Cursul 7

Chapter 6 ***SOLAR SPECTRAL RADIATION UNDER CLOUDLESS SKIES***

As discussed in the previous lecture, the direct spectral irradiance reaching the Earth's surface is determined by the extraterrestrial spectral irradiance attenuated by the atmosphere, with the overall atmospheric transmittance expressed as the product of the spectral transmittances of its individual constituents.

$$G_{n\lambda} = G_{0n\lambda} \prod_{i=1}^{i=j} \tau_{i\lambda} \quad (1)$$

Origins of Diffuse Spectral Radiation

Diffuse radiation is generated by the scattering effects of air molecules and aerosols.

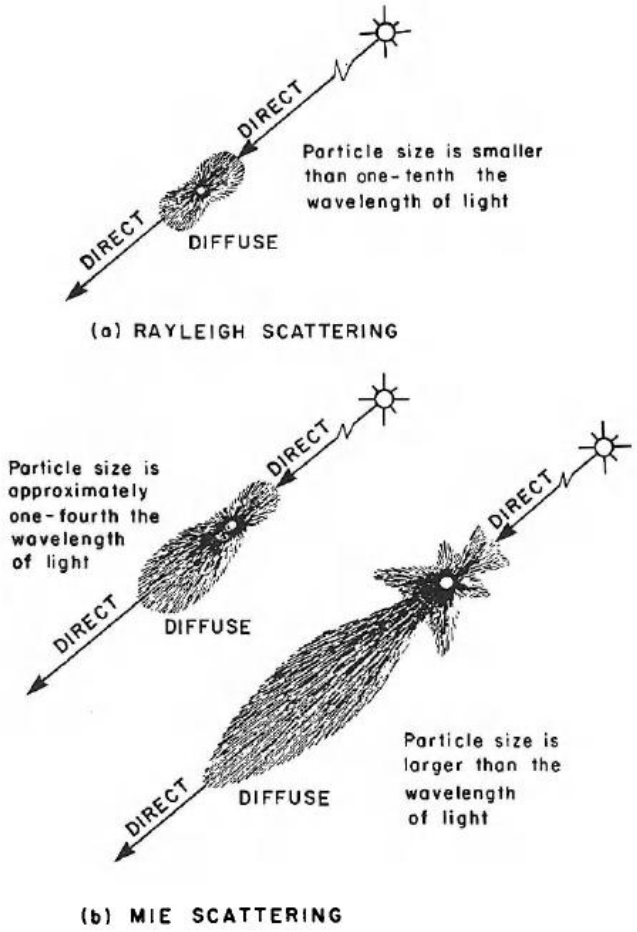


Figure 1. Scattering of electromagnetic radiation: (a) Rayleigh scattering (b) Mie scattering.

The diagram illustrates the **diffuse radiation** resulting from the first interaction of direct solar radiation with atmospheric molecules and particles. This process is known as **primary scattering**.

The scattered radiation can subsequently interact with other atmospheric molecules and particles, undergoing further scattering events. This successive scattering process is known as **multiple scattering**.

Figure 2 shows multiple scattering by air molecules, and Fig. 3 by aerosols. Diffuse radiation generated by primary scattering is dominant.

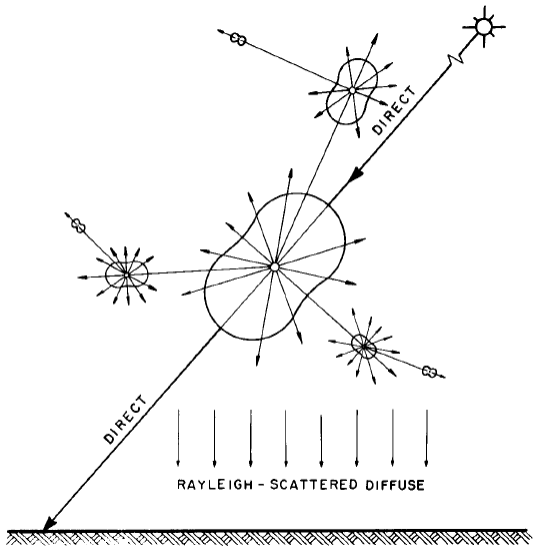


Figure 2. Multiple scattering by air molecules.

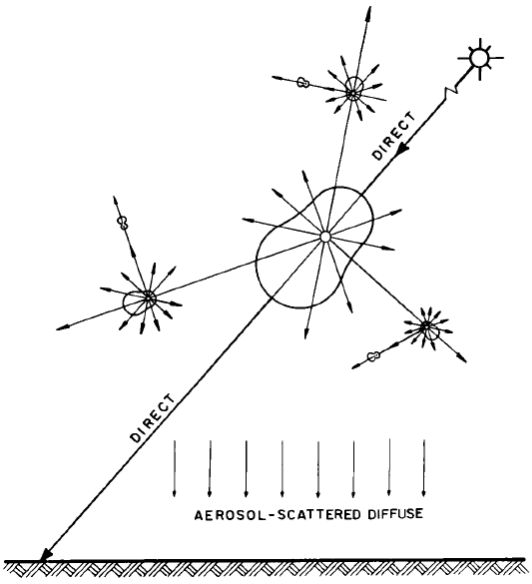


Figure 3. Multiple scattering by aerosols.

The diffuse radiation depends, in part, on the concentration of atmospheric molecules and aerosol particles.

The total radiation resulting from primary and multiple scattering is distributed among three components:

- a fraction is scattered back into space;
- a fraction is absorbed by the atmosphere;
- a fraction reaches the Earth's surface.

Although multiple scattering contributes to the diffuse radiation, its effect is often relatively small. Moreover, the improvement gained from an exact treatment of multiple scattering may be offset by uncertainties in the optical properties of the real atmosphere, particularly those of aerosols and the Earth's surface (albedo).

For this reason, we will consider **single scattering only**.

Our main objective is to develop separate and relatively simple expressions for the diffuse radiation produced by atmospheric molecules (Rayleigh scattering) and aerosol particles (Mie scattering).

Step 1. Primary scattering by air molecules and aerosols

First, consider a **pure Rayleigh atmosphere** irradiated by a unit-intensity parallel monochromatic beam. The diffuse radiation produced by primary scattering is given by:

$$1 - \tau_{r\lambda} \tag{2}$$

Next, consider a **pure aerosol atmosphere** irradiated by the same unit-intensity parallel monochromatic beam.

The attenuation of solar radiation by aerosols results from two simultaneous processes: absorption and scattering. Let us define:

$\tau_{aa\lambda}$ the spectral transmittance of aerosols due to absorption,

$\tau_{as\lambda}$ the spectral transmittance of aerosols due to scattering effects.

The total aerosol transmittance is therefore:

$$\tau_{a\lambda} = \tau_{aa\lambda}\, \tau_{as\lambda} \tag{3}$$

The total attenuation by aerosols is

$$1 - \tau_{a\lambda} = 1 - \tau_{aa\lambda}\, \tau_{as\lambda} \tag{4}$$

The respective contributions of absorption and scattering cannot be determined from the total attenuation alone. To distinguish between these two processes, atmospheric scientists use the

single scattering albedo, ω_0, which represents the fraction of the energy removed from the direct solar beam that is scattered rather than absorbed during the first interaction with aerosol particles ($\omega_0 = \frac{scattering}{scattering+absorption}$).

Accordingly, the energy scattered by aerosols is given by:

$$\omega_0(1 - \tau_{a\lambda}) \tag{5}$$

In practice, determining the single scattering albedo (SSA) is difficult. This parameter depends on the composition, shape, size, and optical properties of aerosol particles.

Aerosols in rural environments generally have higher single scattering albedo values than those in urban-industrial environments, where absorbing particles are more abundant.

In simplified atmospheric models, it is common to assign a fixed SSA value, independent of wavelength, typically ranging from 0.7 to 1.0.

Typical assumed values are:

- urban-industrial regions $\omega_0 \approx 0.6$,
- rural-agricultural areas $\omega_0 \approx 0.9$.

Urban-industrial aerosols generally contain more carbonaceous material, which contributes to stronger light absorption and, consequently, lower SSA values.

Step 2. Scattering by a mixture of air molecules and aerosols

Consider a homogeneous mixture of air molecules (Rayleigh scatterers) and aerosol particles. For simplicity, gaseous absorption is neglected.

Assuming a unit incident solar energy, the total energy attenuated by the atmosphere is given by:

$$1 - \tau_{r\lambda}\,\tau_{a\lambda} \tag{6}$$

We are particularly interested in determining the fraction of this energy that is scattered.

In several simplified models proposed in the literature (e.g. Leckner), aerosol absorption is neglected. Under this assumption, the scattered energy, in the presence of molecular absorbers, is expressed as:

$$\tau_{ma\lambda}(1 - \tau_{r\lambda}\,\tau_{a\lambda}) \tag{7}$$

Furthermore, it is commonly assumed that **one-half of the total scattered energy reaches the Earth's surface**, while multiple reflections between the atmosphere and the ground are neglected.

