

Problem Set 10

1. A population distribution is known to have standard deviation 20. Determine the p -value of a test of the hypothesis that the population mean is equal to 50, if the average of a sample of 64 observations is
(a) 52.5; (b) 55.0; (c) 57.5.

R: a)0.317 b)0.0455 c)0.00269

2. In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of 0.02. Suppose 10 independent measurements yielded the following pH values:

8.18 8.17 8.16 8.15 8.17 8.21 8.22 8.16 8.19 8.18

- (a) What conclusion can be drawn at the $\alpha = 0.10$ level of significance?
(b) What about at the $\alpha = 0.05$ level of significance?

R: p-value 0.0009 reject a),b)

3. IQ is normally distributed in the population according to a $N(100,15^2)$ distribution. We suspect that most MIT students have above average IQ so we frame the following hypotheses.

H_0 =MIT student IQs are distributed identically to the general population = MIT IQ's follow a $N(100,15^2)$ distribution.

H_1 =MIT student IQs tend to be higher than those of the general population = the average MIT student IQ is greater than 100. Notice that H_1 is one sided.

Suppose we test 9 students and find they have an average IQ of $\bar{x} = 112$. Can we reject H_0 at a significance level $\alpha = 0.05$? Compute the p-value.

R: 0.0081975

4. Our data is normal with variance 25. The null hypothesis is that the data is drawn from $N(10, 25)$. That is, the null distribution has $\mu=10$. We have 1 data point $x=20$.

(a) If the alternative hypothesis is one-sided $\mu>10$. What is the p-value for this data?

(b) Suppose our significance level is 0.05. Will we reject the null hypothesis?

R: a)0.0227 b)yes

5. The mean response time of a species of pigs to a stimulus is 0.8 seconds. Twentyeight pigs were given 2 oz of alcohol and then tested. If their average response time was 1.0 seconds with a standard deviation of 0.3 seconds, can we conclude that alcohol affects the mean response time? Use the 5 percent level of significance.

R: TS =3.527668, p-value 0.001521239

6. Suppose that we have data that follows a normal distribution of unknown mean μ and unknown variance. Let the null hypothesis H_0 be that $\mu = 0$. Let the alternative hypothesis H_1 be that $\mu > 0$. Suppose we collect the following data: 1, 2, 3, 6, -1

At a significance level of $\alpha = 0.05$, should we reject the null hypothesis?

R: do not reject p-value 0.065

7. Suppose data is drawn from a normal distribution with unknown mean μ and unknown standard deviation. We make the following hypotheses:

There are 100 data points with a sample mean of 2 and sample variance of 36.

For a t-test let $H_0: \mu=1$ and $H_1: \mu>1$.

- (a) What is the value of the t statistic?
- (b) How many degrees of freedom does the null distribution (the distribution t of the statistic test) have?
- (c) What is the p-value for this test? Use R to compute your answer.
- (d) At a significance level of 0.01 do you reject the null hypothesis in favor of H_1 ?

R: a)1.666 c) 0.04943731