\Omega(1 - \bar{\Omega}^2)\Gamma^5 \cos^3 \bar{r}$   
 $\times (\rho\Omega\partial_t + \rho^{-1}\partial_{\varphi})$ .

Axial vortical effect correction:

$$J_A^\mu = \sigma_A^\omega \omega^\mu, \quad \sigma_A^\omega = \frac{T^2}{6} + \frac{1}{24\pi^2} \left( \omega^2 + 3\mathbf{a}^2 - 6M^2 + \frac{R}{4} \right) + O(M^4). \quad (8)$$



# Geometric approach: point-splitting

- The t.e.v. of an operator  $\hat{A}$  is

$$\langle \hat{A} \rangle_{\beta_0, \Omega} = Z^{-1} \text{tr}(\hat{\varrho} \hat{A}), \quad \hat{\varrho} = e^{-\beta_0 (\hat{H} - \Omega \hat{M}^z)}, \quad Z = \text{tr}(\hat{\varrho}). \quad (9)$$

- T.e.v.s can be obtained via point-splitting, e.g.

$$\langle \hat{J}_A^\mu \rangle_{\beta_0, \Omega} = \lim_{x' \rightarrow x} \text{tr}[\gamma^\mu \gamma^5 S_{\beta_0, \Omega}^F(x, x') \Lambda(x', x)], \quad (10)$$

where  $\Lambda(x, x')$  is the bispinor of parallel transport.

- The Feynman propagator can be obtained from the Wightman functions,

$$\begin{aligned} S_{\beta_0, \Omega}^F(x, x') &= \Theta_c(\tau - \tau') S_{\beta_0, \Omega}^+(x, x') + \Theta_c(\tau' - \tau) S_{\beta_0, \Omega}^-(x, x'), \\ iS_{\beta_0, \Omega}^+(x, x') &= \langle \hat{\Psi}(x) \hat{\bar{\Psi}}(x') \rangle_{\beta_0, \Omega}, \\ iS_{\beta_0, \Omega}^-(x, x') &= -\langle \hat{\bar{\Psi}}(x') \hat{\Psi}(x) \rangle_{\beta_0, \Omega}, \end{aligned} \quad (11)$$

where  $\Theta_c(\tau - \tau')$  satisfies ( $\varepsilon > 0$ ):

Mallik, Sarkar, *Hadrons at finite temperature* (CUP, 2016)

$$\Theta_c(t - t' - i\varepsilon) = 1, \quad \Theta_c(t - t' + i\varepsilon) = 0. \quad (12)$$

- ▶ Taking into account  $\hat{\varrho} = e^{-\beta_0(\hat{H}-\Omega\hat{M}^z)}$ ,  $\hat{\Psi}$  obeys

$$\hat{\varrho}\hat{\Psi}(t, \varphi)\hat{\varrho}^{-1} = e^{-\beta_0\Omega S^z}\hat{\Psi}(t + i\beta_0, \varphi + i\beta_0\Omega). \quad (13)$$

- ▶ The KMS relation reads

VEA, EW, Symmetry **2021**, *13*, 2019

$$\begin{aligned} S_{\beta_0, \Omega}^-(\tau, \varphi; x') &= iZ^{-1}\text{tr}[\hat{\varrho}\hat{\bar{\Psi}}(\tau', \varphi')\hat{\Psi}(\tau, \varphi)] \\ &= iZ^{-1}\text{tr}[\hat{\varrho}e^{\beta_0\Omega S^z}\hat{\Psi}(\tau - i\beta_0, \varphi - i\beta_0\Omega)\hat{\bar{\Psi}}(\tau', \varphi')] \\ &= -e^{\beta_0\Omega S^z}S_{\beta_0, \Omega}^+(\tau - i\beta_0, \varphi - i\beta_0\Omega; x'). \end{aligned} \quad (14)$$

- ▶ The KMS relation above allows  $S_{\beta_0, \Omega}^F$  to be written in terms of  $S_{\infty, \Omega}^F$ :

$$S_{\beta_0, \Omega}^F(x, x') = \sum_j (-1)^j e^{-j\beta_0\Omega S^z} S_{\infty, \Omega}^F(t + ij\beta_0, \varphi + ij\beta_0\Omega; x'). \quad (15)$$

- ▶ In bounded systems,  $S_{\infty, \Omega}^F = S_{\infty, 0}^F$  if no SLS forms inside the boundary.