Since Rayleigh and aerosol scattering have different angular and spectral characteristics, it is desirable to distinguish their respective contributions to the diffuse radiation.

Following Iqbal, we consider the following expressions:

Rayleigh-scattered diffuse radiation:

$$\tau_{ma\lambda}(1-\tau_{r\lambda})\tau_{a\lambda} \tag{8}$$

Aerosol-scattered diffuse radiation:

$$\tau_{ma\lambda}(1-\tau_{a\lambda})\tau_{r\lambda}\,\omega_0 \tag{9}$$

Expressions (7-9) are all empirical and approximate.

We should distinguish between ***multiple scattering*** within the atmosphere and ***multiple reflections*** between the atmosphere and the Earth's surface. In the latter case, part of the radiation reaching the ground is reflected upwards by the surface. Some of this ***upwelling radiation*** is then reflected back towards the ground by the atmosphere. This process may occur repeatedly, adding further contributions to the diffuse radiation reaching the surface. Let's note

$G_{d\lambda}$ the ***diffuse spectral irradiance on a horizontal surface***.

This diffuse irradiance is composed of three parts:

- $G_{dr\lambda}$ the diffuse spectral irradiance produced by ***Rayleigh*** scattering that arrives on the ground after the first pass through the atmosphere;
- $G_{da\lambda}$ the diffuse spectral irradiance produced by ***aerosols*** that arrive on the ground after the first pass through the atmosphere;
- $G_{dm\lambda}$ the diffuse spectral irradiance produced by ***multiple reflections***.

$$G_{d\lambda} = G_{dr\lambda} + G_{da\lambda} + G_{dm\lambda} \tag{10}$$

This ***downward-diffuse radiation*** is also called ***sky-diffuse*** or ***sky radiation***.

Rayleigh-Scattered Spectral Diffuse Irradiance

Regardless of the solar angle of incidence, it is commonly assumed that approximately half of the diffuse radiation is directed towards the Earth's surface, while the other half escapes into space. This approximation is widely used in models based on Rayleigh scattering.

Spectral studies indicate that this assumption is generally reasonable. However, at shorter wavelengths, multiple scattering becomes more significant, and slightly less than half of the diffuse radiation is directed towards the ground.

Consider a plane-parallel layer representing the earth's atmosphere containing all the attenuators we have treated so far. The Rayleigh-scattered diffuse radiation generated by an irradiance of $G_{0n\lambda}\cos\theta_z$ on a horizontal surface at the top of the atmosphere is:

$$G_{0n\lambda} \cos\theta_z \ \tau_{ma\lambda}(1 - \tau_{r\lambda})\tau_{a\lambda} \tag{11}$$

If we assume that half the above amount is directed downwards, the diffuse radiation reaching the ground after the first pass through the atmosphere is:

$$G_{dr\lambda} = G_{0n\lambda} \cos\theta_z \ \tau_{ma\lambda}[0.5(1 - \tau_{r\lambda})\tau_{a\lambda}] \tag{12}$$

Two phrases often employed by meteorologists: ***forward scatterance*** and ***backscatterance***. The former implies the fraction of the *total scattered* energy propagated in the direction of the incident wave. This direction is usually toward the ground. Backscatterance is the fraction of the total scattered energy propagated in a direction opposite to the incident wave, and this direction is usually toward outer space. In the Rayleigh atmosphere treated above, we have assumed that the forward and the backward scatterances are each equal to 0.5.

In Fig. 4 we show some results of the Rayleigh-scattered spectral diffuse irradiance. We assume $\beta = 0.1$ and $\alpha = 1.3$. In this diagram the shaded areas represent the effects of molecular absorption, with ozone increasing from 0 to 0.35 cm, and water vapor from 0 to 2 cm. The effect of absorption is small and is mainly in the ultraviolet and a portion of the visible region. The absorption effect increases, however, with an increase in the zenith angle.

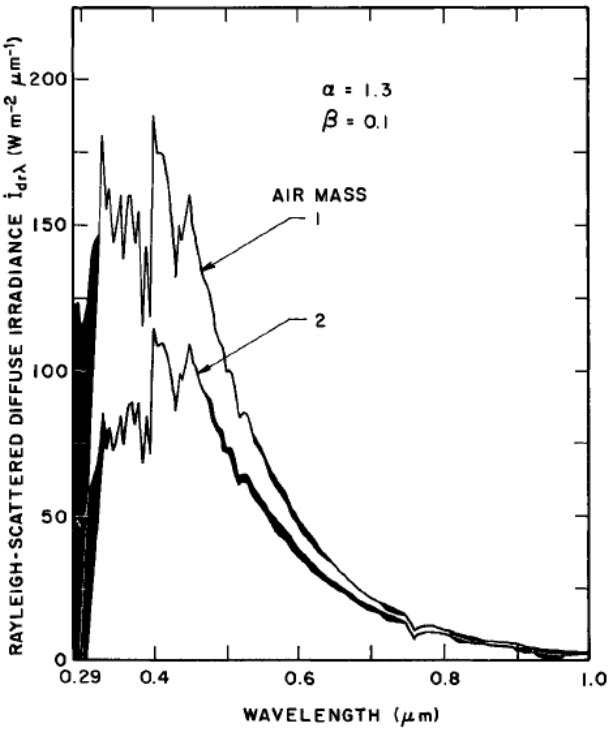
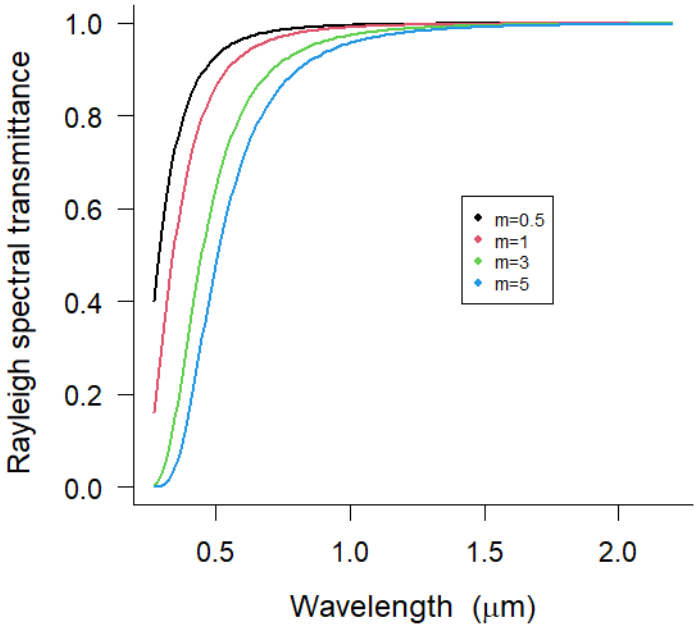


Figure 4. Rayleigh scattered spectral diffuse irradiance. The shaded areas indicate increase in attenuation when ozone is increased from 0 to 0.35 cm and water vapor is increased from 0 to 2 cm.

An increase in the zenith angle reduces the amount of radiation reaching the ground.

We have observed that Rayleigh scattering is negligible at $\lambda > 1\ \mu m$.



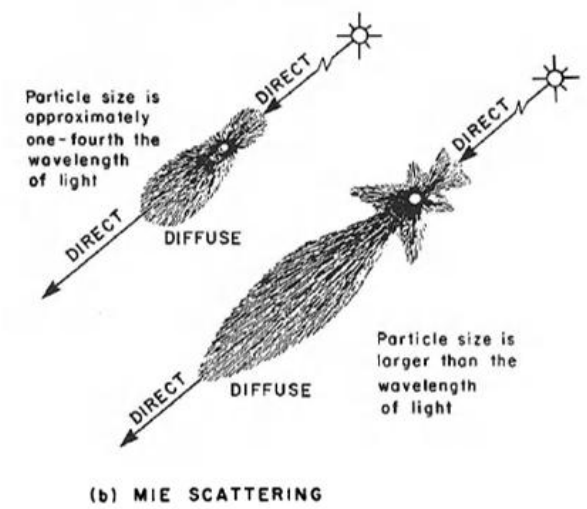
Aerosol-Scattered Spectral Diffuse Irradiance

Aerosols are the biggest single atmospheric element creating severe problems in calculating solar irradiance. One part of the difficulty lies in specifying their number, size, distribution, and optical properties. In general, aerosol-generated diffuse radiation is substantial and hence cannot be ignored. In many industrialized and/or dusty parts of the world, it is much greater than the Rayleigh-scattered diffuse radiation.

