## Problem Set 8

1. Each computer chip made in a certain plant will, independently, be defective with probabitity 0.25. If a sample of 1000 chips is tested, what is the approximate probability that fewer than 200 chips will be defective? R: 0.000113
2. Use R or tables to compute $P\{X \le 10\}$ when $X$ is a binomial random variable with parameters $n = 100$, $p = 0.1$. Now compare this with its (a) Poisson and (b) normal approximation. In using the normal approximation, write the desired probability as $P\{X \le 10.5\}$ so as to utilize the continuity correction.

   R: 0.5831555, a) 0.5830398 b) 0.5661576
3. The temperature at which a thermostat goes off is normally distributed with variance $\sigma^2$ . If the thermostat is to be tested five times, find
   a) $P\left(S^2/\sigma^2 \le 1.8\right)$
   b) $P\left(0.85 \le S^2/\sigma^2 \le 1.15\right)$ R: a)0.87431 b)0.16239
4. Consider two independent samples — the first of size 10 from a normal population having variance 4 and the second of size 5 from a normal population having variance 2. Compute the probability that the sample variance from the second sample exceeds the one from the first. (Hint: Relate it to the F-distribution.) R: $P\left(F_{9,4} < \frac{1}{2}\right) = 0.178196$
5. Twelve percent of the population is left-handed. Find the probability that there are between 10 and 14 left-handers in a random sample of 100 members of this population. That is, find $P\{10 \le X \le 14\}$, where $X$ is the number of left-handers in the sample.

   R: 0.55828
6. The sample mean and sample standard deviation of all San Francisco student scores on the most recent Scholastic Aptitude Test examination in mathematics were 517 and 120. Approximate the probability that a random sample of 144 students would have an average score exceeding
   (a) 507; (b) 517; (c) 537; (d) 550.

   R:a)0.84134 b)1/2 c)0.02275 d)0.0004834
7. Suppose that the life time of Badger brand light bulbs is modeled by an exponential distribution with (unknown) parameter λ. We test 5bulbs and find they have lifetimes of 2,3,1,3, and 4 years, respectively. What is the Maximum Likelihood Estimator for λ?

   R: 5/13
8. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation $\sigma = 0.1$ mg. Suppose that the results of five successive weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141.
   **(a)** Determine a 95 percent confidence interval estimate of the true weight.
   **(b)** Determine a 99 percent confidence interval estimate of the true weight.

   R: a) $(3.0625,\ 3.2378)$ b) $(3.035,\ 3.265)$

9. The standard deviation of test scores on a certain achievement test is 11.3. If a random sample of 81 students had a sample mean score of 74.6, find a 90 percent confidence interval estimate for the average score of all students. R: $(72.5347, 76.6652)$

10. The following are scores on IQ tests of a random sample of 18 students at a large eastern university.

130, 122, 119, 142, 136, 127, 120, 152, 141,
132, 127, 118, 150, 141, 133, 137, 129, 142

(a) Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university.
(b) Construct a 95 percent lower confidence interval estimate.
(c) Construct a 95 percent upper confidence interval estimate.

R:a) $(128.1435,\ 138.3009)$ b) $(-\infty,\ 137.4098)$ c) $(129.0347,\ \infty)$

11. A standardized test is given annually to all sixth-grade students in the state of Washington. To determine the average score of students in her district, a school supervisor selects a random sample of 100 students. If the sample mean of these students' scores is 320 and the sample standard deviation is 16, give a 95 percent confidence interval estimate of the average score of students in that supervisor's district.

R: $(316.8253,\ 323.1747)$

12. The capacities (in ampere-hours) of 10 batteries were recorded as follows:

140, 136, 150, 144, 148, 152, 138, 141, 143, 151

(a) Estimate the population variance $\sigma^2$.

(b) Compute a 99 percent two-sided confidence interval for $\sigma^2$.

(c) Compute a value *v* that enables us to state, with 90 percent confidence, that

$\sigma^2$ is less than *v*. R: a)32.233 b)(12.297, 167.211) c)69.599

13. The following are independent samples from two normal populations, both of which have the same standard deviation σ.

16, 17, 19, 20, 18 and 3, 4, 8

Use them to estimate σ. R: 4

14. The following are the daily numbers of company website visits resulting from advertisements on two different types of media.

| | | | | | |
|---|---|---|---|---|---|
| Type I | 481 572 | 506 561 | 527 501 | 661 487 | 501 524 |
| Type II | 526 537 | 511 582 | 556 605 | 542 558 | 491 578 |

Find a 99 percent confidence interval for the mean difference in daily visits assuming normality with unknown but equal variances.

R: (-74.956, 41.956)