

Tema 8

1. Se dau doua numere complexe $z = 3 + 4i$ si $w = 2 - i$. Calculati si reprezentati intr-o diagram Argand: a) $z + w$ b) $w - z$ c) wz d) z/w e) $z^*w + w^*z$ f) w^2
2. Considerand partile reale si partile imaginare ale produsului $e^{i\theta}e^{i\phi}$, demonstrati formulele pentru $\cos(\theta + \phi)$ si $\sin(\theta + \phi)$.
3. Scriind $\pi/12 = (\pi/3) - (\pi/4)$ si considerand $e^{i\pi/12}$, evaluate $\text{ctg}(\pi/12)$.
4. Evaluati : a) $\text{Re}(e^{2iz})$ b) $(-1 + i\sqrt{3})^{1/2}$ c) $\exp(i^3)$
5. Determinati ecuatiile in termeni de x si y pentru multimile de puncta din diagram Argand care satisfac: a) $\text{Re } z^2 = \text{Im } z^2$ b) $(\text{Im } z^2) / z^2 = -i$
6. Schitati regiunile din diagram Argand in care:
 - a) $\text{Re } z^2 < 0$, $|z^{1/2}| \leq 2$
 - b) $0 \leq \arg z^* \leq \pi/2$
7. Scrieti numerele complexe in forma trigonometrica si exponentiala:

$$z = -2 \qquad z = 2i \qquad z = -i \qquad z = -3 + \sqrt{3}i$$

8. Scrieti in forma algenrica:

$$\text{a) } \sqrt{2}e^{i\frac{3\pi}{4}} \quad \text{b) } 4e^{i\frac{\pi}{2}} \quad \text{c) } 2e^{4\pi i} \quad \text{d) } e^{1+i\frac{\pi}{2}}$$

9. Calculati $z_a = (1+i)^2$ si $z_b = (1+i)^{10}$ folosind forma algebrica a numerelor complexe si folosind forma exponentiala a numerelor complexe.

10. Calculati: $(2-2i)^7$, $(\sqrt{3}-3i)^6$

11. Schitati intr-o diagrama Argand regiunile:

$$\text{a) } |z| < 2 \quad \text{b) } |z+3| < 2 \quad \text{c) } 0 < |z-1| < 2$$

R: 1)a. $5+3i$ b. $-1-5i$ c. $10+5i$ d. $2/5+11i/5$ e. 4 f. $3-4i$ 3) $2+\sqrt{3}$ 4) a. se considera $z = x+iy$ rezulta $e^{-2y} \cos 2x$ b. se exprima $-1+i\sqrt{3}$ in forma polara si rezulta $\sqrt{2}e^{i\frac{\pi}{3}}$ c. $0.54-0.841i$ 5) a. $x^2 - y^2 = 2xy$ b) $x^2 = y^2$