

Main objectives of the course Solar Energy Conversion:

- Calculation of the solar radiation incident on a surface on Earth.
- Parametric clear-sky and decomposition models.
- Conversion of solar energy into electricity.
- Understanding the principles of photovoltaic systems.
- Modelling the conversion of solar irradiance and auxiliary weather variables into solar power using a model chain.

Chapter 1 Sun-Earth geometry

1.1 Sun-Earth distance

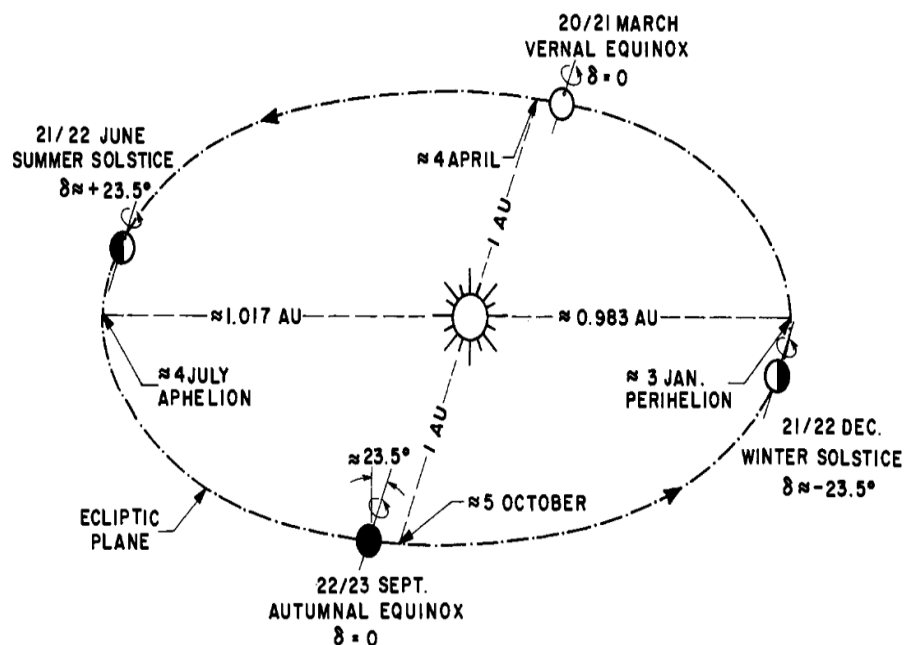


Figure 1. Motion of the Earth around the Sun.

The mean sun-earth distance is $r_0 = D_{SE} = 1 AU = 1.496 \cdot 10^8 km$

The sun-earth distance r for any day of any year is known with considerable accuracy.

Sursa: American Ephemeris and Nautical Almanac

Traditionally, the distance r is expressed in terms of a Fourier series. Spencer [1] developed the following expression for the reciprocal of the square of the radius vector of the earth, here called the *eccentricity correction factor* of the earth's orbit,

$$\begin{aligned} \varepsilon = (r_0/r)^2 = & 1.00011 + 0.034221 \cos \theta_j + 0.00128 \sin \theta_j + 0.000719 \cos 2 \theta_j \\ & + 0.000077 \sin 2 \theta_j \end{aligned} \quad (1)$$

Here, $\theta_j = 2\pi(j-1)/365$ is the day angle.

Where j is the day number of the year (Julian day).

For most engineering and technological applications, a simple expression may be used.

$$\varepsilon = 1 + 0.033 \cdot \cos \left[\frac{2\pi}{365} (j) \right] \quad (2)$$

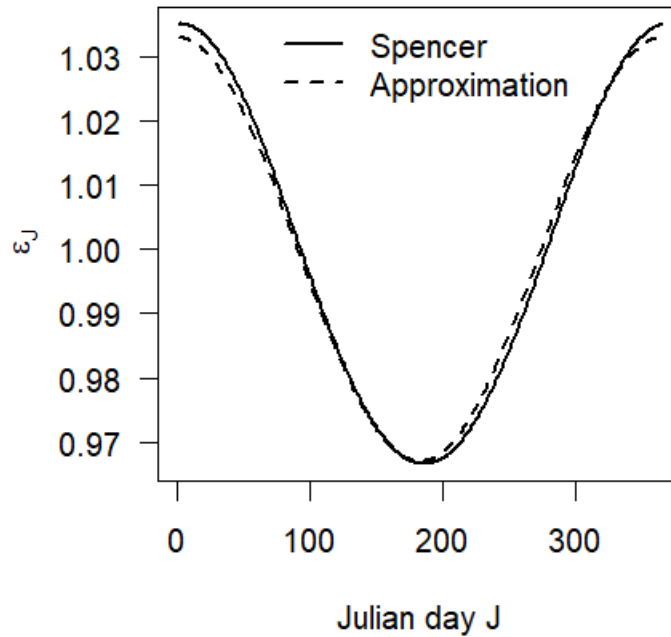


Figure 2. Spencer eccentricity correction factor.