Let us consider again the earth's atmosphere as a plane-parallel layer containing all the attenuating elements and irradiated by a flux of $G_{0n\lambda} \cos\theta_z$ on a horizontal surface. The total amount of aerosol-scattered diffuse radiation is given by the following:

$$G_{0n\lambda} \cos\theta_z \, \tau_{ma\lambda}(1 - \tau_{a\lambda})\tau_{r\lambda} \, \omega_0 \qquad (13)$$

This formulation represents the amount of diffuse radiation streaming out in all directions. We are interested in that portion which is directed toward the ground. We have already seen that aerosol scattering is mainly in the forward direction.



(b) MIE SCATTERING

Therefore, we need to specify a ratio of forward to total energy scattered, that is, forward scatterance. Let this ratio be called F_c such that

$$F_c = \frac{energy\ scattered\ in\ the\ forward\ direction}{total\ energy\ scattered} \tag{14}$$

As F_c is ***forward scatterance***, $1 - F_c$ is backscatterance.

Determination of F_c is difficult. This ratio depends on particle size, shape, and wavelength. Since the word "forward" implies the direction of beam radiation, F_c is a function of the zenith angle as well. Next to ω_0, F_c is the most poorly specified of the aerosol properties.

For the type of aerosols prevailing over the British Isles, Robinson (1962) determined the value of F_c as a function of the zenith angle. Table lists the values of F_c which may be used until more precise studies are available.

F_c, the Ratio of Forward Scattering to Total Scattering[a]

θ_z	0	10	20	30	40	50	60	70	80	85
F_c	0.92	0.92	0.90	0.90	0.90	0.85	0.78	0.68	0.60	0.50

[a] Adapted from Robinson [33].

It follows that aerosol-scattered diffuse radiation reaching the ground after the first pass through the atmosphere is

$$G_{da\lambda} = G_{0n\lambda} \cos\theta_z\, \tau_{ma\lambda}[F_c\omega_0(1 - \tau_{a\lambda})\tau_{r\lambda}] \tag{15}$$

We consider the factors F_c and ω_0 to be invariant with wavelength.

In Fig. 5 we show the spectral variation of aerosol-scattered diffuse irradiance. Nonabsorbing aerosols are assumed—that is, $\omega_0 = 1$. Molecular absorbers are included and their effect, which is small, is shown by the shaded areas, which represent increasing ozone from 0 to 0.35 cm and water vapor from 0 to 2 cm. Radiation is further attenuated at larger air masses, and molecular absorption becomes more important. Aerosol-scattered radiation extends to longer wavelengths than does Rayleigh-scattered radiation.

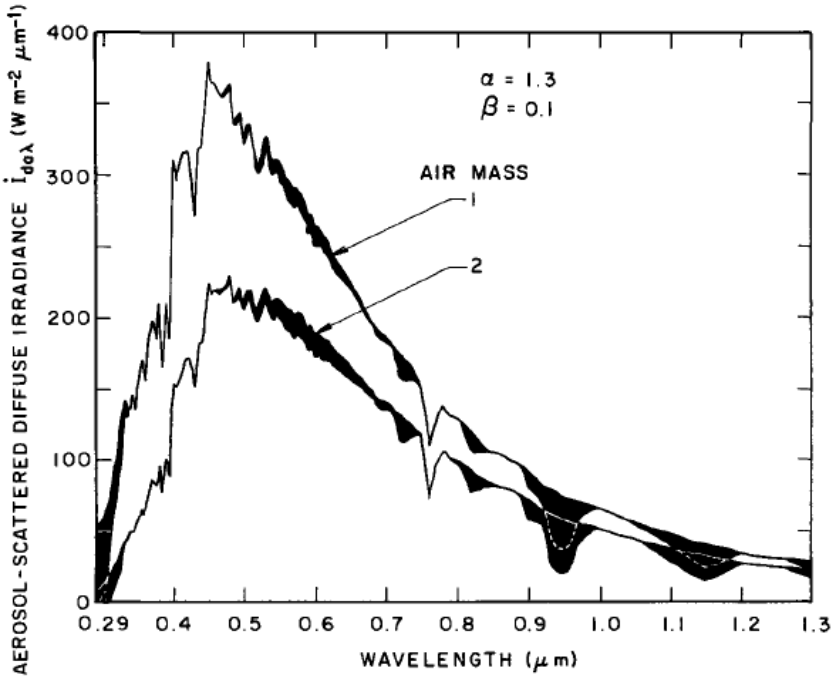


Figure 6.17.1 Aerosol-scattered diffuse irradiance. The shaded areas indicate increase in attenuation when ozone is increased from 0 to 0.35 cm(NTP) and water vapor is increased from 0 to 2 cm.

Figure 5. Aerosol scattered spectral diffuse irradiance. The shaded areas indicate increase in attenuation when ozone is increased from 0 to 0.35 cm and water vapor is increased from 0 to 2 cm.

Because the aerosols have a ***strong forward scattering component***, a substantial portion of the aerosol-scattered diffuse radiation appears to come from a small annular area around the solar disk; this radiation is called ***circumsolar radiation*** or the solar aureole (Fig. 6). The size of the annular area varies with the amount of aerosol loading and aerosol particle size distribution. It is necessary to point out that $G_{da\lambda}$ includes circumsolar radiation.

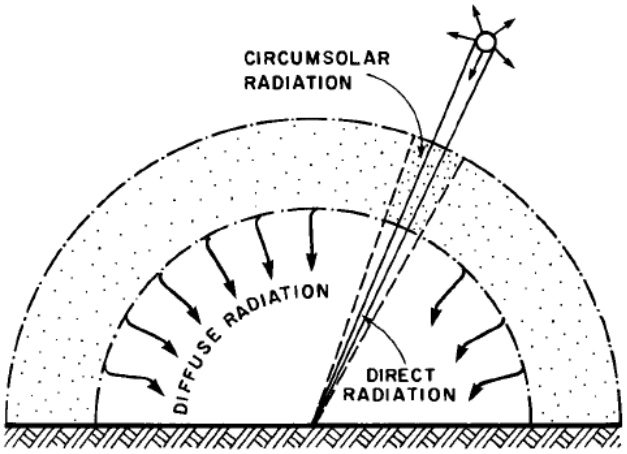


Figure 6. Direct, diffuse and circumsolar radiation.

Atmospheric Albedo

In meteorology, reflectance is also called ***albedo***. In this section we develop a mathematical expression for the albedo of the cloudless atmosphere. ***Albedo*** is defined as the ratio of the energy reflected back to the incident energy. Consider the atmosphere as a plane-parallel layer, irradiated by one unit of parallel monochromatic beam (Fig. 7).

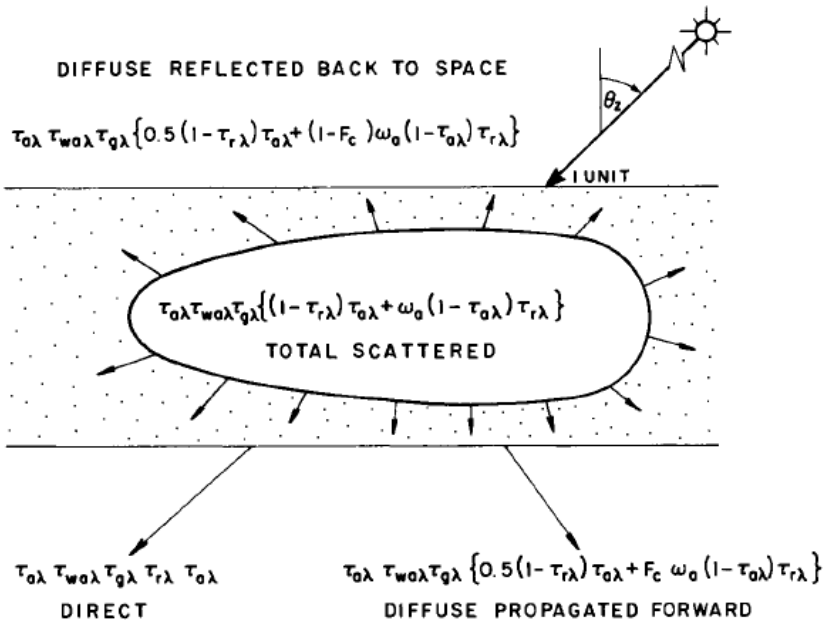


Figure 7. Partition of radiation into direct, diffuse propagated forward, and diffuse reflected back to space.

In this diagram, the partition of the incident energy into various components is shown. Albedo of the atmosphere is the diffuse component reflected back to space. In an approximate form it is

$$\rho_{a\lambda} = \tau_{ma\lambda}[0.5(1-\tau_{r\lambda})\tau_{a\lambda} + (1-F_c)\omega_0(1-\tau_{a\lambda})\tau_{r\lambda}] \quad (16)$$

The first term on the right-hand side of this equation represents the albedo of the Rayleigh atmosphere, and the second that of aerosols.