VEA, EW, PRD **93** (2016) 104014

- ▶ A similar argument holds on adS, if  $\Omega < \ell^{-1}$  (subcritical rotation).

# Thermal expectation values

## ► Introducing the notation

$$\zeta_j = -\frac{1}{\sin^2 \frac{\bar{s}_j}{2}} = \frac{\cos^2 \bar{r}}{\sinh^2 \frac{j\beta_0}{2\ell} - \bar{\rho}^2 \sinh^2 \frac{\Omega j \beta_0}{2}}, \quad (16)$$

we find  $J_A^{\hat{\alpha}} = \sigma_A^\omega \omega^{\hat{\alpha}}$ , where

$$[\Gamma_k = \Gamma(2+k)\sqrt{\pi}/4^k \Gamma(\tfrac{1}{2}+k)]$$

$$\begin{aligned} \sigma_A^\omega &= \frac{\Gamma_k}{2\pi^2 \ell^3 \Omega \Gamma^2 \cos^2 \bar{r}} \sum_{j=1}^{\infty} (-1)^{j+1} \zeta_j^{2+k} \sinh \frac{j\beta_0}{2\ell} \sinh \frac{\Omega j \beta_0}{2} \\ &\quad \times {}_2F_1(k, 2+k; 1+2k; -\zeta_j) \\ &= \frac{T^2}{6} + \frac{1}{24\pi^2} \left( \omega^2 + 3a^2 - 6M^2 + \frac{R}{4} \right) + O(T^{-1}). \end{aligned} \quad (17)$$

- At high temperature, the Minkowski expression is reproduced, including the **temperature-independent terms**.
- When  $T \rightarrow 0$ ,  $\sigma_A^\omega \rightarrow 0$  and the  $O(T^0)$  are also suppressed.
- The **curvature correction** appears in the  $O(T^0)$  term and is proportional to  $R = -12\ell^{-2}$ .

# Critical rotation: $\bar{\Omega} = 1$

- ▶ At critical rotation,  $\Gamma = 1/\sqrt{1 - \bar{\rho}^2}$  and

$$\zeta_j = \frac{\Gamma^2 \cos^2 \bar{r}}{\sinh^2 \frac{j\beta_0}{2\ell}}. \quad (18)$$