Example 1: Calculation of the sun-earth distance r on 16 October.

$$j = 289$$

$$\text{Day angle } \theta_j = \frac{2\pi(288)}{365} = 4.958$$

$$\begin{aligned} \varepsilon = (r_0/r)^2 &= 1.00011 + 0.034221 \cos \theta_j + 0.00128 \sin \theta_j + 0.000719 \cos 2 \theta_j \\ &\quad + 0.000077 \sin 2 \theta_j = 1.006508 \end{aligned}$$

$$r = 149115534723 \text{ m} = 149 \text{ milioane km}$$

$$r = 0.99676 \text{ AU}$$

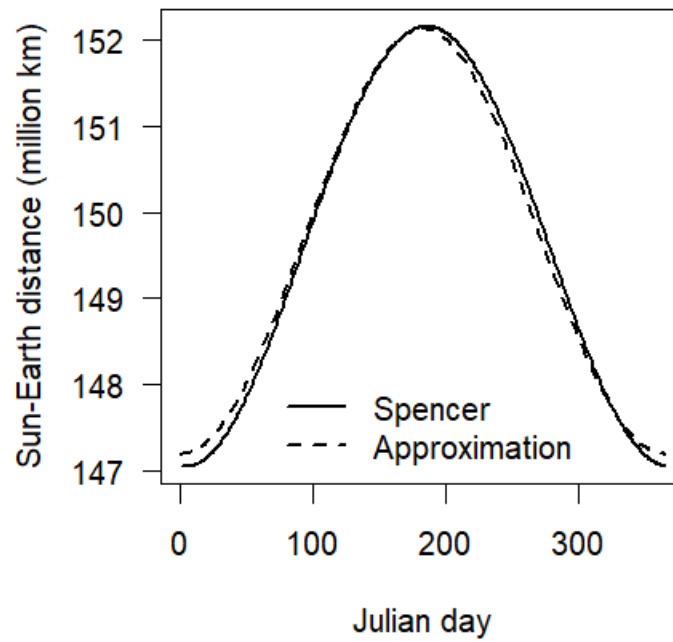


Figure 3. Sun-Earth Distance

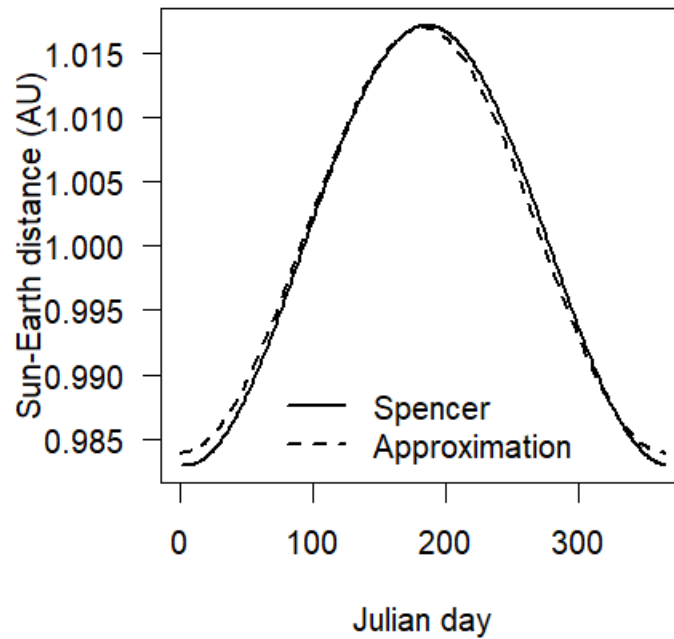


Figure 3. Sun-Earth Distance in AU.

1.2 Solar Declination δ

The plane of revolution of the earth around the sun is called the *ecliptic plane*. The earth itself rotates around an axis called the *polar axis*, which is inclined at approximately 23.5° from the normal to the ecliptic plane (Fig. 4).

The earth's rotation around its axis causes the diurnal changes in radiation income; the position of this axis relative to the sun causes seasonal changes in solar radiation. The angle between the polar axis and the normal to the ecliptic plane, however, remains unchanged. The same is true of the angle between the earth's equatorial plane and the ecliptic plane.

