Set Probleme 13

1. Determinaţi domeniul în care următoarele ecuaţii cu derivate parţiale sunt hiperbolice, parabolice şi eliptice.

   a) $\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}-3\frac{\partial^2 u}{\partial y^2}=0$

   b) $\frac{\partial^2 u}{\partial x^2}-2x\frac{\partial^2 u}{\partial x\partial y}+y\frac{\partial^2 u}{\partial y^2}=0$

   c) $\frac{\partial^2 u}{\partial x^2}+x\frac{\partial^2 u}{\partial y^2}=0$

   d) $\sin^2 x\frac{\partial^2 u}{\partial x^2}+\sin(2x)\frac{\partial^2 u}{\partial x\partial y}+\cos^2 x\frac{\partial^2 u}{\partial y^2}=0$

   e) $8\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial x}+\left[\lg\left(2+x^2\right)\right]u=0$

2. Ecuatia diferentiala cu derivate partiale:

   $$6\frac{\partial^2 u}{\partial x^2}-5\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=14$$

   este liniara neomogena de ordinul doi.

   a) Aratati ca este de tip hiperbolic
   b) Cu schimbarile de variabile: $\xi=x+2y$ si $\eta=x+3y$ deduceti forma canonica a acestei ecuatii.
   c) Determinati solutia generala a ecuatiei date.

   R: b) $\frac{\partial^2 u}{\partial\xi\,\partial\eta}=-14$ c) $u(x,y)=f(x+2y)+g(x+3y)-14(x+2y)(x+3y)$

3. Consideram ecuatia cu derivate partiale liniara neomogena:

   $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=x^2y^2$$

   a) Procedati la schimbarea variabilelor: $\xi=x+iy$, $\eta=x-iy$, $i=\sqrt{-1}$
   b) Determinati solutia generala a ecuatiei omogene asociate (la forma in noile variabile)

c) Determinati prin integrarea ecuatiei obtinute la punctul a) o solutie particulara a ecuatiei neomogene (considerand functiile constante de integrare nule)

d) Scrieti solutia generala a ecuatiei in variabile (ξ,η) si apoi in variabilele initiale (x,y)

R: a) $\frac{\partial^2 u}{\partial\xi\partial\eta}=-\frac{1}{64}\left(\xi^2-\eta^2\right)^2$ b) $u_o(\xi,\eta)=f(\xi)+g(\eta)$,

f,g arbitrare c) $\tilde{u}(\xi,\eta)=-\frac{1}{64}\left(\frac{1}{5}\xi^5\eta-\frac{2}{9}\xi^3\eta^3+\frac{1}{5}\xi\eta^5\right)$

d) $u(x,y)=f(x+iy)+g(x-iy)+\frac{1}{12}x^4\left(y^2-\frac{1}{15}x^2\right)$

4. Arătaţi că funcţia $u(x,t)=\cos\frac{3\pi a}{l}t\sin\frac{3\pi}{l}x$ este soluţie pentru problema mixtă:

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}, \quad t>0, \quad 0<x<l$$

$$u\big|_{x=0}=u\big|_{x=l}=0 , \quad t\geq 0$$

$$u\big|_{t=0}=\sin\frac{3\pi x}{l}, \quad \frac{\partial u}{\partial t}\bigg|_{t=0}=0, \quad 0\leq x\leq l$$