Tema 9

1. Evaluati: a) b) i^i c) $Ln\left[\left(\sqrt{3}+1\right)^3\right]$
2. Rezolvati ecuatia polinomiala, cu etapele indicate:
$$z^7-4z^6+6z^5-6z^4+6z^3-12z^2+8z+4=0$$
   a) Examinati efectul inlocuirii lui z^3 cu 2 in ecuatie.
   b) Factorizati ecuatia si folositi dezvoltarea binomiala $(z-1)^4$
3. Dezvoltarea binomiala pentru $(1+x)^n$ este:
$$(1+x)^n=\sum_{r=0}^{n} C_n^r x^r \quad \text{cu} \quad C_n^r=\frac{n!}{r!(n-r)!}$$
   a) Cu formula de Moivre aratati ca
$$C_n^0-C_n^2+C_n^4-\ldots+(-1)^m C_n^{2m}=2^{n/2}\cos(n\pi/4)\ , \quad n-1\le 2m\le n$$
   b) Determinati un rezultat similar pentru
$$C_n^1-C_n^3+C_n^5-\ldots(-1)^m C_n^{2m+1}\ , \quad n-1\le 2m+1\le n$$
4. Considerand dezvoltarea binomiala $\left(1+e^{i\theta}\right)^n$, aratati ca :
$$\sum_{r=0}^{n} C_n^r \cos(r\theta)=2^n\cos^n(\theta/2)\cos(n\theta/2)$$
$$\sum_{r=0}^{n} C_n^r \sin(r\theta)=2^n\cos^n(\theta/2)\sin(n\theta/2)$$
5. Folosind formula lui de Moivre cu $n=4$ pentu a demonstra ca:
$$\cos 4\theta=8\cos^4\theta-8\cos^2\theta+1$$
   Si pentru a deduce ca $\cos\frac{\pi}{8}=\left(\frac{2+\sqrt{2}}{4}\right)^{1/2}$
6. Cu formula lui de Moivre pentru $n=5$ demonstrate ca:
$$\tan 5\theta=\frac{t^5-10t^3+5t}{5t^4-10t^2+1} \quad \text{unde} \quad t=\tan\theta$$

R: 1. a) 5.11 b) $e^{-\left(\frac{\pi}{2}+2n\pi\right)}$ c) $\ln 8+i\left(6n+\frac{1}{2}\right)\pi$ 2. Radacinile sunt $2^{1/3}e^{2in\pi/3}$ cu n=0,1,2 si $1+3^{1/4}e^{2in\pi/4}$ cu n=0,1,2,3 3. a) considerati $(1+i)^n$ b) $2^{n/2}\sin(n\pi/4)$ 5. Folositi dezvoltarea binomial $(\cos\theta+i\sin\theta)^4$