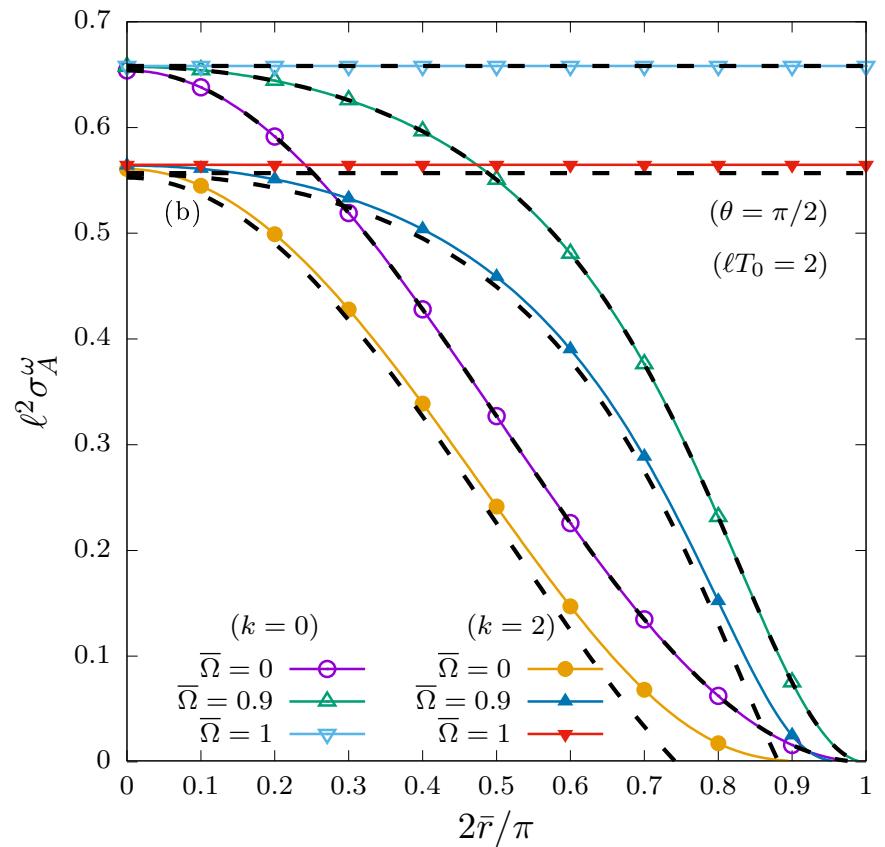
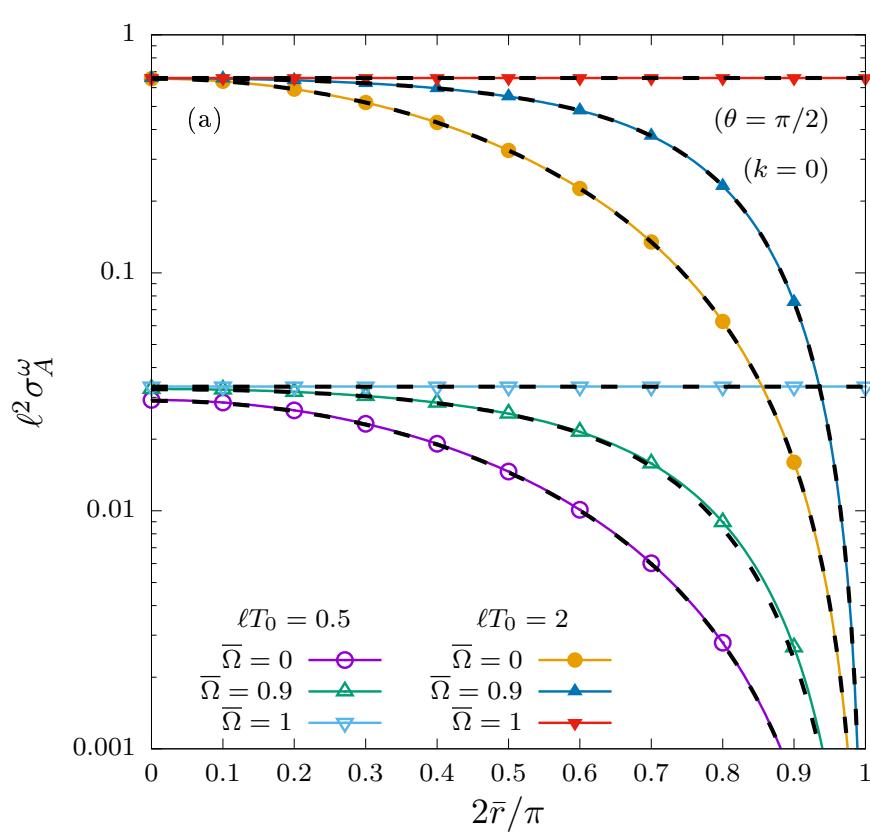
- ▶ In the equatorial plane ( $\theta = \frac{\pi}{2}$ ),

$$\Gamma = \frac{1}{\cos \bar{r}} \quad \Rightarrow \quad T = T_0 \Gamma \cos \bar{r} = T_0, \quad \zeta_j = \operatorname{cosech}^2 \frac{j\beta_0}{2\ell}. \quad (19)$$

- ▶ When  $\bar{\Omega} = 1$ , rotation balances “clumping” in the equatorial plane.
- ▶ In particular,  $\sigma_A^\omega$  is constant in the EP:

