

Set Probleme 2

I. Determinați intervalele de convergență pentru seriile de funcții următoare:

$$\begin{array}{lll}
 1. \sum_{n=1}^{\infty} e^{nx} & 2. \sum_{n=1}^{\infty} \frac{n}{2^{nx}} & 3. \sum_{n=1}^{\infty} \ln^n(1+x^2) \\
 4. \sum_{n=1}^{\infty} \frac{1}{(2n-1)x^n} & 5. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{(x-2)^n} & 6. \sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^5 x^{2n}}
 \end{array}$$

R: 1. $(-\infty, 0)$ 2. $(0, +\infty)$ 4. $(-\infty, -1) \cup (1, \infty)$

I. Cu testul Weierstrass, demonstrați convergența seriilor de funcții:

$$\begin{array}{l}
 1. \sum_{n=1}^{\infty} \frac{\sin nx}{n^2} \text{ pe } \mathbb{R} \\
 2. \sum_{n=1}^{\infty} \frac{\cos nx}{n\sqrt{n}} \text{ pe } \mathbb{R}
 \end{array}$$

II. Determinați intervalele de convergență pentru seriile de puteri:

$$\begin{array}{lll}
 1. \sum_{n=1}^{\infty} \frac{x^n}{n2^n} & 2. \sum_{n=0}^{\infty} \frac{2^n}{n+1} x^n & 3. \sum_{n=0}^{\infty} \frac{n+1}{\sqrt{1+2n}} x^n \\
 4. \sum_{n=1}^{\infty} n^n x^n & 5. \sum_{n=1}^{\infty} nx^n & 6. \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^n \\
 7. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n & 8. \sum_{n=1}^{\infty} (-1)^n (2n+1)^n x^n & 9. \sum_{n=1}^{\infty} \frac{x^n}{n!} \\
 10. \sum_{n=1}^{\infty} \frac{x^n}{n^n} & 11. \sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 5^n} & \\
 12. \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1} (x+2)^n & 13. \sum_{n=1}^{\infty} \frac{(-1)^n n^3}{3^n} (x+3)^n & \\
 14. \sum_{n=0}^{\infty} \frac{(n+1)!}{5^n} (x-2)^n & 15. \sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^2} (x-1)^n &
 \end{array}$$

R: 1. $[-2, 2)$ 4. $x=0$ 5. $(-1, 1)$ 6. $(-1, 1]$ 7. $(-1, 1]$ 9. \mathbb{R} 10. $x=0$ 14. $x=2$ 15. $x=1$

