## Problem Set 11

1. Use least squares to fit a line to the following data: $(0,1),(2,1),(3,4)$.

$$
\mathrm{R}: y=\frac{4}{7}+\frac{6}{7} x
$$

2. Consider the sample space of children under 13 in the United States. The random variable $x$ represents the age of a child. The random variable $Y$ represents the number of lollipops possessed by the child. By catching and releasing three children, we record the sample data:

$$
(1,2),(2,3),(3,3)
$$

of the form ( $\mathrm{x}, \mathrm{y}$ ).
(a) Calculate the sample means for the x data and the y .
(b) Using least squares determine the best-fit line in the xy-plane: $\alpha+\beta x$
a)2, 2.666
b) $1.666+0.5 x$
3. Show that $S S_{R}=\frac{S_{x X} S_{Y Y}-\left(S_{x Y}\right)^{2}}{S_{x x}}$, where

$$
\begin{gathered}
S_{x Y}=\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(Y_{i}-\bar{Y}\right)=\sum_{i=1}^{n} x_{i} Y_{i}-n \bar{x} \bar{Y} \\
S_{x x}=\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}=\sum_{i=1}^{n} x_{i}^{2}-n \bar{x}^{2} \\
S_{Y Y}=\sum_{i=1}^{n}\left(Y_{i}-\bar{Y}\right)^{2}=\sum_{i=1}^{n} Y_{i}^{2}-n \bar{Y}^{2}
\end{gathered}
$$

4. The following data relate the number of units of a good that were ordered as a function of the price of the good at six different locations.

| Number ordered $\left(Y_{i}\right)$ | 88 | 112 | 123 | 136 | 158 | 172 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| Price $\left(x_{i}\right)$ | 50 | 40 | 35 | 30 | 20 | 15 |

a) Fit a linear curve to the data
b) How many units do you think would be ordered if the price were 25 ?

$$
\text { R:a) } y=206.74-2.376 x \text { b) } 147.34
$$

5. The following data set presents the heights of 12 male law school classmates whose law school examination scores were roughly equal. It also gives their first year salaries. Each of them went into corporate law. The height is in inches and the salary in units of $\$ 1,000$.

| Height | Salary |
| :--- | :--- |
| 64 | 91 |
| 65 | 94 |
| 66 | 88 |
| 67 | 103 |
| 69 | 77 |
| 70 | 96 |
| 72 | 105 |
| 72 | 88 |
| 74 | 122 |
| 74 | 102 |
| 75 | 90 |
| 76 | 114 |

(a) Do the above data establish the hypothesis that a lawyer's salary is related to his height? Use the 5 percent level of significance.
b)What was the null hypothesis in part (a)?

$$
\mathrm{R}: p-\mathrm{val} 0.1109807 \text { b) } H_{0}: \beta=0
$$