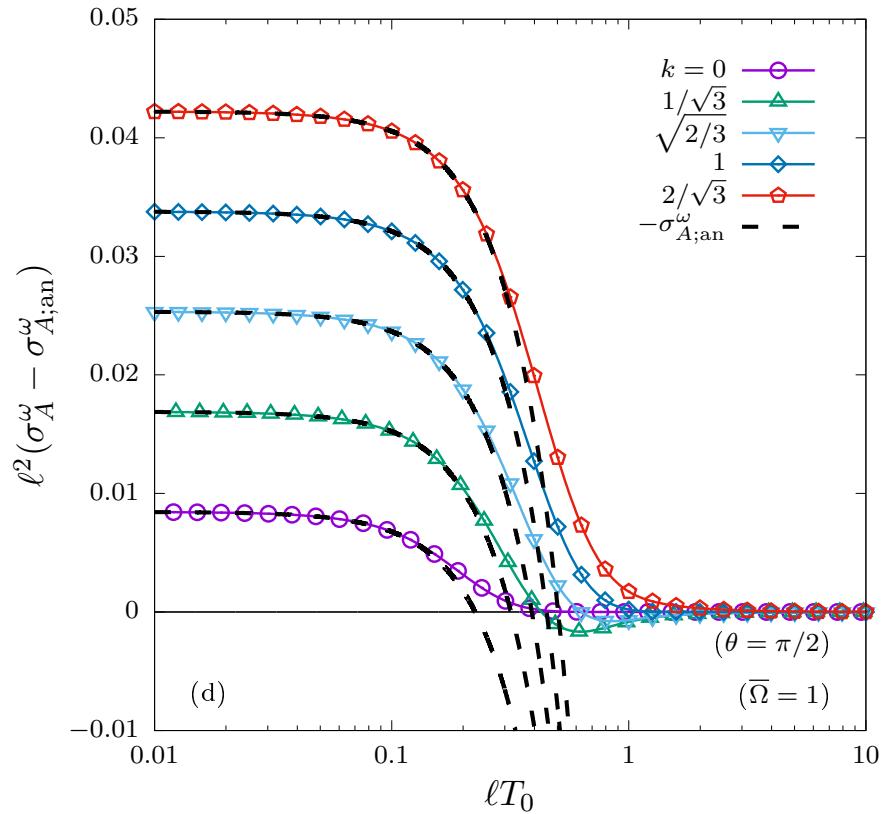
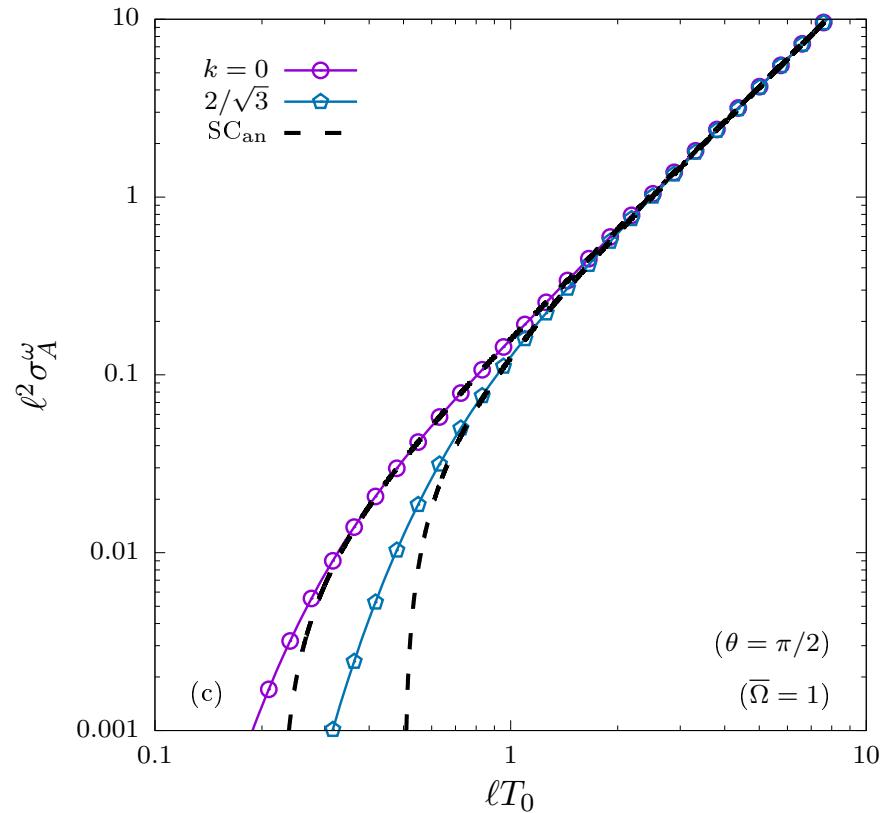
$$\begin{aligned} \lim_{\bar{\Omega} \rightarrow 1} \sigma_A^\omega \left( \theta = \frac{\pi}{2} \right) &= \frac{\Gamma_k}{2\pi^2 \ell^2} \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{(\sinh \frac{j\beta_0}{2\ell})^{2+2k}} \\ &\times {}_2F_1 \left( k, 2+k; 1+2k; -\operatorname{cosech}^2 \frac{j\beta_0}{2\ell} \right). \end{aligned} \quad (20)$$