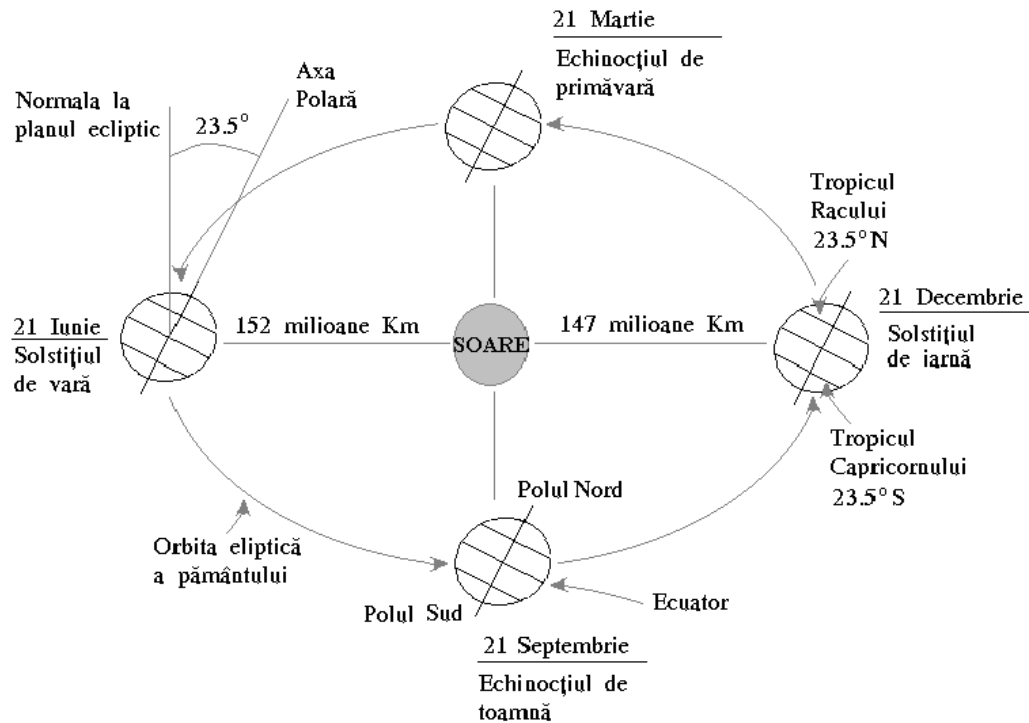


Figure 4. Sun-Earth

However, the angle between a line joining the centers of the sun and the earth to the equatorial plane changes every day, in fact, every instant. This angle is called the *solar declination* δ . It is zero at the *vernal* and *autumnal equinoxes* (literally, equal nights) and has a value of approximately $+23.5^\circ$ at the *summer solstice* and about -23.5° at the *winter solstice*.

<https://www.pveducation.org/pvcdrom/properties-of-sunlight/declination-angle>

Another means of describing the solar declination is by drawing a *celestial sphere* with the earth at the center and the sun revolving around the earth [2]. In the celestial sphere, the celestial poles are the points at which the earth's polar axis, when produced, cuts the celestial sphere. Similarly, the celestial equator is an outward projection of the earth's equatorial plane on the celestial sphere. The intersection of the plane of the earth's equator with the plane of the sun's revolution, the ecliptic, makes an angle of approximately 23.5° . At any given time, the position of the sun relative to the plane of the celestial equator describes the **declination angle**.

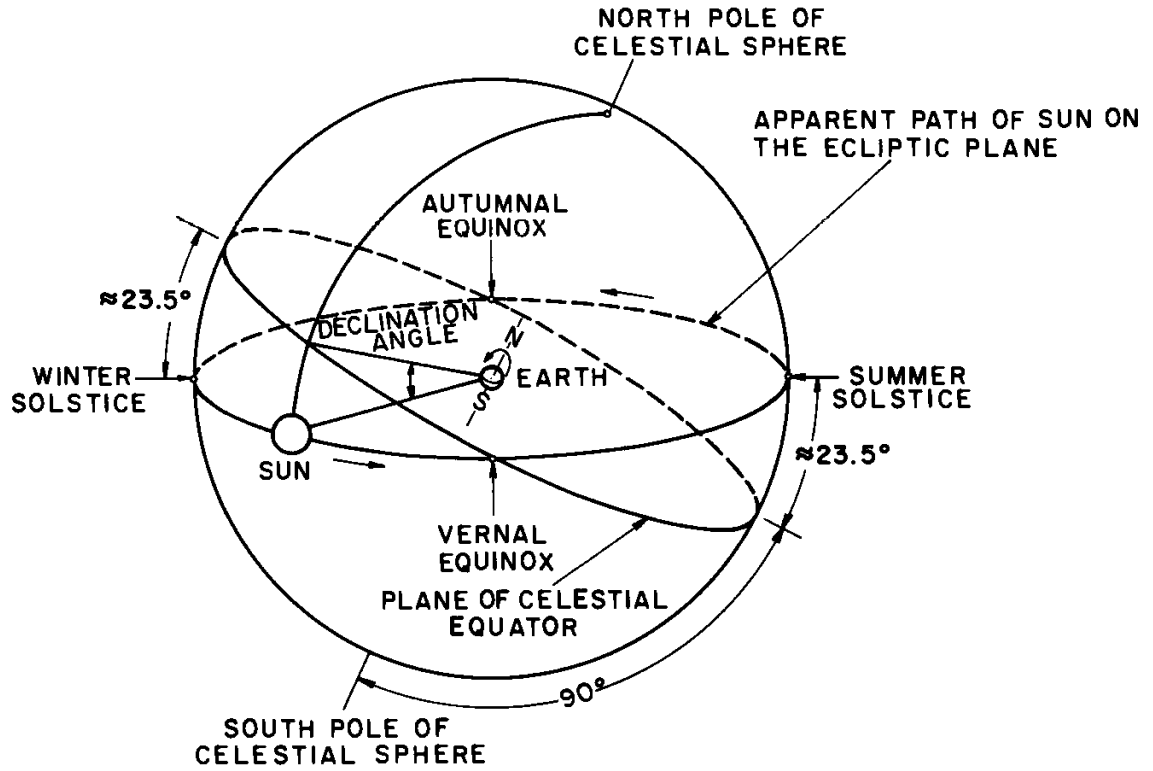


Figure 5. Celestial sphere

The solar declination angle can also be interpreted geometrically. It is equal to the angular difference between the Sun's highest position in the sky (at solar noon) on a given day and its highest position at the equinoxes, as observed from the same location.

At the equinoxes, the declination is zero and the zenith angle at noon $\theta_z = \phi$ and the solar altitude at noon is $h = 90^\circ - \phi$, where ϕ is the observer's latitude. On any other day, the Sun appears higher or lower by an angle equal to the declination, giving the well-known relation

$$h = 90^\circ - \phi + \delta \quad (1)$$

where h is the solar altitude at solar noon and δ is the solar declination.

