

Problem Set 2

Exercise 1. A bag contains seven red balls and three white balls. Three balls are drawn at random and not replaced. Find the probability function for the number of red balls drawn.

Exercise 2. A committee of size 5 is to be selected from a group of 6 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?
R: 240/1001

Exercise 3. A group of 5 boys and 10 girls is lined up in random order—that is, each of the $15!$ permutations is assumed to be equally likely.

- (a) What is the probability that the person in the 4th position is a boy?
- (b) What about the person in the 12th position?
- (c) What is the probability that a particular boy is in the 3rd position?

R: $1/3, 1/3, 1/15$

Exercise 4. Suppose the random variable X has probability density function

$$f(x) = \begin{cases} cx^3, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

a) Determinati valoarea lui c b) calculati $P(0.4 < X \leq 0.8)$.

R: a) 4 b) 0.384

Exercise 5. Find $E[X]$ where X is the outcome when we roll a fair die.

R: $7/2$

Exercise 6. Suppose that you are expecting a message at some time past 5 P.M. From experience you know that X , the number of hours after 5 P.M. until the message arrives, is a random variable with the following probability density function:

$$f(x) = \begin{cases} \frac{1}{1.5} & \text{if } 0 < x < 1.5 \\ 0 & \text{otherwise} \end{cases}$$

What is the expected amount of time past 5 P.M. until the message arrives?

R: 0.75 hours

Exercise 7. Suppose X has the following probability function

$$p(0) = 0.2, \quad p(1) = 0.5, \quad p(2) = 0.3$$

Calculate $E[X^2]$.

R:1.7

Exercise 8. Suppose X has the following probability density function :

$$f(x) = \begin{cases} (-\ln p) p^x, & x \in [0, \infty) \\ 0, & \text{otherwise} \end{cases}$$

where $p \in (0,1)$. Find:

a) The median M of the distribution.

b) The mode of the distribution.

R: a) $-\ln 2 / \ln p$ b) 0

Exercise 9. The sample mean of the annual salaries of a group of 100 accountants who work at a large accounting firm is \$130,000 with a sample standard deviation of \$20,000. If a member of this group is randomly chosen, what can we say about the probability that his or her salary is between \$90,000 and \$170,000?