# Equatorial plane (EP) profiles



- ▶  $T = T_0 \Gamma \cos \bar{r}$  decreases with  $\bar{r}$  slower at larger  $\bar{\Omega}$ .
- ▶ Deviations from asymptotic formula appear at low  $T_0$ , high  $k = \ell M$  and/or high  $\bar{r}$ .
- ▶  $\sigma_A^\omega$  in EP independent of  $\bar{r}$  when  $\bar{\Omega} = 1$  (critical rotation).

# Critical rotation $\bar{\Omega} = 1$ : temperature dependence



- ▶ Asymptotic behaviour reached later for higher  $k$ .
- ▶ All terms including  $O(T^0)$  confirmed by comparison with numerical results.

- $J_A^\mu$  obeys the conservation equation

$$\nabla_\mu J_A^\mu = \frac{1}{\sqrt{-g}} \frac{\partial(J_A^z \sqrt{-g})}{\partial z} = -2MPC,$$

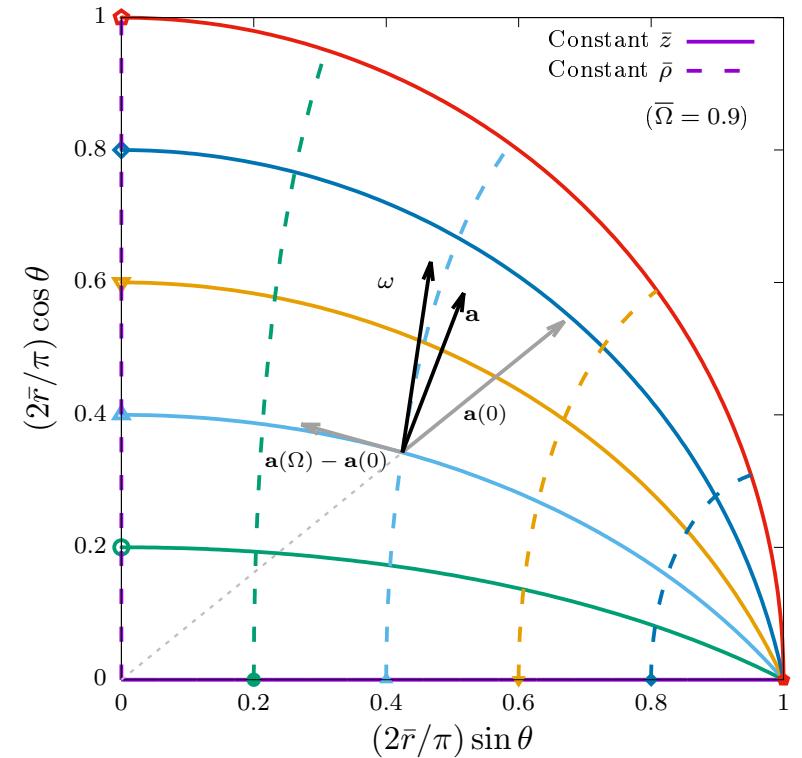
where  $PC = -i\bar{\psi}\gamma^5\psi$  and

$$\bar{z} = \tan \bar{r} \cos \theta, \quad \bar{\rho} = \sin \bar{r} \sin \theta.$$