În funcție de ziua Juliană, unghiul de declinație se poate calcula cu relația Spencer[1]:

$$\delta_j = 0.006918 - 0.399912 \cos \theta_j + 0.070257 \sin \theta_j - 0.006759 \cos 2\theta_j + 0.000907 \sin 2\theta_j + 0.00148 \sin 3\theta_j - 0.002697 \cos 3\theta_j \quad (2)$$

Expressed in radians. This equation estimates δ with a maximum error of 0.0006 rad ($< 3'$) or, if the final two terms are omitted, with a maximum error of 0.0035 rad ($12'$) [2].

More simple and commonly used formula (in degrees):

$$\delta = 23.45 \sin \left[\frac{2\pi}{365} (j + 284) \right] \quad (3)$$

Example 2: Calculation of the solar declination on 16 October.

Using (2) with $j = 289$, $\delta = -8.589^\circ$

Or with the approximation (3) $\delta = -9.966^\circ$

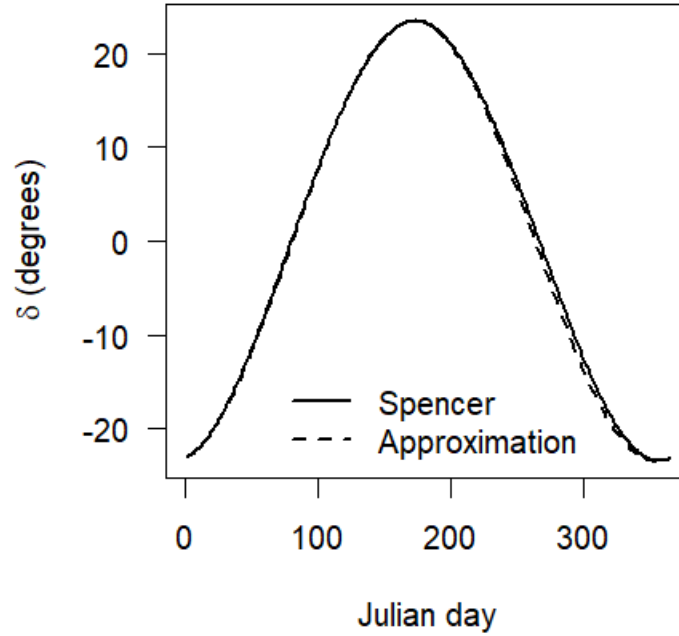


Figure 6. The solar declination angle.

1.3 Equation of Time E_t

Solar time is based on the rotation of the earth about its polar axis and on its revolution around the sun. A solar day is the interval of time (not necessarily 24 h) as the sun appears to complete one cycle about a stationary observer on earth. The solar day varies in length throughout the year. The two principal factors for this variance are the following:

- (1) the earth sweeps out unequal areas on the ecliptic plane as it revolves around the sun
- (2) the earth's axis is tilted with respect to the ecliptic plane

If an observer facing the equator today sets a clock at 12 noon, when the sun is directly over the local meridian, then after a month or so at 12 noon, clock time, the sun may not appear exactly over the local meridian. A discrepancy of as much as 16 min is possible. This discrepancy is called the *equation of time* and is measured relative to a perfectly uniform terrestrial motion. Again, from Spencer [1],

$$\begin{aligned} ET_j = & 0.000075 + 0.001868 \cos \theta_j - 0.032077 \sin \theta_j - 0.014615 \cos 2 \theta_j \\ & - 0.04089 \sin 2 \theta_j \end{aligned} \quad (1)$$

$$\begin{aligned} ET_j = & (0.000075 + 0.001868 \cos \theta_j - 0.032077 \sin \theta_j - 0.014615 \cos 2 \theta_j \\ & - 0.04089 \sin 2 \theta_j) 229.18 \end{aligned} \quad (2)$$

Eq (1) is in radians and (2) in minutes of solar time.

Example: Calculation of the equation of time on 16 October.

$$E_t(289) = 14.622 \text{ min}$$

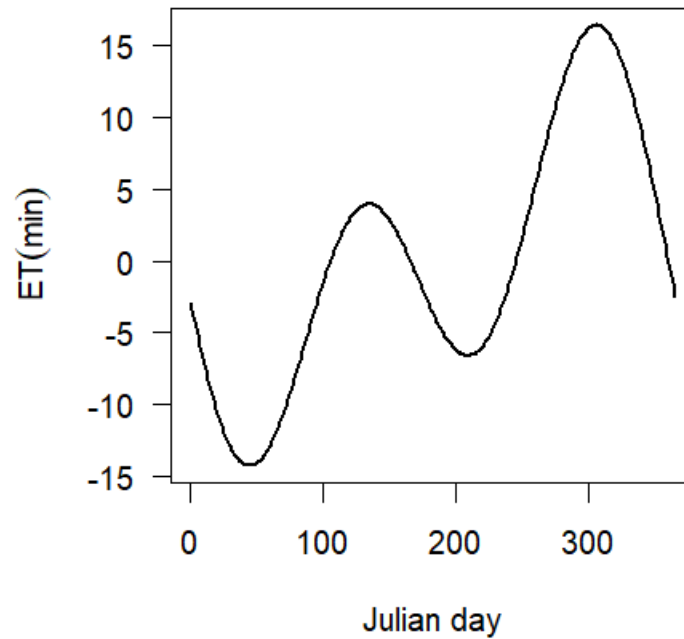


Figure 7. The yearly variation of the equation of time

1.4 Apparent Solar Time

Solar radiation data are often recorded in terms of *local apparent time* (LAT), also called true solar time (TST), also called **apparent solar time** (AST) (timp solar adevarat). On the other hand, some meteorological data such as temperature and wind speed are often recorded in terms of local clock time, which may be *local standard time* or daylight-saving time.

