Problem Set 4

Exercise 0. The applet at http://mathlets.org/mathlets/probability-distributions/ gives a dynamic view of some discrete distributions.

Exercise 1. If a single six-sided die is rolled seven times, what is the probability that a six is thrown exactly four times?

Exercise 2. A coin is tossed five times. Calculate the probability of obtaining more heads than tails.

Exercise 3. An agent sells life insurance policies to five equally aged, healthy people. According to recent data, the probability of a person living in these conditions for 30 years or more is 2/3. Calculate the probability that after 30 years: a) All five people are still living b) At least three people are still living c) Exact two people are still living.

<div align="center">R: 0.132, 0.791, 0.164</div>

Exercise 4. If from nine to ten in the morning one telephone line in every five is engaged in a conversation, what is the probability that when 10 telephone numbers are chosen at random, only two are in use?   R: 0.302

Exercise 5. The color of one's eyes is determined by a single pair of genes, with the gene for brown eyes being dominant over the one for blue eyes. This means that an individual having two blue-eyed genes will have blue eyes, while one having either two brown-eyed genes or one brown-eyed and one blue-eyed gene will have brown eyes. When two people mate, the resulting offspring receives one randomly chosen gene from each of its parents' gene pair. If the eldest child of a pair of brown-eyed parents has blue eyes, what is the probability that exactly two of the four other children (none of whom is a twin) of this couple also have blue eyes? R: 27/128

Exercise 6. Let X~ Bernoulli(1/3). (a) What is $E[X]$?

(b) What is $E[X^2]$?   R:a)1/3 b)1/3

Exercise 7. Let $Y \sim Bin(12, 1/3)$. What is $E[Y]$?   R:4

Exercise 8. Using X and Y from the previous two problems.

(a)What is $E[4X + 7]$?   (b)What is $E[X + Y]$?

(Assume X and Y are random variables on the same sample space.)   R:a)25/3 b)13/3

Exercise 9. In the manufacture of glassware, bubbles can occur in the glass which reduces the status of the glassware to that of a 'second'. If, on average, one in every 1000 items produced has a bubble, calculate the probability that exactly six items in a batch of three thousand are seconds.

R: 0.05

Exercise 10. The mean number of bacteria per millilitre of a liquid is known to be 6. Find the probability that in 1 ml of the liquid, there will be: (a) 0, (b) 1, (c) 6 bacteria   R: 0.00248, 0.0149, 0.1606

Exercise 11. Astronomers treat the number of stars in a given volume of space as a Poisson random variable. The density in the Milky Way Galaxy in the vicinity of our solar system is one star per 16 cubic light-years.

(a) What is the probability of two or more stars in 16 cubic light-years?

(b) How many cubic light-years of space must be studied so that the probability of one or more stars exceeds 0.95?

R: a)0.264 b)48