

Relative Optical Mass m

When monochromatic radiation passes through the atmosphere, its intensity decreases due to absorption and scattering by atmospheric molecules and aerosol particles. The degree of attenuation depends on the type and concentration of these atmospheric constituents, as well as on the distance travelled by the radiation through the atmosphere. This distance is known as the path length or slant path.

The density multiplied by the path length represents the ***mass of a substance in a column of unit*** cross section; this is also called the ***optical mass***. The actual optical mass can be written

$$m_{act} = \int_0^\infty \rho \, ds \quad (1)$$

where ds is the geometrical path length of the light ray from the sun and ρ is the density of the substance at ds . The integration is along a path, s , (called the slant path) traversed by the beam of radiation from the upper limits of the atmosphere to the ground (or to a surface at a certain height). Since refraction is wavelength dependent, the slant path varies with wavelength, and consequently Eq. (1) applies to monochromatic radiation. When the sun is at its zenith, the light path goes straight downward and ds equals the height of an element, dz , where z is the distance along the vertical direction. Thus, the actual optical mass in the vertical direction is as follows:

$$m_{act,v} = \int_0^\infty \rho \, dz \quad (2)$$

This is the mass of a substance in a vertical column of unit cross section.

The ***relative optical mass*** m_r is defined as the ratio of the optical path along the oblique trajectory to the vertical path in the zenith direction. Thus

$$m_r = \frac{\int_0^\infty \rho \, ds}{\int_0^\infty \rho \, dz} \quad (3)$$

Equation (3) should be solved separately for each one of the attenuating components of the atmosphere (water vapor, aerosols, etc).

Ignoring the earth's curvature and assuming that the atmosphere is nonrefractive and completely homogeneous (Fig. 5a), the relative optical mass applied to all the atmospheric constituents is

$$m_r \approx \frac{1}{\cos \theta_z} \quad (4)$$

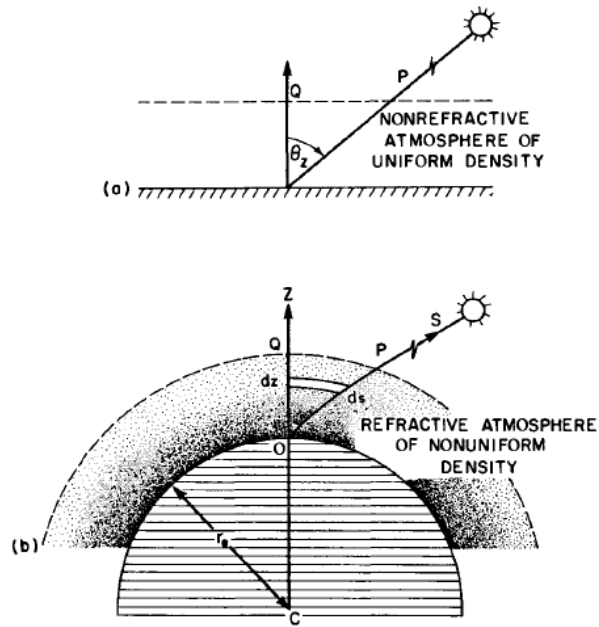
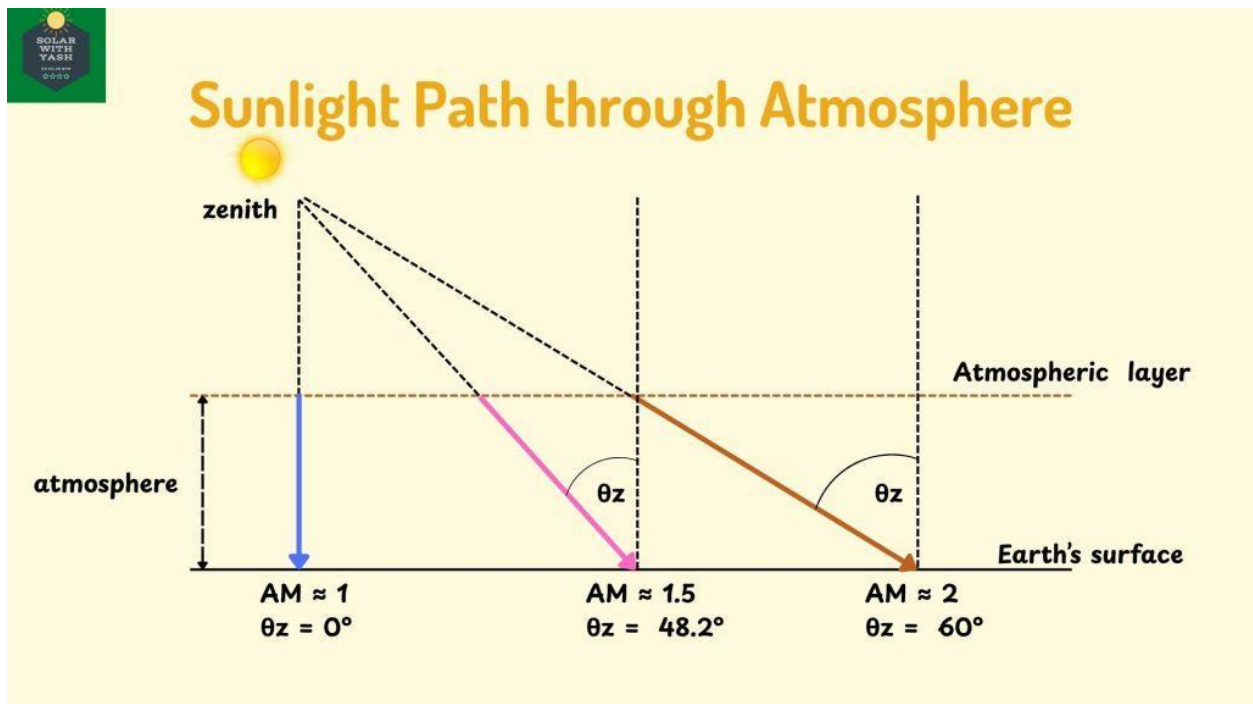