- AVE  $\Rightarrow$  non-vanishing axial flux  $F_A$ :

$$F_A = \ell^{-1} \int_0^1 d\bar{\rho} \int_0^{2\pi} d\varphi \sqrt{-g} J_A^z(\bar{z})$$

$$\begin{aligned} &= \frac{\Gamma_k}{2\pi\ell(1+k)} \sum_{j=1}^{\infty} \frac{(-1)^{j+1} \sinh \frac{\Omega j \beta_0}{2} \left( \sinh \frac{j \beta_0}{2\ell} \right)^{-1-2k}}{\left( \sinh^2 \frac{j \beta_0}{2\ell} - \sinh^2 \frac{\Omega j \beta_0}{2} \right) (1 + \bar{z}^2)^k} \\ &\quad \times {}_2F_1 \left( k, 1+k; 1+2k; -\frac{(1+\bar{z}^2)^{-1}}{\sinh^2 \frac{j \beta_0}{2\ell}} \right). \quad (21) \end{aligned}$$



# Transparency to chiral particles

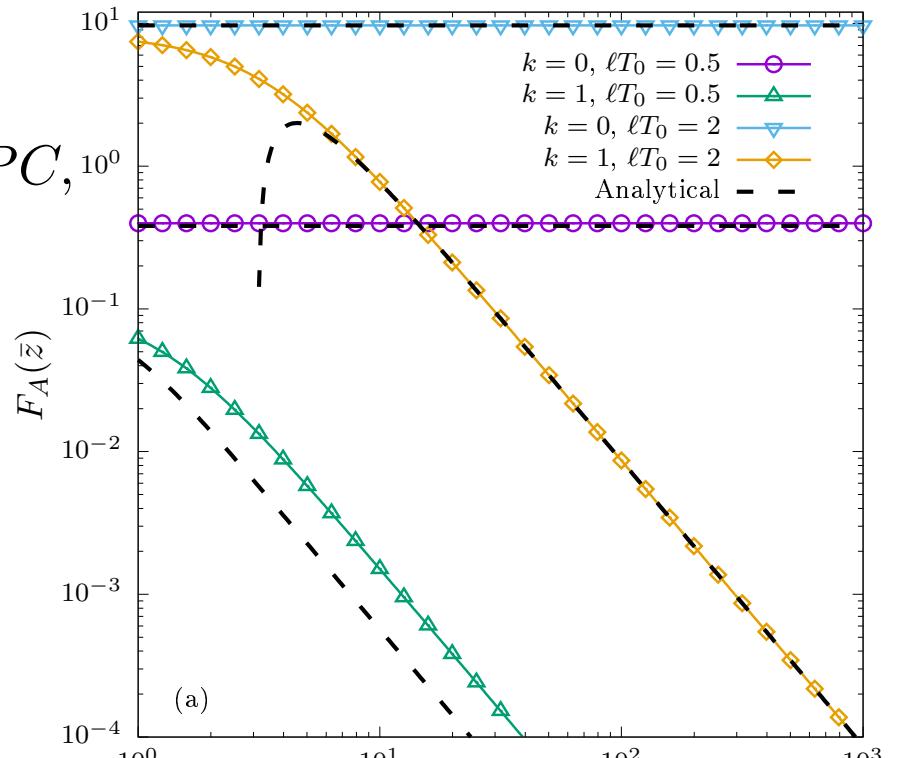
- ▶ The conservation equation entails

$$F_A(\bar{z}) - F_A(0) = -\frac{2M}{\ell} \int_V d^3x \sqrt{-g} PC,$$

showing that  $F_A(\pm\infty) = F_A(0)$  when  $k = 0 \Rightarrow$  adS is transparent to chiral particles.

