Problem Set 6

1. Flip a fair coin 100 times. Estimate the probability of more than 55 heads.
   Answer:

Let X_j be the result of the jth flip, so $X_j = 1$ for heads and $X_j = 0$ for tails (Bernoulli variable). The total number of heads is

$$S = X_1 + X_2 + \ldots + X_{100}$$

We know $E\left[X_j\right] = 0.5$ and $V\left[X_j\right] = 1/4$. Since $n = 100$, we have

$$E[S] = 50;\ V[S] = 25 \text{ and } \sigma_S = 5$$

The standardization on S gives the question asks for P(S>55):

$$P(S > 55) = P\left(\frac{S-50}{5} > \frac{55-50}{5}\right) = P(Z > 1) \approx 0.16$$

2. Estimate the probability of more than 220 heads in 400 fips.

   R: 0.025

3. Estimate the probability of between 40 and 60 heads in 100 fips.

   R: 0.9544997

4. Suppose *X* and *Y* both take values in [0,1] with density $f(x, y) = 4xy$. Show $f(x, y)$ is a valid joint pdf, visualize (in a graph xOy) the event A = \` $X < 0.5$ and $Y > 0.5$' and find its probability.

   R:3/16

5. Suppose (X;Y) takes values on the square $[0,1]\times[1,2]$ with joint pdf $f(x, y) = \frac{8}{3}x^3 y$. Find the marginal pdf's $f_X(x)$ and $f_Y(y)$.

   R: $4x^3$, $\frac{2}{3}y$

6. Suppose (X;Y) takes values on the unitsquare $[0,1]\times[0,1]$ with joint pdf $f(x, y) = \frac{3}{2}\left(x^2 + y^2\right)$. Find the marginal pdf $f_X(x)$ and use it to find $P(X < 0.5)$.

   R: $3x^2 + \frac{1}{2}$, 5/16

7. $f(x,y)=cxy$ is a joint pdf on [0,1]×[0,1]. What is the value of c?

R:4

8. Consider the following joint probability table.

X\Y	1	2	3	4	$f_X(x)$
1	1/24	1/24	1/24	1/24	
2	1/12	1/12	1/12	1/12	
3	1/8	1/8	1/8	1/8	
$f_Y(y)$					

(a) What is the probability that X≤2 and Y≤2?
(b) What is the marginal probability X=1?
(c) Are X and Y independent?

R: a)1/4 b)1/6 c)da

9. Flip a fair coin 3 times. Let X be the number of heads in the first 2 flips and let Y be the number of heads on the last 2 flips (so here is overlap on the middle flip). A) Constructs the joint probability table. b) Compute $C\text{ov}[X,Y]$.

R: a)

X\Y	0	1	2	$p(x_i)$
0	1/8	1/8	0	
1	1/8	2/8	1/8	
2	0	1/8	1/8	
$p(y_j)$				

b)1/4