It is desirable to convert local standard time to apparent solar time.

To carry out this conversion, it is necessary to know the standard meridian for the local time zone. All international standard meridians are multiples of 15° east or west of Greenwich, England. Therefore, all standard times are multiple hours ahead of or behind the Greenwich mean time (GMT).

Apparent solar time for a given standard time can now be written:

local apparent time (AST) = local standard time + longitude correction+ equation of time

$$\text{AST} = \text{local standard time} + 4(L_s - L_e) + E_t \quad (1)$$

L_s = the standard longitude

L_e = the local longitude

The longitude correction, 4 min for every degree, accounts for the difference between the local and the standard meridians. It should be noted that the longitude correction is positive if the local meridian is *east* of the standard and is negative *west* of the standard meridian. The value of the equation of time E_t is added algebraically; it may be positive or negative. The correction for daylight saving time can be made appropriately.

Example: Determination of the apparent solar time (AST) on 16 October at Buenos Aires ($58^\circ 29'$ W) when the standard time is 10:00.

$$E_t(289) = 14.622 \text{ min}$$

Buenos Aires is $1^\circ 31'$ east of the 60^{th} standard meridian. The longitude correction is

$$4(60 - 58.48) = 6.08 \text{ min}$$

Apparent solar time = $10:00 + (6.08 + 14.62) \text{ min} = 10:20:41$



Figure 8. Buenos Aires standard meridian.

1.5 Position of the Sun Relative to Horizontal Surfaces

In order to calculate the solar radiation reaching a horizontal surface on the earth, it is necessary to write down the trigonometric relationships between the solar position in the sky and the surface coordinates on the earth. For instance, we can describe an observer on the earth by drawing again a celestial sphere with the earth as the center (see Fig. 9). At any given time, an observer on the earth's surface has a corresponding position in the celestial sphere called the observer's *zenith*; this is the point of intersection with the celestial sphere of a normal to the earth's surface at the observer's position. The point diagonally opposite to the local zenith is called the *nadir*. The observer's *horizon* is the great circle in the celestial sphere the plane of which passes through the center of the earth normal to the line joining the center of the earth and the zenith. The *zenith angle* θ_z is the angle between the local zenith and the line joining the observer and the sun. It is an angle between 0° and 90° . The *solar altitude* h (also called the solar elevation, unghiul de înaltare al soarelui pe cer) is the sun's angular height above the observer's celestial horizon. It is an angle between 0° and 90° . The solar altitude is the complement of the zenith angle.

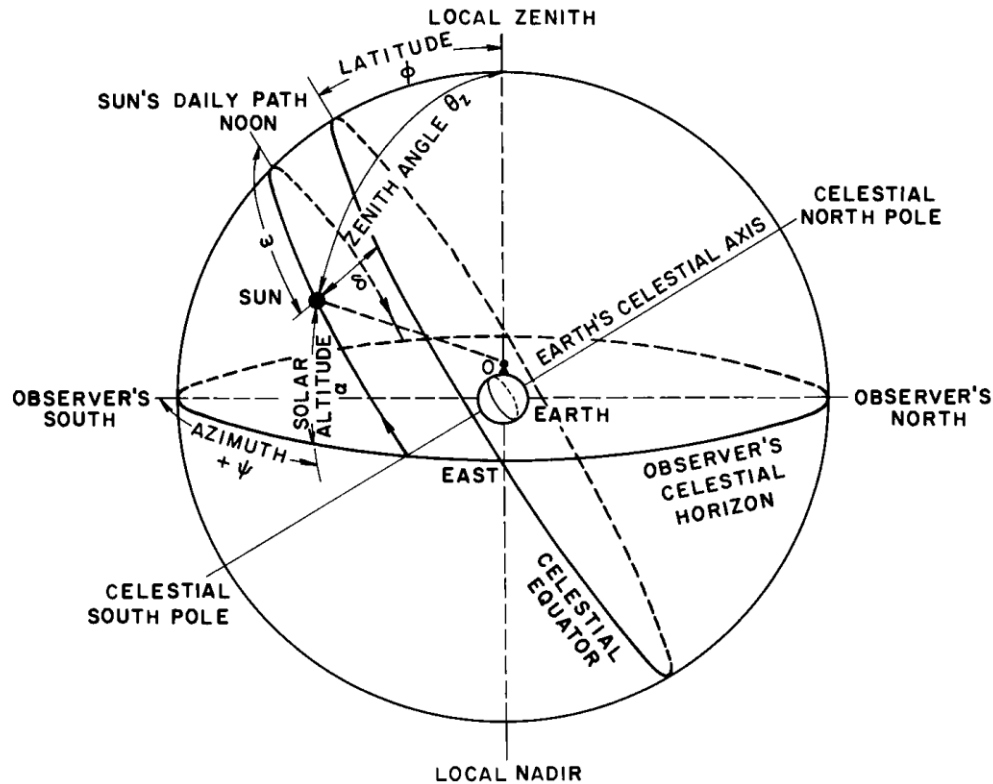


Figure 9. Celestial sphere ($\alpha = h$ solar altitude)