If the incident radiation is of diffuse nature, for instance the ***upwelling radiation*** from the ground, the angle of incidence will vary from 0° to 90°. In such a situation, calculation of atmospheric albedo is quite complicated. It is common to assume a single average angle of incidence corresponding to air mass 1.6-1.9. Here we shall assume m = 1.9.

We rewrite the preceding equation to prevent possible confusion:

$$\rho_{a\lambda}' = \tau_{ma\lambda}'[0.5(1-\tau_{r\lambda}')\tau_{a\lambda}' + (1-F_c')\omega_0(1-\tau_{a\lambda}')\tau_{r\lambda}'] \quad (17)$$

The primes over various quantities indicate that they are to be evaluated at $m = 1.9$, $\theta_z \approx 60^\circ$.

The variation of atmospheric albedo $\rho_{a\lambda}'$ with α and β is shown in Fig. 8 for two specific wavelengths. ***Albedo*** is greater at shorter wavelengths because more energy is scattered in the shorter than in the longer wavelengths. The albedo increases with β and α. Larger values of α represent small average particle size, and we know that smaller particles scatter more energy than do bigger particles. Consequently, the atmospheric albedo increases with α. At long wavelengths, the behavior is reversed: large particles scatter more.

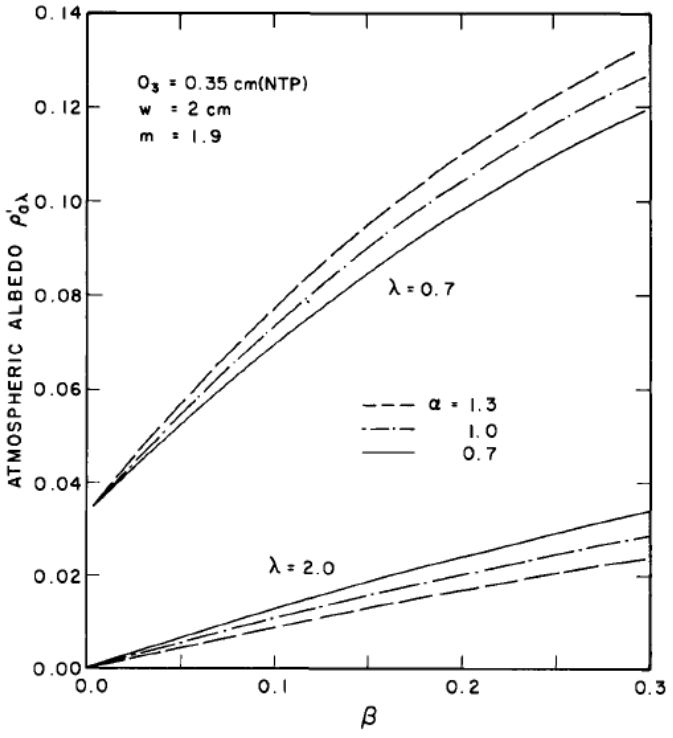


Figure 8. Variation of the atmospheric albedo with wavelength and β.

Multiply Reflected Spectral Diffuse Irradiance

Let consider diffuse irradiance reaching the ground after the ***first pass***, and beam irradiance on a horizontal surface. Let

$$Q_\lambda = G_{dr\lambda} + G_{da\lambda} + G_{n\lambda}\cos\theta_z \tag{18}$$

We are interested in determining the diffuse irradiance resulting from ***multiple reflections*** of Q_λ between the ground and the atmosphere. Let $\rho_{g\lambda}$ be the ground albedo. We assume that this albedo produces a perfectly isotropic diffuse reflection. That is, the upwelling radiation $Q_\lambda\rho_{g\lambda}$, has a uniform intensity in all directions.

Figure 9 shows the process of multiple reflections between the ground and the atmosphere. The upwelling diffuse radiation $Q_\lambda\rho_{g\lambda}$ is reflected back to the ground, and the process of multiple reflections between the ground and the sky continues. The radiation is continuously attenuated at each impact. The total amount of ***multiply reflected radiation*** reaching ground is:

$$G_{dm\lambda} = Q_\lambda\left(\rho_{g\lambda}\,\rho'_{a\lambda} + \rho^2_{g\lambda}\,\rho'^2_{a\lambda} + \rho^3_{g\lambda}\,\rho'^3_{a\lambda} + \cdots\right)$$

$$G_{dm\lambda} = Q_\lambda\,\frac{\rho_{g\lambda}\,\rho'_{a\lambda}}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} \tag{19}$$

The value of $\rho'_{a\lambda}$ is very small, and $\rho_{g\lambda}$ is usually about 0.2. Consequently, the effect of multiple reflections is minor. However, in the event of a snow cover, $\rho_{g\lambda}$ is approximately 0.7 and the effect of multiple reflections assumes important dimensions. We assume $\rho_{g\lambda}$ invariant with wavelength.

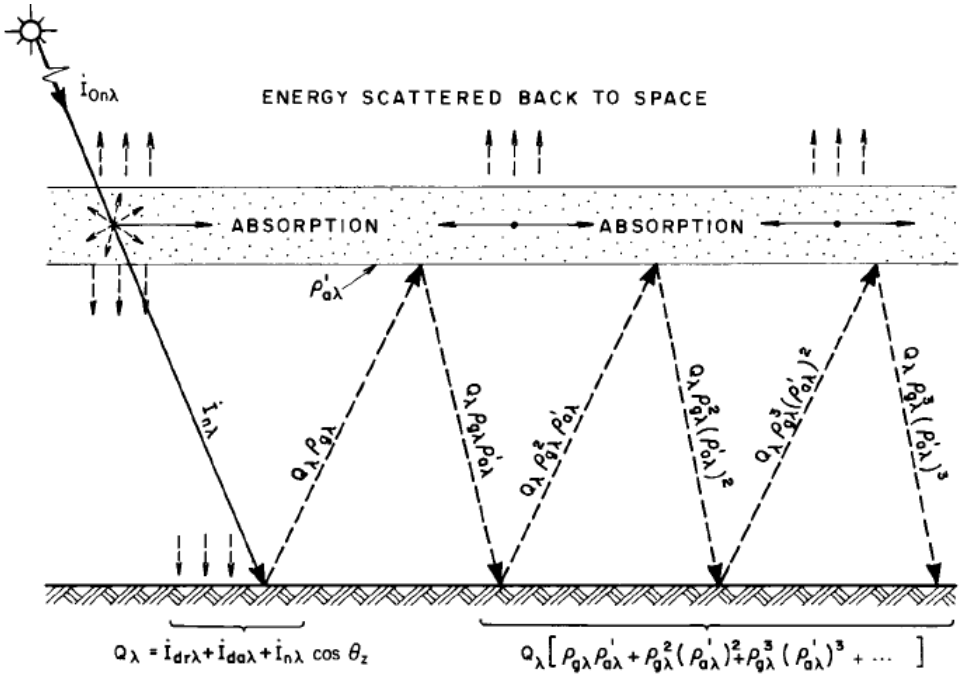


Figure 9. The process of multiple reflection of radiation between the atmosphere and the ground.

Variations of multiply reflected irradiance with ground albedos of 0.2 and 0.7 are shown in Fig. 10. Naturally, higher albedos increase the level of multiply reflected radiation. In the same diagram, the effect of turbidity is also shown. The effect of turbidity appears to vary with wavelength. At lower turbidity *(β = 0.05)*, $G_{dm\lambda}$ is higher at shorter wavelengths *($\lambda < 0.5\mu m$)*. At higher turbidity *(β = 0.2)*, $G_{dm\lambda}$ is higher at longer wavelengths *($\lambda > 0.5\mu m$).*

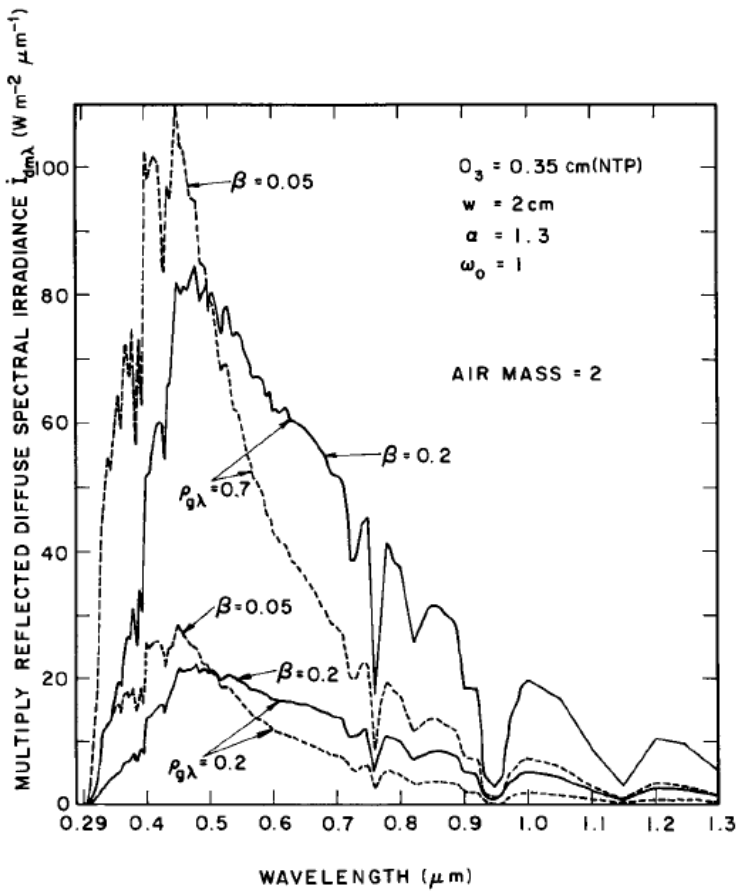


Figure 10. Multiply reflected diffuse spectral irradiance on a horizontal surface as e function of β and ground albedo.