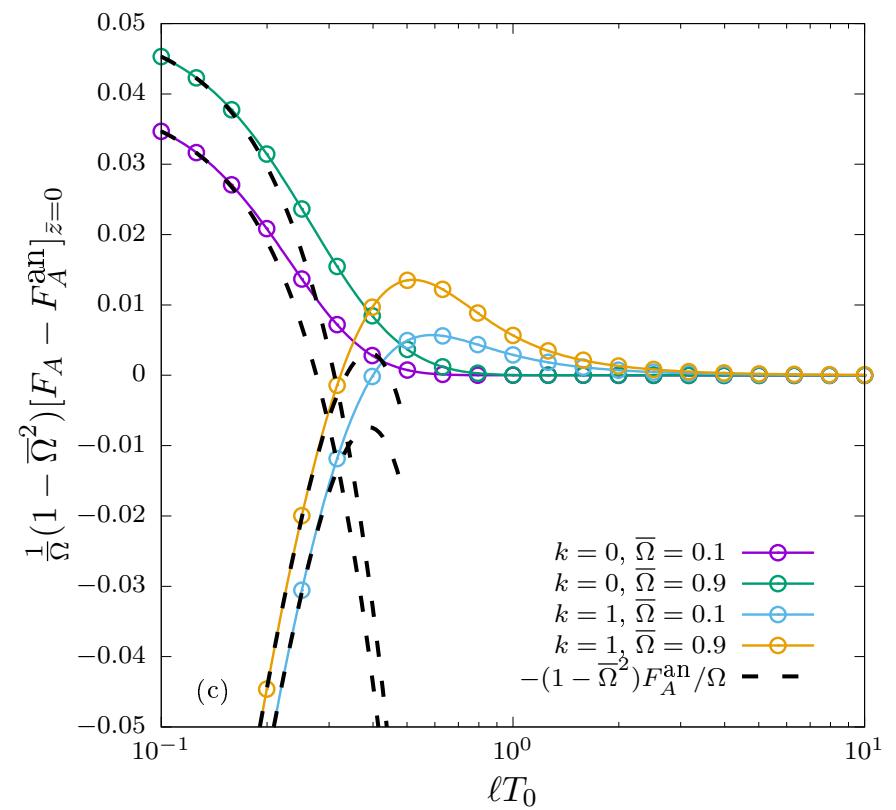
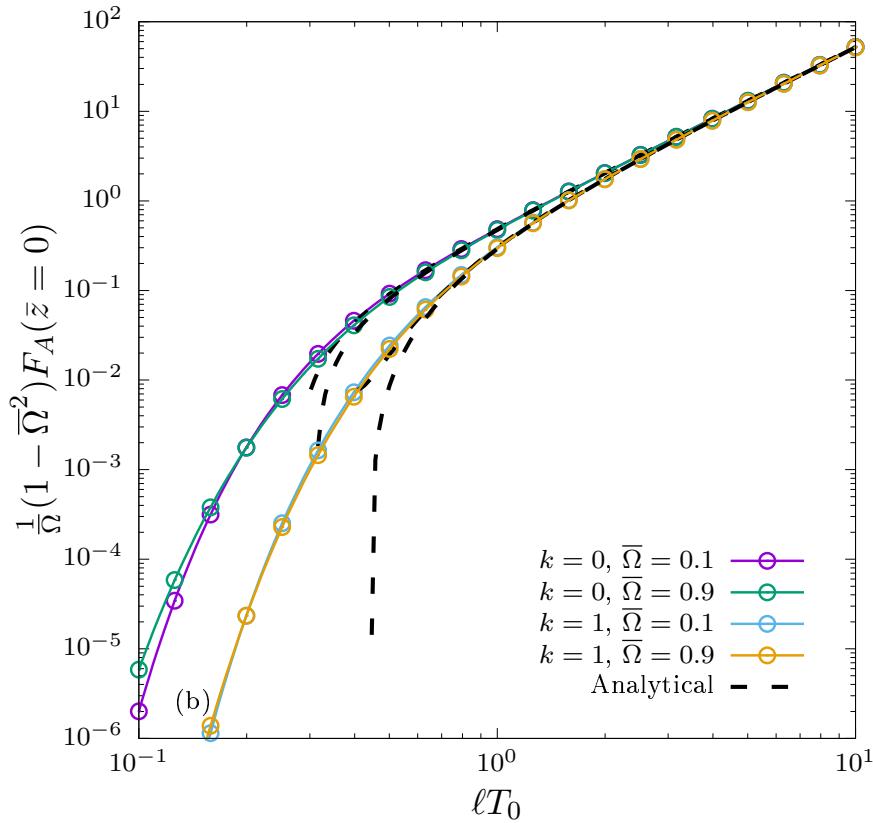
- ▶ At finite mass,  $F_A(\bar{z})$  can be expanded for large  $\bar{z}$  as

