## Set Probleme 1

I. Determinaţi a n-a sumă parţială pentru seria $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$ şi demonstraţi convergenţa acesteia cu definiţia.

R: conv. suma1

II. Cu unul din cele două teste de comparaţie (test I, test II) examinaţi convergenţa seriilor:

1. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$  2. $\sum_{n=1}^{\infty}\frac{\sin^2 \alpha n}{2^n}$, $\alpha \neq 0$  3. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{4^n+7}}$

4. $\sum_{n=1}^{\infty}\frac{1}{2^n+\cos^2 n}$  5. $\sum_{n=1}^{\infty}\left(1-\cos\frac{\pi}{3^n}\right)$  6. $\sum_{n=1}^{\infty} tg\,\frac{\pi}{n}$

R: 1. div. 2.conv. 3. conv. 4.conv. 5.conv. 6. div.

III. Cu testul D'Alembert, examinaţi convergenţa seriilor:

1. $\sum_{n=1}^{\infty}\frac{n^3}{3^n}$  2. $\sum_{n=1}^{\infty}\frac{2^n}{n^2+n}$  3. $\sum_{n=1}^{\infty}\frac{n!}{n^2\cdot 2^n}$

4. $\sum_{n=1}^{\infty} n^3\sin\frac{\pi}{3^n}$  5. $\sum_{n=1}^{\infty} ntg\,\frac{\pi}{2^{n+1}}$  6. $\sum_{n=1}^{\infty}\frac{a^n}{n^n}$, $a\in(0,1)$

R: 1. conv. 2.div. 3. div. 4.conv. 5.conv. 6. conv.

IV. Cu testul Cauchy, examinaţi convergenţa seriilor:

1. $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$  2. $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^{n}$  3. $\sum_{n=1}^{\infty}\left(arctg\,\frac{1}{n}\right)^{n}$

4. $\sum_{n=1}^{\infty}\frac{8^{n+2}}{5^n}$  5. $\sum_{n=1}^{\infty}2^{-n}\left(\frac{n+1}{n}\right)^{n^2}$  6. $\sum_{n=1}^{\infty}\ln^n\frac{2n+1}{n}$

R: 1. conv. 2.conv. 3. conv. 4.conv. 5.div. 6. conv.

V. Cu testul Cauchy integral examinaţi convergenţa seriilor:

1. $\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^2+1}}$  2. $\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{1}{n}$

3. $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{\ln n}}$  4. $\sum_{n=1}^{\infty}\frac{arctg\ n}{n^2+1}$  5. $\sum_{n=4}^{\infty}\frac{1}{(n-2)\ln^2(n-2)}$

R: 1. div. 2.conv. 3. div. 4.conv. 5. conv