Figure 5. The trajectory of a solar ray through the earth's atmosphere (a) Nonrefractive plane parallel atmosphere of uniform density (b) Refractive spherical atmosphere of variable density.



The error in this equation, because the earth's curvature and the refraction of the real atmosphere have been neglected, is 0.25% at $\theta_z = 60^\circ$, and increases to 10% at $\theta_z = 85^\circ$.

However, density is variable with height. Furthermore, because of the curvature of the earth and refraction of the atmosphere, the slant path of the beam radiation will follow the path OP (Fig. 5b). Therefore Eq. (3) must be evaluated through integration along the slant path and the zenith direction. Relative optical mass is obviously a function of the distribution, with height, of atmospheric density and refractive index: the relative mass from a mountain location will be different from that recorded at sea level.

The atmosphere can be considered as a mixture of dry air, water vapor, and aerosols, each contributing differently to the attenuation of solar radiation. Accordingly, the optical air mass is evaluated separately for these atmospheric constituents. We begin with dry air, while ozone is treated separately because its concentration varies with altitude in a distinct manner.

Relative Optical Air Mass m_a

To solve Eq. (3), the density variation of an actual atmosphere is required. However, this information is generally available only for a Standard Atmosphere. Kasten [2], solved the above equation.

F. Kasten, A new table and approximate formula for relative optical air mass. *Arch. Meteorol. Geophys. Bioklimatol. Ser. B* 14, 206-223 (1966).

$$m_r = [\cos \theta_z + 0.15(93.885 - \theta_z)^{-1.253}]^{-1} \quad (5)$$

For ground-level measurements, this formula is accurate to better than 0.1% for zenith angles up to 86° . The greatest deviation, 1.25%, occurs at $\theta_z = 89.5^\circ$. The error is slightly greater for measurements at very high altitudes.

Equation (5) is applicable to a standard pressure of 1013.25 mbars at sea level; for other pressures, it should be modified. Although this task is very complicated, it is common to employ the following approximation to obtain the relative optical air mass for local conditions:

$$m_a = m_r(p/1013.25) \quad (6)$$

where p is local pressure in millibars.

In general, pressure correction must be applied to stations at 2000 m or higher. Even at stations not far above sea level, pressure correction is necessary if the difference between the standard and local pressure is more than 20 mbars. The pressure above sea level may be obtained from

$$\frac{p}{p_0} = \exp(-0.0001184 z) \quad (7)$$

Relative Optical Water-Vapor Mass m_w

Equation (3) holds here as well; however, the density term in this equation now refers to the density of water vapor. The vertical water-vapor density profile is very different from that of dry air. Furthermore, since water vapor is concentrated mainly in the lower layers of the atmosphere, the limits of integration in this equation need not go to upper limits of the atmosphere. Kasten, based on a study by Schnaidt, has developed the following formula:

$$m_w = [\cos \theta_z + 0.0548(92.650 - \theta_z)^{-1.452}]^{-1} \quad (8)$$

The uncertainty involved in accurate determination of w is much greater than that in determination of its relative optical mass. Therefore, it is common practice in literature to use m_r , Eq. (5), instead of above Eq 8. or $m_r \approx \frac{1}{\cos \theta_z}$.

Values of Optical Masses for Air, Water Vapor, and Ozone

Zenith angle θ_z (deg)	Optical mass			
	$m'_r = \sec \theta_z$ Eq. (5.6.4)	Air	Water vapor	Ozone
		m_r (Kasten) Eq. (5.7.2)	m_w (Schnaidt) Eq. (5.8.1)	m_o (Robinson) Eq. (5.9.1)
0	1.00	1.00	1.00	1.00
30	1.15	1.15	1.15	1.15
60	2.00	1.99	2.00	1.99
65	2.37	2.35	2.36	2.34
70	2.92	2.90	2.92	2.86
75	3.86	3.81	3.85	3.70
80	5.76	5.58	5.71	5.23
85	11.47	10.32	11.11	8.36
86	14.34	12.34	13.65	9.28
87	19.11	15.22	17.61	10.25
88	28.65	19.54	24.52	11.17
89	57.30	26.31	38.74	11.86
90	∞	36.51	75.12	12.12

Relative Optical Ozone Mass m_o

The concentration profile of ozone in the atmosphere is very different from the profiles of dry air or water vapor. Let us assume that all ozone is concentrated in a thin layer centered at a height z_3 . Robinson, through simple geometry, has shown the following:

$$m_o = \frac{1 + z_3/r_e}{[\cos^2 \theta_z + 2 z_3/r_e]^{1/2}} \quad (9)$$

r_e is the mean earth radius (6370 km).