The solar azimuth ψ is the angle in the observer's horizontal plane between the North-South line and the projection of the observer to Sun's direction onto the horizontal plane. It is measured west positive, east negative. The hour angle ω is the angle measured between the observer's meridian and the solar meridian. The hour angle represents the Sun's angular displacement from the local meridian and is due to the Earth's rotation at a rate of 15° per hour.

$$\omega = \frac{2\pi}{24}(t - 12) \quad (1)$$

where t is the apparent solar time (AST).

The position of the sun relative to an observer on earth can be fully described by two angles: the solar zenith angle (θ_z) and the azimuth angle (ψ). The calculation formulas for these two angles are given as

$$\cos \theta_z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega = \sinh \quad (2)$$

$$\sin \psi = \frac{-\cos \delta \sin \omega}{\cos h} \quad (3 \text{ Dazhi Yang}[4])$$

There are two things to take note in using these formulas. One of those is that these formulas give only approximations rather than represent the exact astronomical expressions. Secondly, there may be some computation issues resulting from the trigonometry involved. More specifically, ψ in Eq. (9) follows a zero north–east–positive convention, in that, its range is from 0° to 360° , whereas in many solar applications another convention is assumed.

$$\psi = (sign \omega) |\arccos((sinh \sin \phi - \sin \delta) / cosh cos \phi)| \quad (3 \text{ Duffie and Beckman}[3])$$

Thus ψ is negative when the hour angle ω is negative and positive when the hour angle is positive.

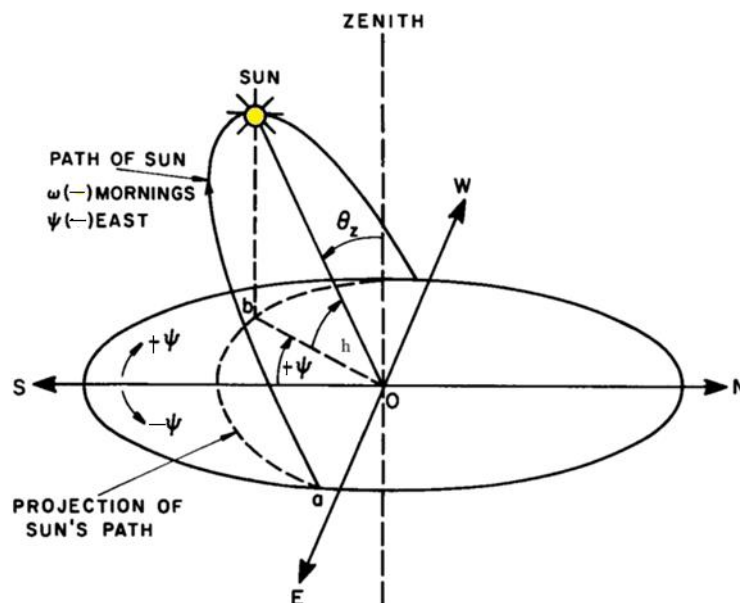


Figure 10. Sun's angles [2]

Equation (8) can be solved for the sunrise hour angle ω_s . At sunrise, $\theta_z = 90^\circ$. Therefore, Eq. (8) results in

$$\omega_s = \text{acos}(-\tan \phi \tan \delta) \quad (4)$$

Note that the sunrise hour angle is equal to the sunset hour angle except for the sign difference. From the preceding equation, day length N_d can easily be computed. The day length is $2\omega_s$, and when expressed in hours is as follows:

$$N_d = \frac{2}{15} \text{acos}(-\tan \phi \tan \delta) \quad (5)$$

(a) At the equator, $\phi = 0$; therefore, $\omega_s = \pi/2$, and day length is independent of solar declination (or season) and is always equal to 12 h. (b) At the equinoxes, $\delta = 0$; therefore, $\omega_s = \pi/2$, and day length is independent of latitude and is equal to 12 h.

Example: Calculation of (a) the zenith angle and solar azimuth at 11:00 AST, (b) the sunrise hour angle, and (c) the day length on 16 October at New York ($40^\circ 7' \text{ N}$).

At 11:00 solar time, $\omega = -15^\circ$

$$\delta = -8.589$$

$$\cos \theta_z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega$$

$$\cos \theta_z = 0.634$$

$$\theta_z = 50.64^\circ \quad h = 39.36^\circ$$

$$\cos \psi = (\sinh \sin \phi - \sin \delta) / \cosh \cos \phi$$

$$\cos \psi = -0.943$$

$$\psi = -19.32^\circ$$

$$\omega_s = \text{acos}(-\tan \phi \tan \delta)$$

$$\omega_s = 82.39^\circ$$

$$\text{The day length } N_d = \frac{2}{15} \omega_s = 11.025h$$

Derivation of the Zenith Angle Formula

$$\cos \theta_z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega$$

In Figure 11, ϕ is the latitude of point P , where the observer is located and for which we calculate $\cos \theta_z$.