Diffuse Spectral Irradiance on the Ground

If we combine the above equations, the diffuse spectral irradiance on a horizontal surface

$$G_{d\lambda} = G_{dr\lambda} + G_{da\lambda} + G_{dm\lambda} \tag{20}$$

$$G_{d\lambda} = G_{dr\lambda} + G_{da\lambda} + (G_{dr\lambda} + G_{da\lambda} + G_{n\lambda}\cos\theta_z)\frac{\rho_{g\lambda}\,\rho'_{a\lambda}}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} \tag{21}$$

$$G_{d\lambda} = (G_{dr\lambda} + G_{da\lambda})\frac{1}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} + G_{n\lambda}\cos\theta_z\,\frac{\rho_{g\lambda}\,\rho'_{a\lambda}}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} \tag{22}$$

We can now examine the variation of diffuse irradiance with some climatic and physical parameters. Air mass, turbidity parameters α and β, and ground albedo are the dominant factors that modify diffuse irradiance.

We have already observed that an increase in air mass reduces diffuse irradiance.

Figure 11 shows the effect of a change in turbidity parameter β. Naturally, greater turbidity results in higher amounts of diffuse radiation.

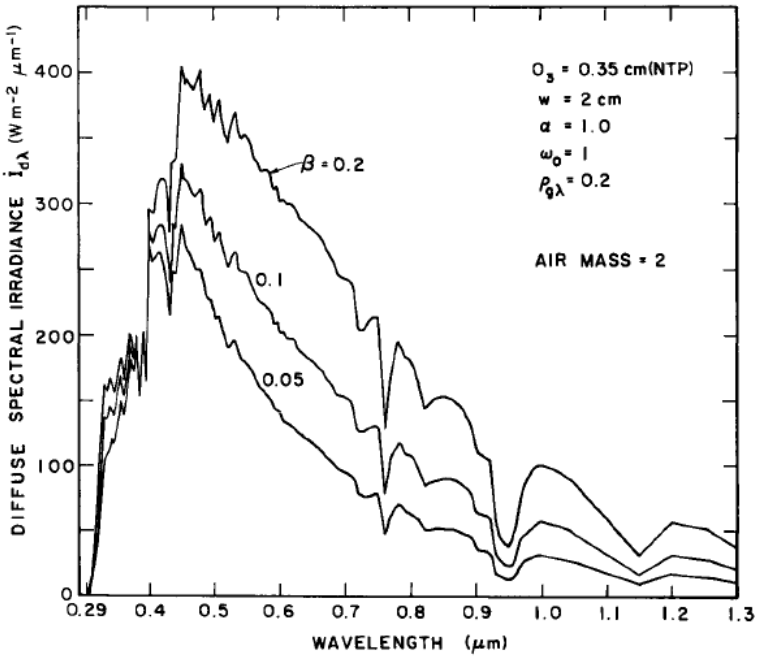
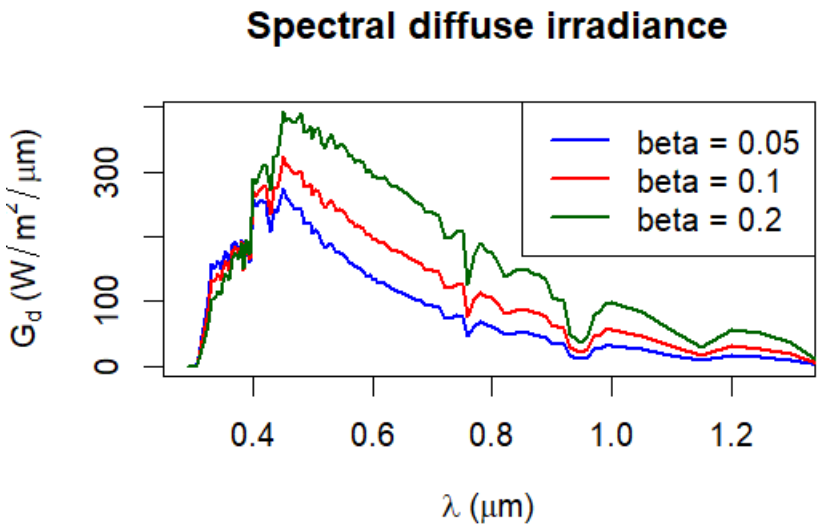


Figure 11. Diffuse spectral irradiance on a horizontal surface as a function of β.




```
lambda <- as.numeric(tabel_ETR_Coef_Leckner_Davos_Smart2[[1]])
>   G0n <- as.numeric(tabel_ETR_Coef_Leckner_Davos_Smart2$Gext_D) * 10
>   # Parametri geometrici si atmosferici
>   J <- 166              # 15 iunie
>   teta_z <- 60
>   ma <- 1 / cos(teta_z * pi / 180)
>   alfa <- 1.0
>   ww <- 2
>   lO3 <- 0.35
>   ro_g <- 0.2
>   ssa <- 1
>   Fc <- 0.78
>   ej <- 1.00011 +
+     0.034221 * cos(2 * pi * (J - 1) / 365) +
+     0.00128 * sin(2 * pi * (J - 1) / 365) +
+     0.000719 * cos(4 * pi * (J - 1) / 365) +
+     0.000077 * sin(4 * pi * (J - 1) / 365)


# Coeficienti de absorbtie
>   k_o <- as.numeric(tabel_ETR_Coef_Leckner_Davos_Smart2$Ko)[ok][ord]
>   k_w <- as.numeric(tabel_ETR_Coef_Leckner_Davos_Smart2$Kw)[ok][ord]
>   k_g <- as.numeric(tabel_ETR_Coef_Leckner_Davos_Smart2$Kg)[ok][ord]

>   # Functii transmitance
>   tau_o <- function(k, l, m) {
+     exp(-k * l * m)
+   }
>   tau_w <- function(k, w, m) {
+     exp((-0.2385 * k * w * m) /
+           (1 + 20.07 * k * w * m)^0.45)
+   }
>   tau_g <- function(k, m) {
+     exp((-1.41 * k * m) /
+           (1 + 118.3 * k * m)^0.45)
+   }
>   tau_r <- function(x, m) {
+     exp(-0.008735 * m * x^(-4.08))
+   }
```

```
>   tau_a <- function(x, a, b, m) {
+     exp(-m * b * x^(-a))
+   }

>   # Functia care calculeaza iradianta difuza pentru o valoare beta
>   calculeaza_difuza <- function(beta) {
+
+     # Transmisii atmosferice
+     tr <- tau_r(lambda, ma)
+     ta <- tau_a(lambda, alfa, beta, ma)
+
+     to <- tau_o(k_o, lO3, ma)
+     tw <- tau_w(k_w, ww, ma)
+     tg <- tau_g(k_g, ma)
+
+     # Iradianta extraterestra pe plan orizontal
+     E0h <- ej * G0n * cos(teta_z * pi / 180)
+
+     # Componenta difuza Rayleigh
+     Gdr <- E0h * to * tw * tg *
+       (0.5 * (1 - tr) * ta)
+
+     # Componenta difuza produsa de aerosoli
+     Gda <- E0h * to * tw * tg *
+       (Fc * ssa * (1 - ta) * tr)
+
+     # Albedo atmospheric  m=1.9 impus
+     tr19 <- tau_r(lambda, 1.9)
+     ta19 <- tau_a(lambda, alfa, beta, 1.9)
+
+     ro_a_prime <- tau_o(k_o, lO3, 1.9) *
+       tau_w(k_w, ww, 1.9) *
+       tau_g(k_g, 1.9) *
+       (0.5 * (1 - tr19) * ta19 +
+          (1 - Fc) * ssa * (1 - ta19) * tr19)
+
+     # Iradianta directa pe plan orizontal
+     beam <- E0h * to * tw * tg * tr * ta
+
+     # Iradianta difuza totala, incluzand reflexiile multiple
+     difuz <- (Gdr + Gda) * (1 / (1 - ro_g * ro_a_prime)) +
+       beam * (ro_g * ro_a_prime / (1 - ro_g * ro_a_prime))
+
+     return(difuz)
+   }

>   # Calcul pentru cele trei valori beta
>   beta_values <- c(0.05, 0.1, 0.2)
>   difuz_beta <- sapply(beta_values, calculeaza_difuza)
>   colnames(difuz_beta) <- paste0("beta = ", beta_values)

>   # Reprezentare grafica
>   matplot(lambda, difuz_beta,
+           type = "l",
+           lty = 1,
+           lwd = 2,
+           xlim = c(0.29,1.3),
+           col = c("blue", "red", "darkgreen"),
+           xlab = expression(lambda~"("*mu*m*")"),
+           ylab = expression(G[d]~"("*W/m^2/mu*m*")"),
+           main = "Spectral diffuse irradiance")

>   legend("topright",
+          legend = colnames(difuz_beta),
```

```
+          col = c("blue", "red", "darkgreen"),
+          lty = 1,
+          lwd = 2)
```