Table also lists the optical mass for ozone. At $\theta_z = 80^\circ$, m_o is substantially different from the other optical masses. In this table, ozone is assumed to be concentrated at a height of 22 km.

Again, the uncertainty involved in the accurate determination of ozone layer thickness is much greater than that in determination of its relative optical mass. Therefore, use of above Eq. 9 is rare. In this text we shall replace m_o by m_r and use Eq. (5). Note, however, that m_w and m_o ***should not be further corrected*** for any variation of local pressure from the standard pressure.

Example: Optical masses (a) m_a , (b) m_w , (c) m_o calculated when the sun is 15° above the horizon at a station located 2000 m above sea level, assuming that ozone is concentrated at 20 km above sea level. The values obtained are compared with $m_r \approx \frac{1}{\cos \theta_z}$.

Zenith angle $\theta_z = 75^\circ$

```
z<-2000
> teta_z<- 75
> ## A)
```

$$p = p_0 \exp(-0.0001184 z)$$

```
> pres<-function(x)
+   return(1013.25*exp(-0.0001184*x))
> p<-pres(z)
> p      ##in mbars
[1] 799.6053
```

Kasten: $m_r = [\cos \theta_z + 0.15(93.885 - \theta_z)^{-1.253}]^{-1}$

```
> m_r<-function(x)
+   return<-(cos(x*pi/180)+0.15*(93.885-x)^(-1.253))^(-1)
> mr<-m_r(teta_z)
> mr
[1] 3.808134
```

$$m_a = m_r(p/1013.25)$$

```
> m_a<-m_r(teta_z)*(p/1013.25)
> m_a
[1] 3.005186
```

$$m_w = [\cos \theta_z + 0.0548(92.650 - \theta_z)^{-1.452}]^{-1}$$

```
> m_w<-function(x)
+   return<-(cos(x*pi/180)+0.0548*(92.65-x)^(-1.452))^(-1)
> mw<-m_w(teta_z)
> mw
[1] 3.851082
```

```
> ##C)
> z3<- 20
> re<-6370
```

$$m_o = \frac{1 + z_3/r_e}{[\cos^2\theta_z + 2 z_3/r_e]^{1/2}}$$

```
> m_o<-function(x)
+   return<-(1+z3/re)/((cos(x*pi/180))^2+2*z3/re)^0.5
> mo<-m_o(teta_z)
> mo
[1] 3.706022
```

$$m_r \approx \frac{1}{\cos \theta_z}$$

```
> mr_aprox<-1/cos(teta_z*pi/180)
> mr_aprox
[1] 3.863703
```

Relative Optical Aerosol Mass m_a

Aerosols are the most uncertain parameters in calculating solar radiation on the ground. They are highly variable in size, distribution, composition, and optical properties.

Optical mass depends on the extinction process because each process is mostly active over a specific altitude range, in general. For processes whose majority of constituents are concentrated at high altitude, like ozone, optical mass tends to be low. Vice versa, the optical mass is high when the constituents are concentrated near the surface, like aerosols or water vapors. The cases of air molecules are intermediate, since they are dispersed over the whole atmosphere. A general equation can be used to obtain the optical mass of each process (Guemard, 1995) is:

$$m_i = [\cos \theta_z + a_{i1}\theta_z^{a_{i2}}(a_{i3} - \theta_z)^{a_{i4}}]^{-1} \quad (10)$$

The a_{ij} coefficients are provided in table [3].

Extinction process	a_{i1}	a_{i2}	a_{i3}	a_{i4}	$m_i @ Z=90^\circ$
Rayleigh	4.5665E-1	0.07	96.4836	-1.6970	38.136
Ozone	2.6845E+2	0.5	115.420	-3.2922	16.601
Nitrogen dioxide [†]	6.0230E+2	0.5	117.960	-3.4536	17.331
Mixed gases	4.5665E-1	0.07	96.4836	-1.6970	38.136
Water vapor	3.1141E-2	0.1	92.4710	-1.3814	71.443
Aerosols	3.1141E-2	0.1	92.4710	-1.3814	71.443

[†] For stratospheric NO₂ only; use the water vapor mass for tropospheric NO₂ and a weighted average for a combination of the two (see Section 4.2.3).

Chapter 6 ***SOLAR SPECTRAL RADIATION UNDER CLOUDLESS SKIES***

This chapter introduces the fundamental processes of atmospheric absorption and scattering that govern the attenuation of solar radiation as it passes through the atmosphere. Simplified formulations are presented to describe the effects of the main atmospheric constituents on direct solar radiation. An algorithm is then developed to calculate the direct and diffuse spectral irradiance reaching the Earth's surface. The formulation allows key atmospheric parameters including ozone content, precipitable water vapor, atmospheric turbidity, and surface albedo to be varied independently, facilitating the investigation of their individual effects on solar radiation.

