Problem Set 9

1. There are three types of coins which are indistinguishable apart from their probability of landing heads when tossed.

- Type A coins are fair, with probability 0.5 of heads
- Type B coins have probability 0.6 of heads
- Type C coins have probability 0.9 of heads

You have a drawer containing 4 coins: 2 of type A, 1 of type B, and 1 of type C. You reach into the drawer and pick a coin at random.

Suppose you toss the coin 5 times and it lands tails all 5 times. What are the posterior probabilities?

R: $P(A|D) = 0.859106$, $P(B|D) = 0.140756$, $P(C|D) = 0.00013745$

2. Stubborn priors. Suppose you run an experiment to test 3 different hypotheses: A, B, and C. You are so convinced that B is the correct hypothesis that your prior is

$$P(A) = 0 \quad P(B) = 1 \quad P(C) = 0$$

You collect data D with likelihoods

$$P(D|A) = 0.8 \qquad P(D|B) = 0.01 \qquad P(D|C) = 0.7$$

(a) Compute the posterior probabilities: $P(A|D)$, $P(B|D)$, $P(C|D)$

(b) Will any amount of data change your belief that B is the correct hypothesis?

3. There are three types of coins which have different probabilities of landing heads when tossed.

Type A coins are fair, with probability 0.5 of heads

Type B coins are bent and have probability 0.6 of heads

Type C coins are bent and have probability 0.9 of heads

You have a drawer containing 4 coins: 2 of type A, 1 of type B, and 1 of type C. You pick a coin at random. Assuming the first two tosses were heads, what is the posterior predictive probability of heads on the third toss? R: 0.7155