$$\begin{aligned} F_A(\bar{z}) &= \frac{\Omega}{1 - \bar{\Omega}^2} \frac{\Gamma(1+k)}{2\sqrt{\pi}\Gamma(\frac{1}{2}+k)} \\ &\times \left( \frac{\ell T_0}{\sqrt{1+\bar{z}^2}} \right)^{2k} \left\{ \left[ \zeta(2+2k) \left( 1 - \frac{1}{2^{1+2k}} \right) (2\ell T_0)^2 \right. \right. \\ &\quad \left. \left. - \frac{\zeta(2k)}{6} \left( 1 - \frac{1}{2^{2k-1}} \right) (3+2k+\bar{\Omega}^2) + O(T_0^{-2}) \right] + O(z^{-2}) \right\} \quad (22) \end{aligned}$$



- ▶ Since  $F_A(\bar{z}) \simeq z^{-2k}$ , a finite mass always ensures that  $F(\pm\infty) = 0 \Rightarrow$  adS is opaque to non-chiral particles.

# High temperature asymptotics



$$F_A(\bar{z}) = \frac{\Omega}{1 - \bar{\Omega}^2} \left\{ \frac{\pi \ell^2 T_0^2}{6} - \frac{3 + \bar{\Omega}^2}{24\pi} - \frac{k^2(1 + \bar{z}^2)}{2\pi} \ln \frac{\pi \ell T_0}{\sqrt{1 + \bar{z}^2}} \right. \\ \left. - \frac{k(1 + \bar{z}^2)}{4\pi} [1 + k - 2kC - 2k\psi(1 + k)] + O(T_0^{-1}) \right\}. \quad (23)$$

All terms are confirmed by comparison with numerical results.

- ▶ New KMS relation allows  $S_{\beta_0, \Omega}^F$  to be written in terms of  $S_{\text{vac}}^F$ .
- ▶ Exact expression for  $S_{\text{vac}}^F$  allows vortical effects to be studied on adS.
- ▶ Axial vortical effect confirmed and curvature correction revealed.
- ▶ Axial flux  $F_A$  of massless particles originates from southern hemisphere and escapes through northern hemisphere.
- ▶ For massive particles,  $F_A \sim \bar{z}^{-2k}$  and no flux penetrates the boundary of adS.
- ▶ Possible extensions: supercritical rotation, finite chemical potential, thermodynamics...
- ▶ This work was supported through a grant of the Ministry of Research, Innovation and Digitization, CNCS - UEFISCDI, project number PN-III-P1-1.1-TE-2021-1707, within PNCDI III.

# Appendix

- ▶ FORQ: Facets of rotating quark-gluon plasma (July 2023 - June 2026)
- ▶ Project led by Maxim N. Chernodub in Timișoara
- ▶ 2 postdocs in July 2023 (adverts will be out soon...)
- ▶ One PhD student in October 2023
- ▶ 3 more postdocs in 2024
- ▶ 2 more PhD students in 2024
- ▶ 6 MSc students in 2024, 2025...