6.1 Attenuation of Direct Solar Radiation: Basic Formulation

Solar radiation is attenuated as it travels through the Earth's atmosphere by two main processes: absorption and scattering. Absorption transfers energy to the atmosphere in the form of heat, whereas scattering changes the direction of propagation of solar radiation, producing diffuse radiation. Consequently, the solar radiation reaching the Earth's surface consists of two components: direct (beam) radiation, which arrives directly from the Sun, and diffuse radiation, which results from scattering within the atmosphere.

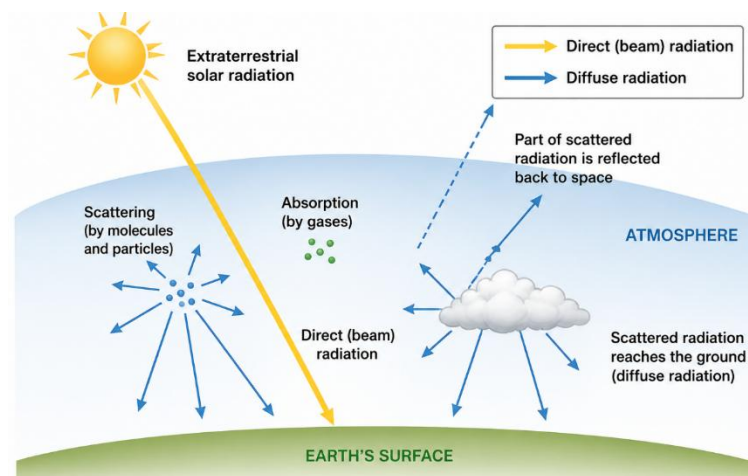


Figure 1. The main components of solar radiation

To estimate the solar radiation available at the Earth's surface, it is necessary to understand how the atmosphere attenuates solar radiation through absorption and scattering.

Atmospheric transmittance

Consider a beam of parallel monochromatic radiation $G_{0n\lambda}$ entering a homogeneous medium, as in Figure 2.

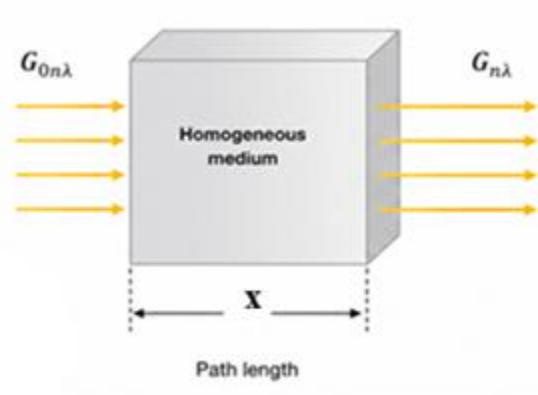


Figure 2. Attenuation of a parallel monochromatic beam

$G_{n\lambda}$ is the flux emerging from the medium after traversing a distance x . (The subscript n indicates that the flux is measured normally to the rays.)

According to Lambert's law, the attenuation of light is proportional to the distance traversed in the medium:

$$G_{n\lambda} = G_{0n\lambda} \exp(-\alpha_{\lambda} x) \quad (1)$$

Where,

α_{λ} is the monochromatic extinction or attenuation coefficient,

x is the path length

This simple law applies whether the attenuation is caused by absorption or scattering.



Lambert's law applies to **monochromatic radiation** only.

Now consider the homogeneous medium in Figure 2 to be the Earth's atmosphere. The incident monochromatic irradiance, $G_{0n\lambda}$, represents the extraterrestrial solar irradiance at the mean Sun–Earth distance, and z the distance traversed by the ray through the atmosphere.

Applying Lambert's law gives

$$G_{n\lambda} = G_{0n\lambda} \exp(-\alpha_{\lambda} z) \quad (2)$$

If H is the path length when Sun is at the zenith, substituting this quantity into Lambert's law gives

$$G_{n\lambda} = G_{0n\lambda} \exp\left(-\alpha_\lambda H \frac{z}{H}\right) \quad (3)$$

We then define

$\alpha_\lambda H = k_\lambda$ the vertical optical depth

$\frac{z}{H} = \frac{1}{\cos \theta_z} = m$ the relative optical air mass

Remarks:

This is a simplification that assumes a plane-parallel atmosphere. The approximation is valid if the solar zenith angle is not too large.

An air mass of 1 (usually noted AM1) denotes an overhead sun ($\theta_z = 0^\circ$).

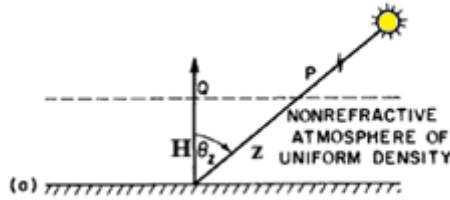


Figure 3. The trajectory of a solar ray through the earth's atmosphere for a nonrefractive plane parallel atmosphere of uniform density

Finally,

$$G_{n\lambda} = G_{0n\lambda} \exp(-k_\lambda m) \quad (4)$$

This form of Lambert's law is commonly used in radiative transfer.

