Problem Set 3

Exercise 1. X has a Bernoulli distribution—that is, X takes on values 0 and 1 with probability $1-p$ and p, respectively. Find the mean and variance for the Bernoulli distribution. R: $E[X]=p$, $V[X]=p(1-p)$

Exercise 2. Let X have the probability density function

$$f(x)=\frac{1+\alpha x}{2} \quad , \quad -1\le x\le 1 \ , \quad -1\le \alpha \le 1$$

Find $E[X]$ and $V[X]$. R: $\alpha/3$, $\frac{1}{3}-\frac{1}{9}\alpha^2$

Exercise 3. Let X be a random variable having the kth moment:

$$\mu_k = E\left[X^k\right]=\frac{1}{k+1} \ , \quad k=1,2,3$$

Find the third central moment $\nu_3 = E\left[(X-\mu_1)^3\right]$ R:0

Exercise 4. A Poisson-distributed integer variable with mean λ has the probability function: $f(x)=\Pr(X=x)=\dfrac{\lambda^x e^{-\lambda}}{x!}$.

a) Find the moment generating function (MGF) for the Poisson distribution.
b) Using MGF, find the mean and variance for the Poisson distribution.

R:a) $M_X(t)=e^{\lambda(e^t-1)}$ b) λ , λ

Exercise 5. The MGF for the Gaussian distribution is given by

$$M_X(t)=e^{\mu t+\frac{1}{2}\sigma^2 t^2}$$

Let X a Gaussian distribution with mean μ_X and variance σ_X^2 and Y a Gaussian distribution with mean μ_Y and variance σ_Y^2 . If X and Y are independent find the MGF of $Z=X+Y$. R: $M_Z(t)=e^{(\mu_X+\mu_Y)t}e^{(\sigma_X^2+\sigma_Y^2)\frac{t^2}{2}}$

Exercise 6. Let X be a discrete random variable that takes on values 0, 1, 2 with probabilities ½, 3/8, 1/8, respectively. Find the moment-generating function of X, $M_X(t)$, and verify that $E[X]=M'(0)$ and that $E\left[X^2\right]=M''(0)$.

R: $M_X(t)=\frac{1}{2}+\frac{3}{8}e^t+\frac{1}{8}e^{2t}$