

Problem Set 5

1. Suppose X has mean 2 and variance 3. Compute the following.

(a) $V[3X]$ (b) $V[3X + 8]$ (c) $E[X^2]$

R: 27,27,7

2. Let X be a random variable with range $[0,1]$ and PDF $f(x) = Cx^2$. What is the value of C ? Find $P(X \leq 1/2)$. Find the cumulative distribution function for the PDF. Take the graphs of $f(x)$ and $F(x)$.

R: 3, 1/8

3. Let $X \sim U(0, 2)$ uniform distribution, so $f_X(x) = \frac{1}{2}$. (a) Find the cumulative probability function $F_X(x)$. (b) What is the range, cdf and pdf of $Y = X^2$?

R: a) $x/2$ b) $[0,4], F_Y(y) = P(Y \leq y) = P(X^2 \leq y) =$

$$= P(X \leq \sqrt{y}) = F_X(\sqrt{y}) = \sqrt{y}/2, f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{1}{2} \frac{1}{2\sqrt{y}} = \frac{1}{4\sqrt{y}}$$

4. Let X an exponential distribution with parameter λ . Find $E[X]$.

R: $1/\lambda$

5. Let $Z \sim N(0,1)$. Find $E[Z]$.

R: 0

6. Let $X \sim U(0,1)$ uniform distribution on $[0,1]$. Find $V[X]$.

R: $1/12$

7. Let $Z \sim N(0,1)$. Show $V[Z] = 1$.

8. Find the mean and variance of $X \sim U(0,4)$.

($U(0,4)$ is the uniform distribution.)

(i) Mean (ii) Variance

R: 2, $4/3$

9. An IQ test produces scores that are normally distributed with mean value 100 and standard deviation 14.2. The top 1 percent of all scores are in what range?

R: >133

10. The number of years a radio functions is exponentially distributed with parameter $\lambda = 1/8$. If Jones buys a used radio, what is the probability that it will be working after an additional 10 years?

11. Earthquakes occur in a given region in accordance with a Poisson process with rate 5 per year.

(a) What is the probability there will be at least two earthquakes in the first half of 2015?

(b) What is the probability that there will be no earthquakes during the first 9 months of 2016?

R: a) 0.7127 b) 0.0235