As the radiation propagates through the atmosphere, it is attenuated by several independent processes, including molecular scattering, aerosol extinction, ozone absorption, water vapor absorption and absorption by uniformly mixed gases. Since these extinction processes act *independently*, the total slant optical depth of the atmosphere is equal to the sum of the optical depth associated with each individual process:

$$k_\lambda m = \sum_{i=1}^5 k_{i\lambda} m_i \quad (5)$$

$k_{i\lambda}$ is the monochromatic optical depth corresponding to atmospheric attenuation process i , with $i = o$ for ozone absorption, $i = r$ for Rayleigh scattering, $i = g$ for absorption by uniformly mixed gases, $i = w$ for water vapor absorption, and $i = a$ for aerosol extinction.

m_i is the optical air mass of process i .

The transmittance of the atmosphere is the ratio of radiation emerging from a medium to incident radiation. The monochromatic transmittance can be written as a product of five specific transmittances:

$$\tau_\lambda = \frac{G_{n\lambda}}{G_{0n\lambda}} = e^{-k_\lambda m} = e^{-\sum_{i=1}^5 k_{i\lambda} m_i} = \prod_{i=1}^5 e^{-k_{i\lambda} m_i} = \prod_{i=1}^5 \tau_{i\lambda} \quad (6)$$

where $\tau_{i\lambda}$ is the transmittance is due to a single process i and τ_λ is due to all the processes combined.

The transmittance due to a single process is given by the following:

$$\tau_{i\lambda} = \exp(-k_{i\lambda} m_i) \quad (7)$$

Lambert's law accurately represents attenuation processes of scattering and absorption by atmospheric gases and aerosols.

Direct irradiance

The direct spectral irradiance at the Earth's surface can be obtained by applying an exponential attenuation factor associated with each atmospheric extinction process to the extraterrestrial spectral irradiance. If the attenuation mechanisms act independently, the direct spectral irradiance is expressed as

$$G_{n\lambda} = G_{0n\lambda} \prod_{i=1}^5 \tau_{i\lambda} = G_{0n\lambda} \tau_{o\lambda} \tau_{r\lambda} \tau_{g\lambda} \tau_{w\lambda} \tau_{a\lambda} \quad (8)$$

The total broadband (summed over all wavelengths) direct normal irradiance on the ground can now be written as follows:

$$G_n = \int_0^\infty G_{n\lambda} d\lambda \quad (9)$$

The problem of computing direct normal irradiance on the ground reduces the problem of determining the atmospheric specific transmittances $\tau_{i\lambda}$.

Scattering of Direct Solar Radiation

When an electromagnetic wave strikes a particle, part of the incident energy is scattered in different directions. This scattered energy is referred to as **diffuse radiation**. If the particle is spherical and much smaller than the wavelength of the incident radiation, the scattering is of the **Rayleigh** type. When the particle size is comparable to the wavelength of the incident radiation, the scattering is of the **Mie** type.

Figure 4 illustrates the differences between Rayleigh and Mie scattering. In Rayleigh scattering (Fig. 4a top), the scattered intensity is symmetric in the forward and backward directions. The scattering is greatest in the forward and backward directions and reaches a minimum at 90° to the direction of the incident radiation. Moreover, Rayleigh scattering is more pronounced at shorter wavelengths. In Mie scattering (Fig. 4b bottom), a larger fraction of the scattered energy is directed forward than backward. As the particle size increases, the forward scattering becomes increasingly dominant, and the angular distribution of the scattered radiation becomes more strongly peaked in the forward direction.

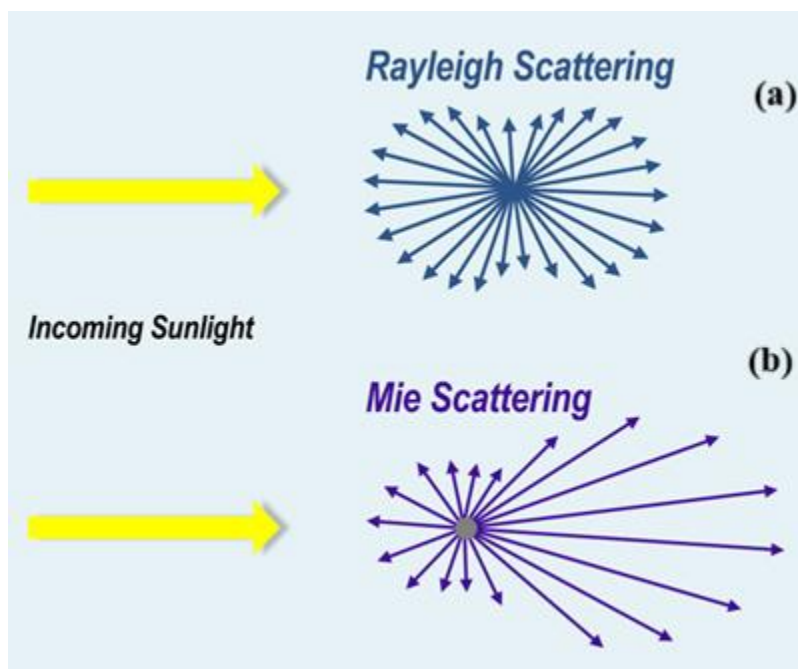


Figure 4. Schematic illustration of (a) Rayleigh scattering by particles much smaller than the wavelength of incident radiation and (b) Mie scattering by particles with sizes comparable to the wavelength.