The observer P is located on the **meridian** OPS. Since the Sun lies in the **meridian plane** OES, the angle formed by the lines OE and OS is precisely the **declination angle** δ . The angle between the two meridians mentioned above is the **hour angle** ω .

The zenith angle θ_z , is formed by the lines OP and OS.

We choose a reference frame $Oxyz$ such that the Ox axis lies in the equatorial plane and intersects the local meridian; Oz is directed along the polar axis, while Oy also lies in the equatorial plane and is perpendicular to the Oxz plane.

In the chosen reference frame, the coordinates of points P and C are (R being the Earth's radius):

$$\begin{aligned} x_P &= R \cos \phi & x_C &= R \cos \delta \cos \omega \\ y_P &= 0 & y_C &= R \cos \delta \sin \omega \\ z_P &= R \sin \phi & z_C &= R \sin \delta \end{aligned} \quad (6)$$

Note: OC must first be projected onto the equatorial plane, and then onto the Ox and Oy axes.

So,

$$PC^2 = [R \cos \delta \cos \omega - R \cos \phi]^2 + [R \cos \delta \sin \omega]^2 + [R \sin \phi - R \sin \delta]^2 \quad (7)$$

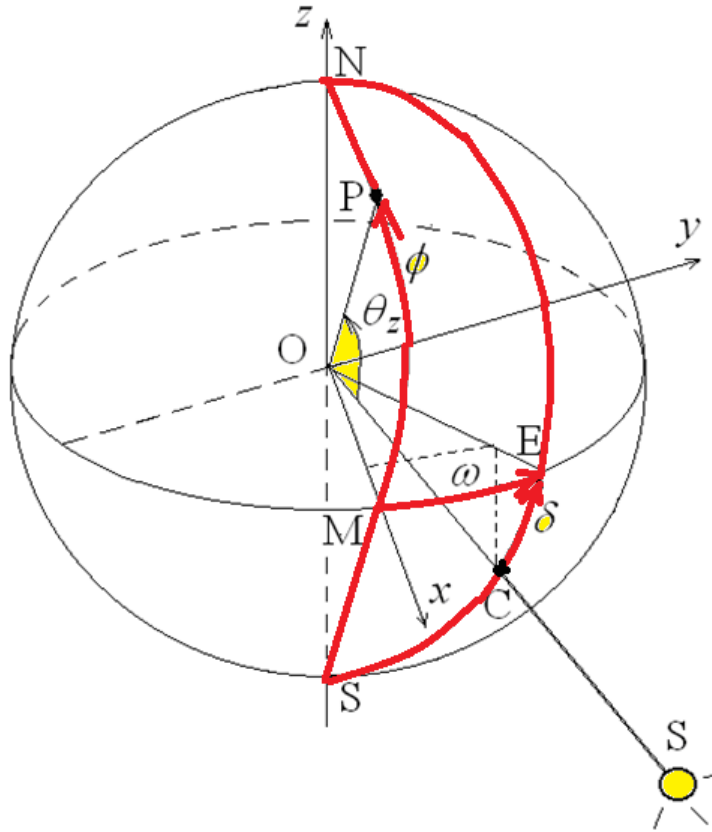


Figure 11. Compute Zenith Angle.

$$PC^2 = 2R^2 \cdot [1 - \cos \phi \cos \delta \cos \omega - \sin \phi \sin \delta] \quad (8)$$

On the other hand, in triangle OPC , the length PC can be expressed using the Pythagorean theorem:

$$PC^2 = OP^2 + OC^2 - 2 \cdot OP \cdot OC \cdot \cos \theta_z = 2R^2 (1 - \cos \theta_z) \quad (9)$$

By comparing Eqs. (12) and (13), we obtain the trigonometric expression for the **solar zenith angle**, θ_z :

$$\cos \theta_z = \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta \quad (10)$$

References

- [1] Spencer JW (1971) Fourier Series Representation of the Position of the Sun, *Search* 2, 162-172.
- [2] Iqbal M (1983) *An Introduction to Solar Radiation*, ACADEMIC PRESS.
- [3] *Solar Engineering of Thermal Processes*, Fourth Edition, John A. Duffie and William A. Beckman, 2013 by John Wiley & Sons.
- [4] A Tutorial Review of the Solar Power Curve: Regressions, Model Chains, and Their Hybridization and Probabilistic Extensions, Dazhi YANG, Xiang'ao XIA, and Martin János MAYER, *ADVANCES IN ATMOSPHERIC SCIENCES*, VOL. 41, JUNE 2024, 1023–1067.

