

## Tema 6

1. Calculați limitele:

$$\text{a) } \lim_{x \rightarrow 0} \frac{\sin 2x}{x}$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{\sin 2x}$$

$$\text{c) } \lim_{x \rightarrow \pi} \frac{\sin 3x}{\sin 2x}$$

$$\text{d) } \lim_{x \rightarrow 1} \frac{1-x^2}{\sin(\pi x)}$$

$$\text{e) } \lim_{x \rightarrow 0} \frac{\ln(1+x^2)}{\sin^2(3x)}$$

$$\text{f) } \lim_{x \rightarrow 0} \frac{\sin 5x - \sin x}{3x}$$

$$\text{g) } \lim_{x \rightarrow 0} \frac{\sqrt[n]{1+x} - \sqrt[n]{1-x}}{x}$$

$$\text{h) } \lim_{x \rightarrow \pi} \frac{\sin x}{1 - \frac{x^2}{\pi^2}}$$

2. Calculați limitele:

$$\text{a) } \lim_{x \rightarrow \infty} \left( \frac{x^2+1}{x^2-1} \right)^{x^2}$$

$$\text{b) } \lim_{x \rightarrow \infty} \left( \frac{x^2-2x+1}{x^2-4x+4} \right)^x$$

$$\text{c) } \lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \cdot \sin 2x}$$

$$\text{d) } \lim_{x \rightarrow 0} \left( \frac{\cos x}{\cos 2x} \right)^{\frac{1}{x^2}}$$

3. Calculați limitele:

$$\text{a) } \lim_{x \rightarrow -\infty} \frac{x-1}{\sqrt{x^2-4}}$$

$$\text{b) } \lim_{x \rightarrow -\infty} \frac{-2x+1}{\sqrt{9x^2+1}}$$

$$\text{c) } \lim_{x \rightarrow \infty} \frac{\sqrt{x+\sqrt{x}}}{2\sqrt{x}}$$

$$\text{d) } \lim_{x \rightarrow 0} \frac{3x + \sin 2x}{x}$$

$$\text{e) } \lim_{x \rightarrow 0} \frac{\sin nx}{\sin mx}, \quad n, m \in \mathbb{Z}^*$$

$$\text{f) } \lim_{x \rightarrow 0} \frac{\sqrt{1+\operatorname{tg} x} - \sqrt{1-\operatorname{tg} x}}{x}$$

$$\text{g) } \lim_{x \rightarrow 0} \frac{e^{5x} - 1}{6x}$$

$$\text{h) } \lim_{x \rightarrow 0} \frac{\ln(1+8x)}{\ln(1+2x)}$$

$$\text{i) } \lim_{x \rightarrow 0} \frac{\ln(1+\sin x)}{\ln(1+\operatorname{tg} x)}$$

$$\text{j) } \lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}}$$

$$\text{k) } \lim_{x \rightarrow \infty} \frac{a^x + b^x}{c^x + d^x} \quad \text{unde } 0 < a < b < c < d$$

4. Calculați limitele folosind criteriul cleselui:

$$\text{a) } \lim_{x \rightarrow -\infty} \frac{[x]}{x} \quad \text{unde } [x] \text{ este partea întreaga}$$

$$\text{b) } \lim_{x \rightarrow \infty} \frac{x + \cos x}{x}$$

5. Să se stabilească dacă funcțiile următoare au limită în punctul  $x_0$  indicat.

$$\text{a) } f(x) = \begin{cases} 5x+1, & x > 3 \\ 3x+2, & x \leq 3 \end{cases} \quad x_0 = 3$$

$$\text{b) } f(x) = \frac{x+1}{x^2-4}, \quad x \in \mathbb{R} \setminus \{\pm 2\} \quad x_0 = 2$$