Problem Set 7

1. Suppose $X_1, X_2, \ldots, X_{100}$ are i.i.d. with mean 1/5 and variance 1/9. Use the central limit theorem to estimate $P\left(\sum X_i < 30\right)$.

   R: 0.9986

2. The average IQ in a population is100 with standard deviation 15. What is the probability that a randomly selected group of 100 people has an average IQ above 115? R: 7.619×10^{-24} practice zero!

3. Suppose that X_1, X_2, X_3 are independent with the common probability function

   $$P(X_i = 0) = 0.2\ ,\ P(X_i = 1) = 0.3\ ,\ P(X_i = 3) = 0.5,\ \ i = 1,2,3$$

   Let $\bar{X}_2 = \frac{X_1 + X_2}{2}$ and $\bar{X}_3 = \frac{X_1 + X_2 + X_3}{3}$. Determine: a) $E\left[\bar{X}_2\right]$, $V\left[\bar{X}_2\right]$ b) $E\left[\bar{X}_3\right]$, $V\left[\bar{X}_3\right]$.

   R:a) 1.8, 0.78 b)1.8, 0.52

4. If 10 fair dice are rolled , approximate the probability that the sum of the values obtained (which ranges from 10 to 60) is between 30 and 40 inclusive.

   R: 0.6922

5. If *X* is a chi-square random variable with 6 degrees of freedom, find
   a) $P(X \le 6)$
   b) $P(3 \le X \le 9)$

   R: use R or tables a)0.5768 b)0.6352

6. If *X* and *Y* are independent chi-square random variables with 3 and 6 degrees od freedom, respectively, determine the probability that $X + Y$ will exceed 10.

   R: use R or tables 0.3504

7. If *T* is a *t*-distribution with 8 degrees of freedom , find a) $P(T \ge 1)$, b) $P(T \le 2)$ c) $P(-1 < T < 1)$

   R: use R or tables a)0.1732 b)0.9597 c)0.6534

8. A highway department has enough salt to handle a total of 80 inches of snowfall. Suppose the daily amount of snow has a mean of 1.5 inches and a standard deviation of 0.3 inches. Approximate the probability that the salt on hand will suffice for the next 50 days. What assumption did you make?

   R: 0.9907