The effect of a variation in α is demonstrated in Fig. 12. A small average particle size represented by high values of α produces greater amounts of diffuse irradiance.

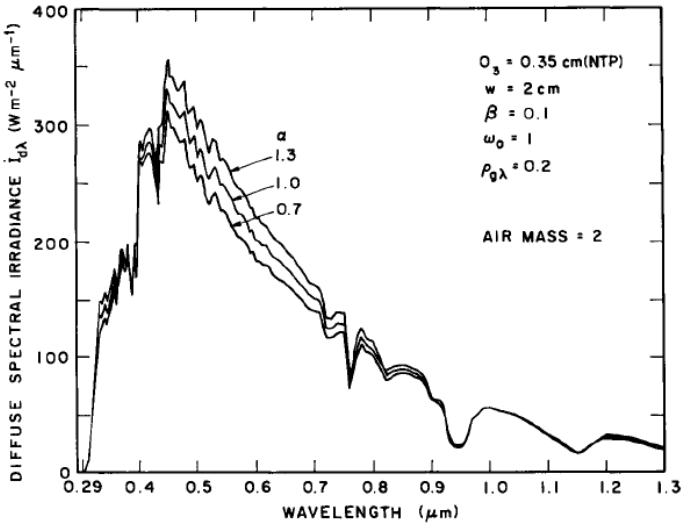


Figure 12. Diffuse spectral irradiance on a horizontal surface as a function of α.

The influence of ground albedo on spectral diffuse irradiance can be examined in Fig. 13. An increase in ground albedo from 0.2 to 0.7 substantially increases diffuse irradiance. However, this increase is not confined to the ultraviolet and the visible portion of the spectrum.

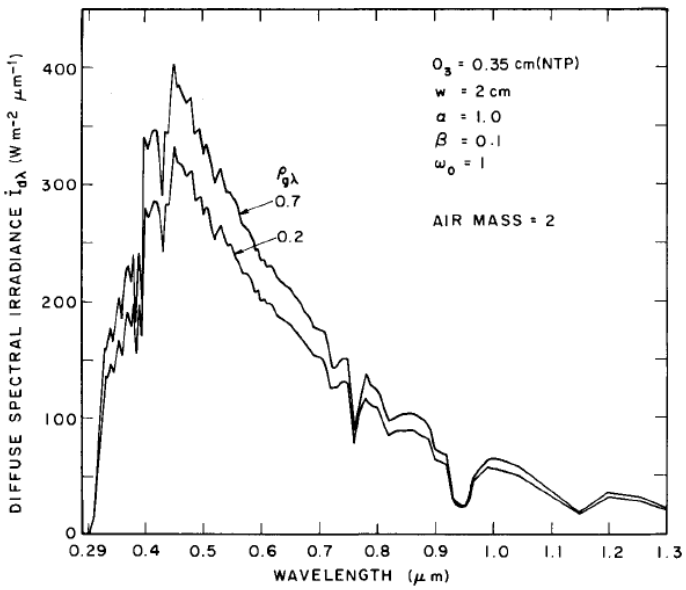


Figure 13. Diffuse spectral irradiance on a horizontal surface as a function of ground albedo.

In Fig. 14, a comparison between $G_{dr\lambda}$, $G_{da\lambda}$, and $G_{dm\lambda}$ is given. At $\beta = 0.1$ and $\rho_{g\lambda} = 0.2$, the multiply reflected component is the smallest and the aerosol-scattered component the largest in magnitude. The flux $G_{da\lambda}$ depends primarily on the magnitude of β, while $G_{dm\lambda}$ depends on both β and $\rho_{g\lambda}$. Naturally, at $\beta = \rho_{g\lambda} = 0$, $G_{da\lambda}$, and $G_{dm\lambda}$ both reduce to zero.

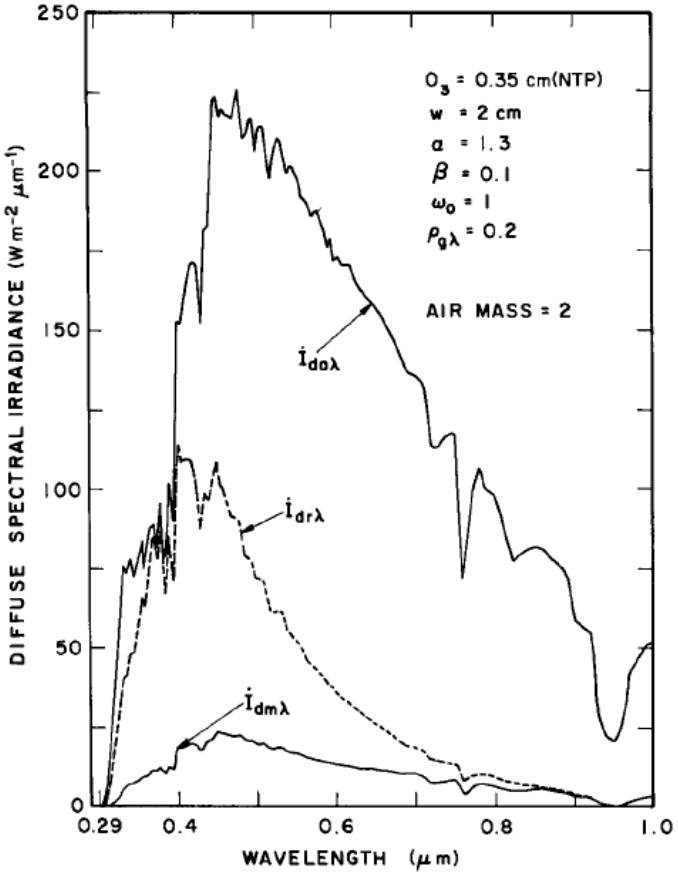


Figure 14. The various components of the spectral diffuse irradiance on a horizontal surface.

Global Spectral Irradiance on the Ground

Global irradiance is the sum of the beam plus diffuse irradiance on a horizontal surface. Let G_λ is the global spectral irradiance on a horizontal surface and

$$G_\lambda = G_{n\lambda} \cos\theta_z + G_{d\lambda} \tag{23}$$

$$G_\lambda = G_{n\lambda} \cos\theta_z + (G_{dr\lambda} + G_{da\lambda}) \frac{1}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} + G_{n\lambda} \cos\theta_z \frac{\rho_{g\lambda}\,\rho'_{a\lambda}}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} \tag{24}$$

$$G_\lambda = (G_{n\lambda} \cos\theta_z + G_{dr\lambda} + G_{da\lambda}) \frac{1}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} \tag{25}$$

The variation of global solar irradiance with β, α, and ground albedo is shown in Figs. 15-17. In the study of the direct normal radiation, we have observed that an increase in the magnitude of β decreases the amount of direct spectral irradiance. On the other hand, the amount of diffuse irradiance increases as β increases. However, the decrease in direct irradiance is far greater than the increase in diffuse irradiance. Consequently, global radiation decreases with an increase in β (Fig. 15). However, this change is not as drastic as in the case of direct or diffuse irradiance. Furthermore, the effect of a change in β on global radiation is minimal at $\theta_z = 0$, and this effect increases as the zenith angle increases.