In a clean dry atmosphere, about half of the energy thus scattered goes back into space and the other half reaches the ground as scattered radiation. In an atmosphere containing dust particles, more scattered energy reaches the ground because of greater forward scattering.

A. Rayleigh Scattering of Air Molecules

Scattering by air molecules is described by Rayleigh's theory, which assumes that the scattering particles are spherical, have diameters smaller than approximately 0.2 times the wavelength of the incident radiation, and scatter independently of one another.

A key result of Rayleigh's theory is that the monochromatic optical depth coefficient $k_{r\lambda}$, varies approximately as λ^{-4} , a dependence that has been confirmed experimentally. Based on Penndorf's theoretical formulation of molecular scattering, Leckner proposed the following approximation for the Rayleigh scattering optical depth [4]:

$$k_{r\lambda} = 0.008735 \lambda^{-4.08} \quad (10)$$

Accordingly, the spectral transmittance due to Rayleigh scattering can be written as a Lambert law

$$\tau_{r\lambda} = \exp(-0.008735 \lambda^{-4.08} m) \quad (11)$$

Figure 5 illustrates the dependence of Rayleigh transmittance on wavelength and optical air mass. The spectral transmittance increases rapidly with wavelength and decreases with increasing optical air mass. At wavelengths greater than approximately 1 μm , molecular scattering becomes negligible. It can be concluded that Rayleigh scattering primarily affects the short-wave portion of the solar spectrum.

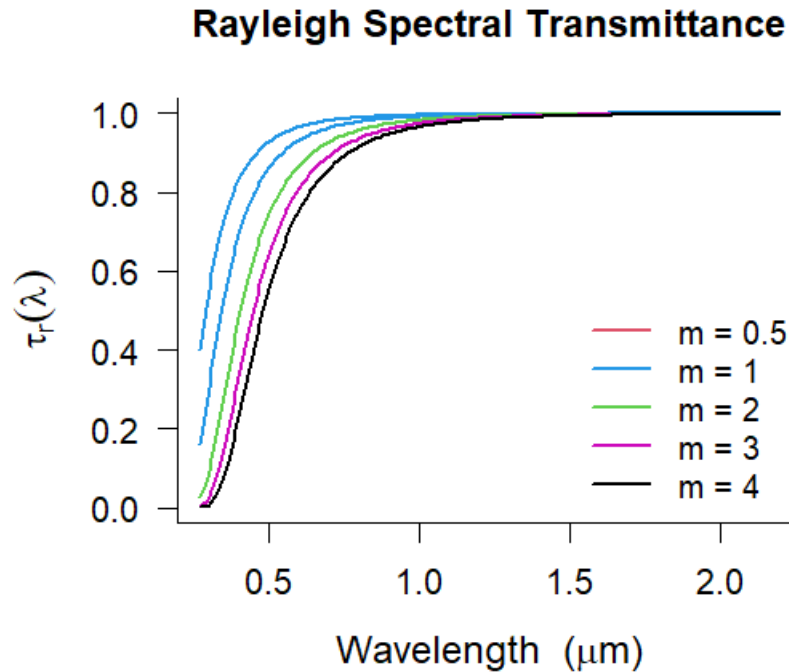


Figure 5. Rayleigh transmittance

```

tau_r<-function(x,m)
+   return<-exp(-0.008735*m*x^(-4.08))

> wave<-seq(0.27,2.2,0.01)

> m_values <- c(0.5, 1, 2, 3, 4)

> # Culori distincte pentru cele cinci curbe
> colors <- c(2, 4, 3, 6, 1)

> par(mar = c(5, 7, 4, 2)) # (jos, stânga, sus, dreapta)
> plot(
+   wave, tau_r(wave, m_values[1]),
+   type = "l", # linie continuă
+   lwd = 2, # grosimea liniei
+   col = 4, # culoare caldă, solară
+   xlab = expression("Wavelength ("*mu*"m)"), # axa X: lungimea de un
dă în micrometri
+   ylab = expression(tau[r](lambda)),
+   main = "Rayleigh Spectral Transmittance",
+
+   cex.lab = 1.2, # dimensiune etichete axe
+   cex.axis = 1.1, # dimensiune marcaje
+   las = 1, # cifrele pe axa Y orizontale
+   bty = "l", # ramă tip L (stânga și jos)
+   ylim =c(0,1)
+ )

> # Adăugăm celelalte curbe
> for (i in 2:length(m_values)) {
+   lines(
+     wave, tau_r(wave, m_values[i]),
+     col = colors[i],
+     lwd = 2
+   )
+ }

> # Legendă
> legend(
+   "bottomright",
+   legend = paste("m =", m_values),
+   col = colors,
+   lwd = 2,
+   cex = 1.0,
+   bty = "n"
+ )

