Tema 14

1. a) Calculati toate derivatele partiale de ordinul intai ale functiilor:

i) $f(x,y)=x^2y$ ii) $f(x,y)=x^2+y^2+4$ iii) $f(x,y)=\sin\left(\frac{x}{y}\right)$

iv) $f(x,y)=arctg\left(\frac{y}{x}\right)$ v) $r(x,y,z)=\sqrt{x^2+y^2+z^2}$

b) Pentru i) si ii) calculati $\partial^2 f/\partial x^2$, $\partial^2 f/\partial y^2$, $\partial^2 f/\partial x\partial y$

2. Determinati care din urmatoarele diferentiale sunt diferentiale exacte:

a) $(3x+2)ydx+x(x+1)dy$ b) $ytgx\,dx+xtgy\,dy$

c) $y^2(\ln x+1)dx+2xy\ln x\,dy$ d) $y^2(\ln x+1)dy+2xy\ln x\,dx$

e) $\frac{x}{x^2+y^2}dy-\frac{y}{x^2+y^2}dx$

3.Aratati ca diferentiala $df=x^2dy-(y^2+xy)dx$ nu este exacta, dar $dg=\frac{1}{xy^2}df$ este exacta.

4. Ecuatia $3y=z^3+3xz$ defineste in mod implicit z in functie de x si y. Evaluati derivatele partiale secunde $\partial^2 z/\partial x^2$, $\partial^2 z/\partial y^2$ si verificati ca z este solutie a ecuatiei: $x\frac{\partial^2 z}{\partial y^2}+\frac{\partial^2 z}{\partial x^2}=0$

5. Functia $G(t)$ este definita de $G(t)=F(x,y)=x^2+y^2+3xy$ unde $x(t)=at^2$ si $y(t)=2at$. Folosind chain rule calculate derivate totala dG/dt .

R: 1.a) i)2xy,x^2 ii)2x,2y iii)$\frac{1}{y}\cos\left(\frac{x}{y}\right)$, $-\frac{x}{y^2}\cos\left(\frac{x}{y}\right)$ iv)$-y/(x^2+y^2)$, $x/(x^2+y^2)$ v)x/r, y/r, z/r b)i)2y,0,2x ii)2,2,0 2.a)nu b)nu c)da d)nu e)da 3.$2x\neq-2y-x$ pentru dg $y^{-2}=y^{-2}$ 4. $\frac{\partial z}{\partial x}=-\frac{z}{x+z^2}$, $\frac{\partial z}{\partial y}=\frac{1}{x+z^2}$, $\frac{\partial^2 z}{\partial x^2}=\frac{2xz}{(x+z^2)^3}$, $\frac{\partial^2 z}{\partial y^2}=-\frac{2z}{(x+z^2)^3}$ 5. $2a^2t(2t+1)(t+4)$.