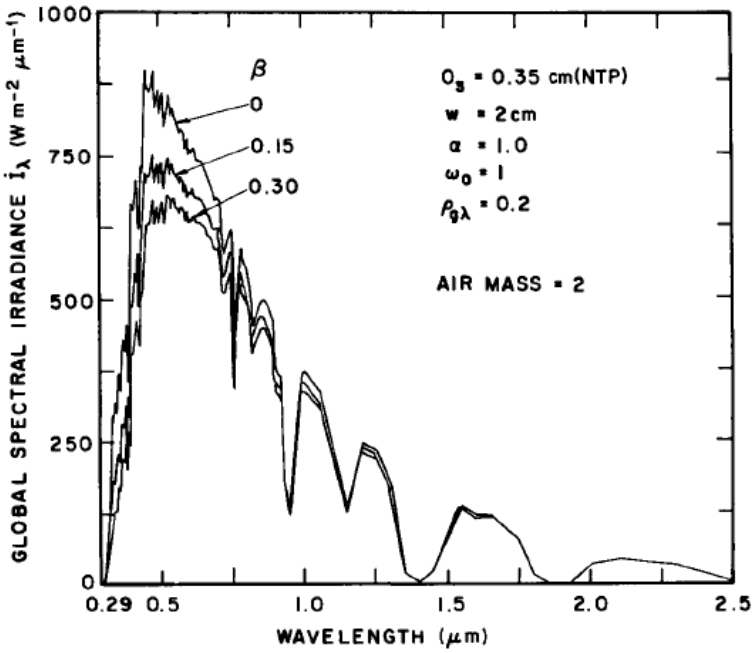


Figure 15. Global spectral irradiance on a horizontal surface as a function of β.

The effect of a variation in α is demonstrated through Fig. 16. Global radiation remains almost insensitive to changes in α. Small variations that do occur in global irradiance are confined to the ultraviolet and the visible portion of the spectrum. An increase in α results in slightly lower values of global irradiance.

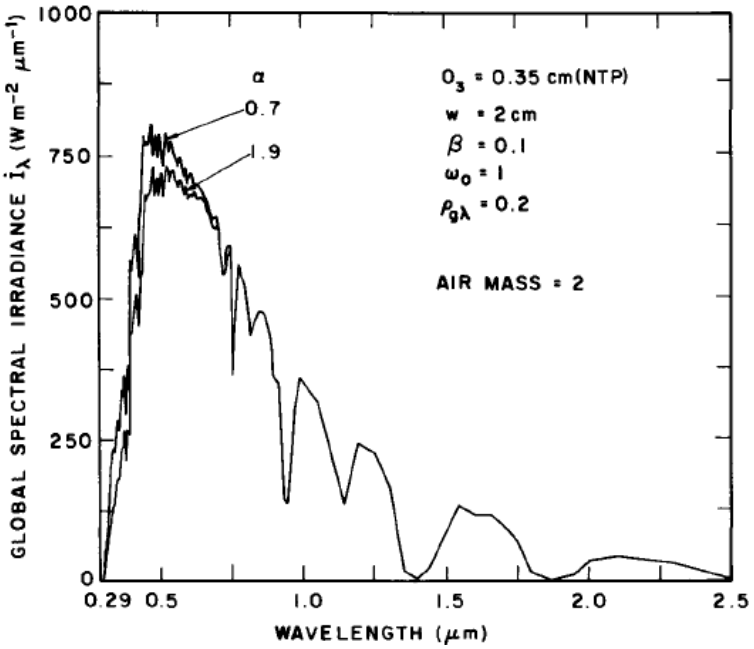


Figure 16. Global spectral irradiance on a horizontal surface as a function of α.

An increase in ground albedo increases the amount of global spectral irradiance because of the multiple reflection effects (Fig. 17). Again, small changes in global irradiance are confined mainly to $\lambda < 1\mu m$.

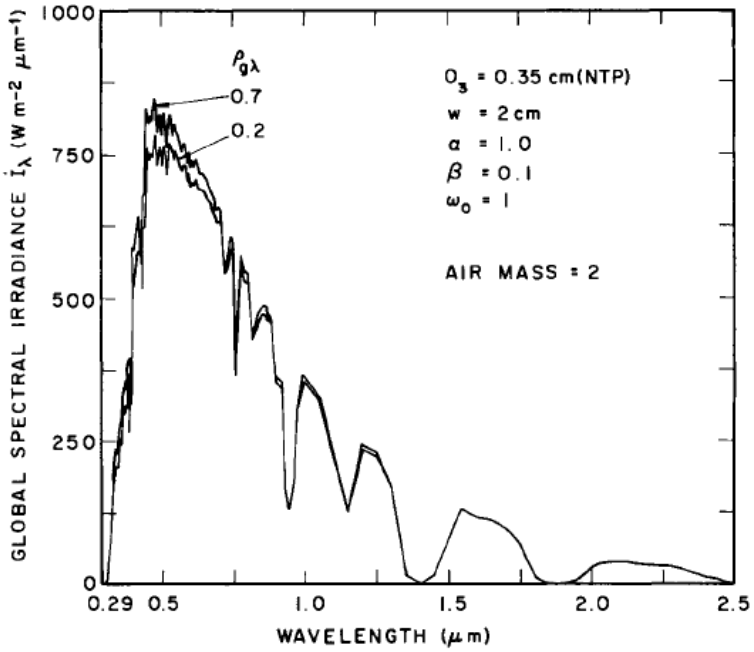


Figure 17. Global spectral irradiance on a horizontal surface as a function of ground albedo.

It is instructive to look at the relative magnitudes of diffuse and direct components of global radiation. In Fig. 18 we present these plots for a typical group of the atmospheric parameters. At wavelengths longer than $1\,\mu m$, global radiation is almost entirely composed of the direct component. At wavelengths shorter than $1\,\mu m$, the fractional contribution of diffuse radiation to global radiation depends on air mass, turbidity, and ground albedo. Other atmospheric parameters such as ozone and water vapor have negligible effects.

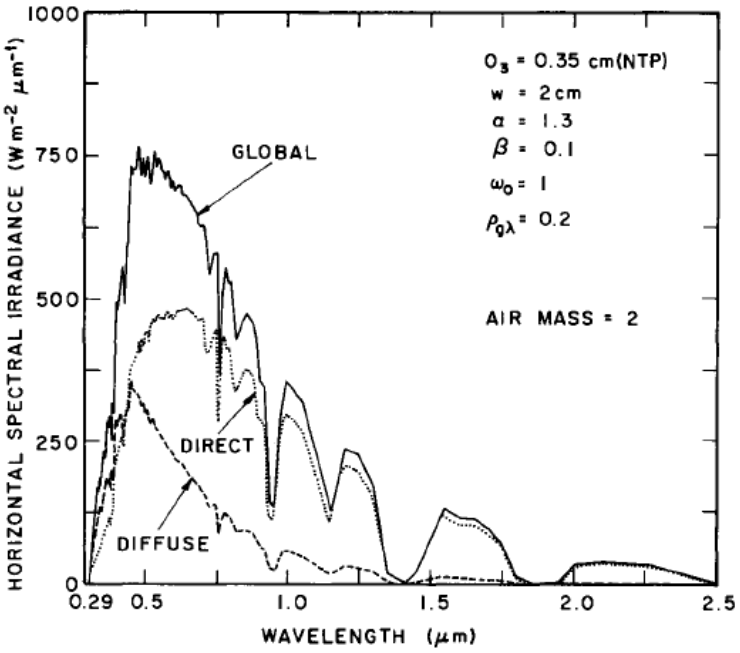


Figure 18. Global, direct and diffuse spectral irradiance on a horizontal surface.

The algorithm to calculate beam radiation is quite accurate. On the other hand, the formulation to compute diffuse radiation is approximate. Because (in most instances) beam radiation is far greater than diffuse, the global radiation values should be quite accurate.