Codul R

```
AU <- 1.496e11      # astronomical unit, m
> J <- 289
> # Spencer correction factor for orbital eccentricity
> correction_spencer <- function(J) {
+   1.00011 +
+   0.034221 * cos(2*pi*(J-1)/365) +
+   0.00128 * sin(2*pi*(J-1)/365) +
+   0.000719 * cos(4*pi*(J-1)/365) +
+   0.000077 * sin(4*pi*(J-1)/365)
+ }

# Approximate correction factor
> correction_approx <- function(J) {
+   1 + 0.033 * cos(2*pi*J/365)
+ }
> # Sun-Earth distance
> r_spencer <- function(J) {
```

```

+ AU / sqrt(correction_spencer(J))
+ }
> r_approx <- function(J) {
+   AU / sqrt(correction_approx(J))
+ }
> r <- r_spencer(J)
> r
[1] 149115534723
> r / AU
[1] 0.9967616

> all_year <- 1:365
> r_spencer <- r_spencer(all_year)
> r_approx <- r_approx(all_year)
> plot(all_year, r_spencer/1e9,
+       type = "l", lwd = 2,
+       xlab = "Julian day",
+       ylab = "Sun-Earth distance (million km)",
+       las = 1)
> lines(all_year, r_approx/1e9,
+       lwd = 2, lty = 2)
> legend("bottom",
+       legend = c("Spencer", "Approximation"),
+       lty = c(1, 2),
+       lwd = 2,
+       bty = "n")

f1 <- correction_spencer(all_year)
> f2 <- correction_approx(all_year)
> plot(all_year, f1,
+       type = "l", lwd = 2,
+       xlab = "Julian day J",
+       ylab = expression(epsilon[J]),
+       las = 1)
> lines(all_year, f2,
+       lwd = 2, lty = 2)
> legend("top",
+       legend = c("Spencer", "Approximation"),
+       lty = c(1, 2),
+       lwd = 2,
+       bty = "n")
> # Spencer

> AngleDeclin <- function(x)
+   return((0.006918 -
+           0.399912*cos(2*pi*(x-1)/365) +
+           0.070257*sin(2*pi*(x-1)/365) -
+           0.006758*cos(4*pi*(x-1)/365) +
+           0.000907*sin(4*pi*(x-1)/365) +
+           0.00148*sin(6*pi*(x-1)/365) -
+           0.002697*cos(6*pi*(x-1)/365))*180/pi)
> # Formula aproximativa
> AngleDeclin_aprox <- function(x)
+   return(23.45*sin(2*pi*(x+284)/365))
> J<-289
> AngleDeclin(J)
[1] -8.589403
> AngleDeclin_aprox(J)
[1] -9.966258
> # Unghiul de declinatie pentru toate zilele anului
> all_year <- 1:365

```

```

> delta_Spencer <- AngleDeclin(all_year)
> delta_aprox <- AngleDeclin_aprox(all_year)

# Reprezentare grafica
> plot(all_year, delta_Spencer,
+      type = "l",
+      lwd = 2,
+      xlab = "Julian day",
+      ylab = expression(delta~"(degrees)"),
+      las = 1)
> lines(all_year, delta_aprox,
+      lwd = 2,
+      lty = 2)
> legend("bottom",
+      legend = c("Spencer", "Approximation"),
+      lty = c(1, 2),
+      lwd = 2,
+      bty = "n")
> EqTime <- function(x)
+   return((0.000075 +
+           0.001868*cos(2*pi*(x-1)/365) -
+           0.032077*sin(2*pi*(x-1)/365) -
+           0.014615*cos(4*pi*(x-1)/365) -
+           0.04089*sin(4*pi*(x-1)/365)) * 229.18)
> J <- 289
> EqTime(J)
[1] 14.62219
> all_year <- 1:365
> EquationTime <- EqTime(all_year)
> # Reprezentare grafica
> plot(all_year, EquationTime,
+      type = "l",
+      lwd = 2,
+      xlab = "Julian day",
+      ylab = expression(ET(min)),
+      las = 1)

> ##example
> J<-289
> StandardTime<-10
> LongS<-60
> Long_deg <- 58
> Long_min <- 29
> Long <- Long_deg + Long_min/60
> Long
[1] 58.48333
> EqTime(J)
[1] 14.62219

AST<-function(x)
+ return(StandardTime*60 + 4*(LongS-Long) + EqTime(x))
> ast<-AST(J)
> ast
[1] 620.6889
> ore <- floor(ast / 60)
> minute <- floor(ast %% 60)
> secunde <- round((ast %% 1) * 60)
> sprintf("%02d:%02d:%02d", ore, minute, secunde)
[1] "10:20:41"

> ###Aplicatie the zenith angle and solar azimuth
> J<-289

```

```

> Lat_deg <- 40
> Lat_min <- 7
> Lat <- Lat_deg + Lat_min/60
> Lat
[1] 40.11667
> AST<-11

> HourAngle<-function(x)
+   return((x-12)*2*pi/24)
> omega<-HourAngle(AST)
> omega
[1] -0.2617994
> omega*180/pi
[1] -15

> AngleDeclin(J)
[1] -8.589403

> ## Solar zenith angle
> delta<-AngleDeclin(J)*pi/180
> phi<-Lat*pi/180

> CosTetaZ<-sin(phi)*sin(delta)+cos(phi)*cos(delta)*cos(omega)
> CosTetaZ
[1] 0.6341567

> Tetaz<-acos(CosTetaZ)
> Tetaz
[1] 0.883879
> Tetaz*180/pi
[1] 50.64254

> CosPsi<-(CosTetaZ*sin(phi)-sin(delta))/(sin(Tetaz)*cos(phi))
> CosPsi
[1] 0.9436374
> Psi<-acos(CosPsi)
> Psi*180/pi
[1] 19.32834

> ### Sunrise and day length
> omega_s<-acos(-tan(phi)*tan(delta))
> omega_s_deg<-omega_s*180/pi

> Nd<-2*omega_s_deg/15
> Nd
[1] 11.0251

```