```

The strong wavelength dependence of Rayleigh scattering also explains several familiar atmospheric optical phenomena. Since blue light (short wavelengths) is scattered much more efficiently than red light (long wavelengths), the daytime sky appears blue. At sunrise and sunset, sunlight travels through a much longer slant atmospheric path, causing most of the blue light to be scattered out of the direct beam. Consequently, the transmitted sunlight reaching the observer is enriched in longer wavelengths, producing the characteristic red and orange colors of sunrise and sunset.



Figure 6. Sunset

B. Angstrom's Turbidity Formula for Aerosols

Aerosol extinction due to scattering, absorption, or their combined effect varies smoothly with wavelength and does not exhibit distinct absorption bands or spectral lines. In most atmospheric conditions, scattering is the dominant attenuation mechanism, while aerosol absorption is generally less significant.

Because the individual contributions of aerosol scattering and absorption are difficult to separate, Ångström proposed a single empirical relationship, commonly known as the **Ångström turbidity formula**, to describe aerosol optical depth:

$$k_{a\lambda} = \beta \lambda^{-\alpha} \quad (12)$$

where β is the Ångström turbidity coefficient, α is the wavelength exponent, and the wavelength λ is expressed in micrometers (μm).

The turbidity coefficient β provides a measure of the aerosol loading in the atmospheric column. Its value typically ranges from nearly 0 for exceptionally clear atmospheres to about 0.5 or even higher under heavily polluted or dusty conditions. The wavelength exponent α is related to the aerosol particle size distribution. Larger values indicate a greater proportion of fine particles, whereas smaller values correspond to atmospheres dominated by coarse particles. In theory, α approaches 4 when the particles become as small as air molecules, corresponding to Rayleigh scattering, and approaches 0 for very large particles whose scattering is nearly independent of wavelength. Under most natural atmospheric conditions, α ranges between approximately 0.5 and 2.5, with a commonly adopted average value of 1.3.

The parameters β and α can be determined simultaneously with a dual wavelength sun photometer. Measurements are typically performed at wavelengths where molecular absorption is absent or minimal, most commonly at 0.38 and 0.5 μm . At 0.38 μm molecular absorption is negligible, while ozone absorption at 0.5 μm is weak. Alternatively, the turbidity coefficient β can

be determined at $\lambda = 1 \mu m$, where the influence of the wavelength exponent α is eliminated and only weak absorption by water vapour is present.

The spectral transmittance due to aerosol scattering,

$$\tau_{a\lambda} = \exp(-\beta\lambda^{-\alpha}m) \quad (13)$$

is represented in Figure 7, for various parameter combinations of the Ångström turbidity coefficient (β) and Ångström exponent (α) describing the properties of atmospheric aerosols.

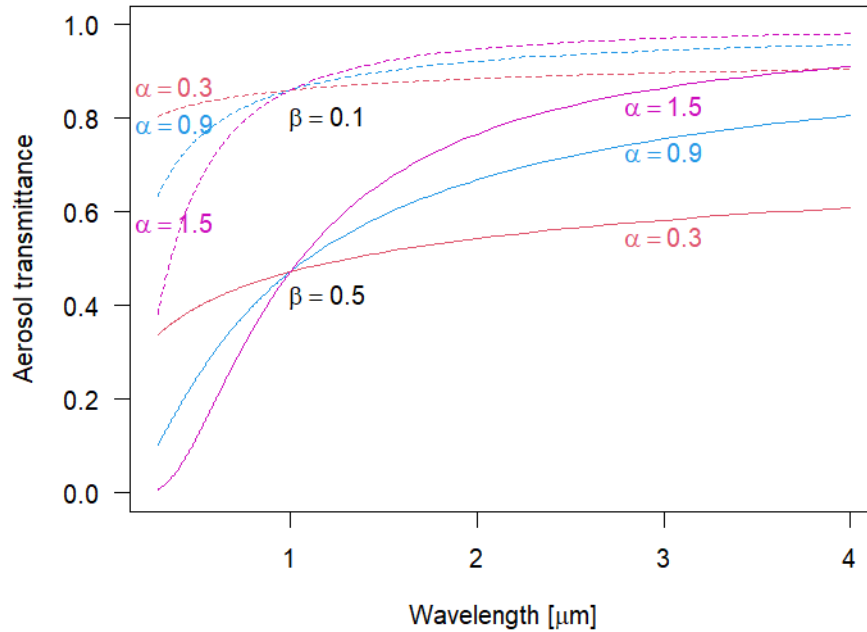


Figure 7. Aerosols transmittance, from the paper: George Stirban, Eugenia Paulescu, Mathematical Assessment of Aerosol Impact on the Diffuse-to-Global Ratio of Solar UV Radiation, *Mathematics*, (2026) Volume 14, Issue 1, Nr. 50.