EXAMPLE. The spectral diffuse and the global irradiance, under standard sea-level conditions, on a horizontal surface calculated at (a) $\lambda = 0.7\,\mu m$ and (b) $\lambda = 2.0\,\mu m$ on 15 June, at a location where the skies are cloudless and the sun is at 60° from its zenith position, for $\beta = 0.1$, $\alpha = 1.3$, $l_{O3} = 0.3\,cm$, w = 2 cm, and ground albedo $\rho_{g\lambda} = 0.2$, assuming $\omega_0 = 0.95$.

$$G_{d\lambda} = (G_{dr\lambda} + G_{da\lambda})\frac{1}{1-\rho_{g\lambda}\,\rho'_{a\lambda}} + G_{n\lambda}\cos\theta_z\,\frac{\rho_{g\lambda}\,\rho'_{a\lambda}}{1-\rho_{g\lambda}\,\rho'_{a\lambda}}$$

$$G_\lambda = (G_{n\lambda}\cos\theta_z + G_{dr\lambda} + G_{da\lambda})\frac{1}{1-\rho_{g\lambda}\rho'_{a\lambda}}$$

$$\theta_z = 60^\circ \rightarrow m = 2$$

$$\varepsilon_j = 0.968$$

a) $\lambda = 0.7\ \mu m$

$G_{0n\lambda} = 1427.5\frac{W}{m^2\mu m}$

$ko = 0.023$

$kg = 0$

$kw = 0.024$

$$G_{dr\lambda} = G_{0n\lambda}\cos\theta_z\ \tau_{ma\lambda}[0.5(1-\tau_{r\lambda})\tau_{a\lambda}]$$

$$G_{dr\lambda} = \varepsilon_j G_{0n\lambda}\cos\theta_z\ \tau_{o\lambda}\tau_{w\lambda}\tau_{g\lambda}[0.5(1-\tau_{r\lambda})\tau_{a\lambda}]$$

$$G_{dr\lambda} = 17.638\ Wm^{-2}\mu m^{-1}$$

$\theta_z = 60^\circ \rightarrow F_c = 0.78$ din table cu date Robinson

$G_{da\lambda} = G_{0n\lambda}\cos\theta_z\,\tau_{ma\lambda}[F_c\omega_0(1-\tau_{a\lambda})\tau_{r\lambda}]$ $F_c = 0.78$ din table 6.17.1

$G_{da\lambda} = G_{0n\lambda}\cos\theta_z\,\tau_{o\lambda}\tau_{w\lambda}\tau_{g\lambda}[F_c\omega_0(1-\tau_{a\lambda})\tau_{r\lambda}]$

$$G_{da\lambda} = 125.875\ Wm^{-2}\mu m^{-1}$$

To obtain the multiply reflected component of the diffuse irradiance, we should first obtain the albedo of the atmosphere

$$\rho_{a\lambda}' = \tau_{ma\lambda}'[0.5(1-\tau_{r\lambda}')\tau_{a\lambda}' + (1-F_c')\omega_0(1-\tau_{a\lambda}')\tau_{r\lambda}']$$

The primes over various quantities indicate that they are to be evaluated at $m = 1.9$, $\theta_z \approx 60^\circ$.

$\rho'_{a\lambda} = 0.074$

$$G_{d\lambda} = (G_{dr\lambda} + G_{da\lambda})\frac{1}{1-\rho_{g\lambda}\rho'_{a\lambda}} + G_{n\lambda}\cos\theta_z\ \frac{\rho_{g\lambda}\rho'_{a\lambda}}{1-\rho_{g\lambda}\rho'_{a\lambda}}$$

$G_{n\lambda}\cos\theta_z = 453.77\ Wm^{-2}\mu m^{-1}$

$$G_{d\lambda} = 152.50\ Wm^{-2}\mu m^{-1}$$

$$G_\lambda = (G_{n\lambda}\cos\theta_z + G_{dr\lambda} + G_{da\lambda})\frac{1}{1-\rho_{g\lambda}\rho'_{a\lambda}}$$

$$G_\lambda = 606.28\, Wm^{-2}\mu m^{-1}$$

(b) $\lambda = 2.0\ \mu m$

$$G_{0n\lambda} = 118.5\frac{W}{m^2\mu m}$$

ko=0

$kg = 2.9$

$kw = 3.0$

$$G_{dr\lambda} = 0.0139\, Wm^{-2}\mu m^{-1}$$

$$G_{da\lambda} = 1.687\, Wm^{-2}\mu m^{-1}$$

$\rho'_{a\lambda} = 0.008$

$G_{n\lambda}\cos\theta_z = 26.91\, Wm^{-2}\mu m^{-1}$

$$G_{d\lambda} = 1.749\, Wm^{-2}\mu m^{-1}$$

$$G_\lambda = 28.664\, Wm^{-2}\mu m^{-1}$$

```
J<-166  ##15 iunie
>   teta_z<-60
>   ma<-1/cos(teta_z*pi/180)
>   ma
[1] 2
>   alfa<-1.3
>   beta<-0.1
>   ww<-2
>   lO3<-0.3
>   ro_g<-0.2
>   ssa<-0.95
>   Fc<-0.78

>   Corectie_Spencer_excentr<-function(x)
+     return<-(1.00011+0.034221*cos(2*pi*(x-1)/365)+0.00128*sin(2*pi*(x-1)/36
5)+0.000719*cos(2*2*pi*(x-1)/365)+0.000077*sin(2*2*pi*(x-1)/365))
>   ej<-Corectie_Spencer_excentr(J)
>   ej
[1] 0.9683586

>   lw<-0.7
>   G0n_lw<-1427.5  ## w/m2 micro m
>   k_o<-0.023
>   k_g<-0
>   k_w<-0.024
```

```
>   tau_o<-function(k,l,m)
+     return<-exp(-k*l*m)
>   tau_w<-function(k,w,m)
+     return<-exp((-0.2385*k*w*m)/(1+20.07*k*w*m)^0.45)
>   tau_g<-function(k,m)
+     return<-exp((-1.41*k*m)/(1+118.3*k*m)^0.45)
>   tau_r<-function(x,m)          ## x lungime de unda
+     return<-exp(-0.008735*m*x^(-4.08))
>   tau_a<-function(x,a,b,m)    ## x lungime de unda
+     return<-exp(-m*b*x^(-a))

>   Gdr_lw <- ej*G0n_lw*cos(teta_z*pi/180)*tau_o(k_o,lO3,ma)*tau_w(k_w,ww,ma)
*tau_g(k_g,ma)*(0.5*(1-tau_r(lw,ma))*tau_a(lw,alfa,beta,ma))
>   Gdr_lw
[1] 17.63865
>   Gda_lw<-ej*G0n_lw*cos(teta_z*pi/180)*tau_o(k_o,lO3,ma)*tau_w(k_w,ww,ma)*t
au_g(k_g,ma)*(Fc*ssa*(1-tau_a(lw,alfa,beta,ma))*tau_r(lw,ma))
>   Gda_lw
[1] 125.8756
>   ro_a_prime<-tau_o(k_o,lO3,1.9)*tau_w(k_w,ww,1.9)*tau_g(k_g,1.9)*(0.5*(1-t
au_r(lw,1.9))*tau_a(lw,alfa,beta,1.9)+(1-Fc)*ssa*(1-tau_a(lw,alfa,beta,1.9))*
tau_r(lw,1.9))
>   ro_a_prime
[1] 0.0741193
>   beam<-ej*G0n_lw*cos(teta_z*pi/180)*tau_o(k_o,lO3,ma)*tau_w(k_w,ww,ma)*tau
_g(k_g,ma)*tau_r(lw,ma)*tau_a(lw,alfa,beta,ma)
>   beam
[1] 453.7783
>   difuz<-(Gdr_lw+Gda_lw)*(1/(1-ro_g*ro_a_prime))+beam*(ro_g*ro_a_prime/(1-r
o_g*ro_a_prime))
>   difuz
[1] 152.5017
>   global<-(beam+Gdr_lw+Gda_lw)*(1/(1-ro_g*ro_a_prime))
>   global
[1] 606.28


###b) lambda 2 micro m
>   lw<-2.0
>   G0n_lw<-118.5  ## W/m2 micro m
>   k_o<-0.0
>   k_g<-2.9
>   k_w<-3.0

>   Gdr_lw <- ej*G0n_lw*cos(teta_z*pi/180)*tau_o(k_o,lO3,ma)*tau_w(k_w,ww,ma)
*tau_g(k_g,ma)*(0.5*(1-tau_r(lw,ma))*tau_a(lw,alfa,beta,ma))
>   Gdr_lw
[1] 0.01390891
>   Gda_lw<-ej*G0n_lw*cos(teta_z*pi/180)*tau_o(k_o,lO3,ma)*tau_w(k_w,ww,ma)*t
au_g(k_g,ma)*(Fc*ssa*(1-tau_a(lw,alfa,beta,ma))*tau_r(lw,ma))
>   Gda_lw
[1] 1.68762
>   ro_a_prime<-tau_o(k_o,lO3,1.9)*tau_w(k_w,ww,1.9)*tau_g(k_g,1.9)*(0.5*(1-t
au_r(lw,1.9))*tau_a(lw,alfa,beta,1.9)+(1-Fc)*ssa*(1-tau_a(lw,alfa,beta,1.9))*
tau_r(lw,1.9))
>   ro_a_prime
[1] 0.00828313
>   beam<-ej*G0n_lw*cos(teta_z*pi/180)*tau_o(k_o,lO3,ma)*tau_w(k_w,ww,ma)*tau
_g(k_g,ma)*tau_r(lw,ma)*tau_a(lw,alfa,beta,ma)
>   beam
[1] 26.91585
>   difuz<-(Gdr_lw+Gda_lw)*(1/(1-ro_g*ro_a_prime))+beam*(ro_g*ro_a_prime/(1-r
o_g*ro_a_prime))
>   difuz
```

```
[1] 1.749016
>   global<-(beam+Gdr_lw+Gda_lw)*(1/(1-ro_g*ro_a_prime))
>   global
[1] 28.66487
```