The two groups of three curves correspond to distinct turbidity conditions: a low turbidity value ($\beta = 0.1$), characteristic of a clean atmosphere, and a high turbidity value ($\beta = 0.5$), associated with an enhanced aerosol load in the atmospheric column. In both groups, lower transmittance is observed in the left side of the figure, indicative of enhanced scattering in the ultraviolet range.

Each group consists of three curves: one for coarse aerosols ($\alpha = 0.3$), one for mixed aerosols ($\alpha = 0.9$), and one for fine aerosols ($\alpha = 1.5$). In both groups the scattering is particularly

pronounced for fine aerosols ($\alpha = 1.5$), such as those of urban-industrial origin. The wavelength of $1 \mu m$ represents a turning point, with fine aerosols contributing predominantly to scattering at shorter wavelengths, while coarse aerosols dominate scattering at longer wavelengths.

Both Rayleigh scattering and aerosol scattering are significant in the ultraviolet range.

At this stage, we can evaluate the monochromatic atmospheric transmittance due solely to **continuum attenuation**, i.e., attenuation produced by Rayleigh scattering and aerosol extinction. Since these processes are assumed to act independently, the total continuum transmittance is obtained as the product of the individual transmittances:

$$\tau_{c\lambda} = \tau_{r\lambda} \tau_{a\lambda} = \exp(-(k_{r\lambda} + k_{a\lambda})m) \quad (14)$$

Example 1:

Calculate the spectral transmittance due to continuum attenuation at wavelengths of $0.7 \mu m$ and $2.0 \mu m$ for a cloudless atmosphere with $\theta_z = 60^\circ$, $\beta = 0.1$, and $\alpha = 1.3$.

(a) Wavelength $\lambda = 0.7 \mu m$

Rayleigh transmittance

$$\tau_{r\lambda} = \exp(-0.008735 \lambda^{-4.08} m) = \exp(-0.008735 (0.7)^{-4.08} 2.0) = 0.92786$$

Aerosol transmittance

$$\tau_{a\lambda} = \exp(-\beta \lambda^{-\alpha} m) = \exp(-0.1 (0.7)^{-1.3} \times 2.0) = 0.7276$$

Therefore,

$$\tau_{c\lambda} = \tau_{r\lambda} \tau_{a\lambda} = 0.92786 \times 0.7276 = 0.675$$

(b) Wavelength $\lambda = 2.0 \mu m$

$$\tau_{r\lambda} = 0.9989$$

$$\tau_{a\lambda} = 0.9219$$

$$\tau_{c\lambda} = \tau_{r\lambda} \tau_{a\lambda} = 0.92088$$

```
##Ex 1
> # Parametri
> m <- 2
> alpha <- 1.3
> b <- 0.1
> wave <- c(0.7, 2.0)
> # Transmittanta Rayleigh
```

```

> tau_r <- function(lambda, m) {
+   exp(-0.008735 * m * lambda^(-4.08))
+ }

> # Transmitanta aerosolilor
> tau_a <- function(lambda, alpha, b, m) {
+   exp(-m * b * lambda^(-alpha))
+ }

> # Calcule
> TauR <- tau_r(wave, m)
> TauA <- tau_a(wave, alpha, b, m)
> TauC <- TauR * TauA

> # Tabel comparativ
> rezultate <- data.frame(
+   Wavelength_um = wave,
+   Rayleigh = TauR,
+   Aerosols = TauA,
+   Combined = TauC
+ )
> print(round(rezultate, 4))
  Wavelength_um Rayleigh Aerosols Combined
1           0.7    0.9279    0.7276    0.6751
2           2.0    0.9990    0.9220    0.9210

```

The continuum transmittance at $2.0 \mu m$ is approximately 36% higher than at $0.7 \mu m$ (0.921 versus 0.675). This analysis refers exclusively to the *direct component* of solar radiation. Although scattering attenuates the direct beam, it simultaneously redistributes part of the incident solar radiation throughout the sky, thereby increasing the diffuse irradiance reaching the Earth's surface. The diffuse component will be examined separately in a subsequent course.

In this course, only the *continuum attenuation of the direct solar beam* has been considered, namely Rayleigh scattering by air molecules and extinction by atmospheric aerosols. In the next course, we shall examine the *selective absorption* of solar radiation by atmospheric gases, including ozone, water vapour, and the uniformly mixed gases, whose attenuation occurs in specific spectral bands.

Example 2: Parameters β and α , determined for the U.S. standard atmosphere, in which the vertical aerosol optical depths at 0.38 and $0.5 \mu m$ are 0.3538 and 0.2661 , respectively.

$$\begin{aligned}
 k_{a\lambda} &= \beta \lambda^{-\alpha} & 0.3538 &= \beta (0.38)^{-\alpha} \\
 & & 0.2661 &= \beta (0.5)^{-\alpha} \\
 \alpha &= 1.038 & \beta &= 0.1296
 \end{aligned}$$

```

a <- - diff(log(AOD)) / diff(log(waves))
> b <- AOD[1]* (waves[1])^a
> a
[1] 1.037979
> b
[1] 0.1295932

```

References

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