Introduction to Supersymmetry

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Lecture 1

- Motivations, 2 components spinors
- Supersymmetry: formalism

  transformations, algebra, representations,

  superspace, chiral and vector super fields, invariant actions,

  R-symmetry, extended supersymmetry,

  spontaneous supersymmetry breaking

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Motivations of SUSY

- natural elementary scalars
- gauge coupling unification: theory perturbative up to the GUT scale
- LSP: natural dark matter candidate
- extension of space-time symmetry: new Grassmann dimensions
- attractive mechanism of Electroweak Symmetry Breaking
- prediction of light Higgs $(\lesssim 135\ GeV)$
- rich spectrum of new particles at the (multi)-TeV scale

2-component Weyl spinors

spin-1/2 irreps of Lorentz group: 2-dim Left and Right

$$\chi_{L,R} \to \left(1 - i\vec{\alpha}\cdot\vec{\sigma} \mp \vec{\beta}\cdot\vec{\sigma}\right)\chi_{L,R} \;\Rightarrow\; L : \chi_\alpha \;,\; R : \chi_{\dot{\alpha}}$$

infinitesimal rotation boost Pauli matrices

charge conjugation matrix $C = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$C : L \to R$ $\quad$ $C\chi^* \equiv \bar{\chi}^c$ transforms as Right $\quad$ parity: $\chi \to \bar{\chi}^c$ $\quad \Rightarrow$

describe R-spinor as charge conjugate of a L-spinor

Dirac-Weyl basis: $\psi = \begin{pmatrix} \chi_\alpha \\ \bar{\eta}^c_{\dot{\alpha}} \end{pmatrix}$ $\quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$ $\quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\sigma^\mu_{\alpha\dot{\alpha}} = (1, \vec{\sigma})$ $\quad \bar{\sigma}^\mu_{\dot{\alpha}\alpha} = (1, -\vec{\sigma}^*)$ $\quad C\sigma^\mu = (\bar{\sigma}^\mu)^T\, C$

Dirac and Majorana masses

$$\psi_L = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix} \quad \psi_R = \begin{pmatrix} 0 \\ \bar{\eta}^c_{\dot{\alpha}} \end{pmatrix} \quad \Rightarrow$$

$$\begin{aligned} \mathcal{L}_F &= i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi \\ &= i\chi^*\,\bar{\sigma}^\mu\partial_\mu\chi + i\eta^*\,\bar{\sigma}^\mu\partial_\mu\eta - (m\chi\eta + \text{h.c.}) \quad \chi\eta \equiv \chi^T C\eta = \epsilon_{\alpha\beta}\chi^\alpha\eta\beta \end{aligned}$$

charge or fermion number $\chi : +1 \quad \eta : -1 \quad \Rightarrow$

- Dirac mass invariant
- Majorana mass: $\eta = \chi \ \Rightarrow \ \epsilon_{\alpha\beta}\chi^\alpha\chi\beta$ violates charge

Majorana fermion: $\psi = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^c_{\dot{\alpha}} \end{pmatrix}$

Supersymmetry

$\delta\phi = \xi\psi$ $\quad\xi$: Weyl spinor $\Rightarrow$ conserved spinorial charge Q_α

$$[Q_\alpha, \phi] = \psi \quad , \quad [Q_\alpha, H] = 0 \quad \Rightarrow \quad \left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha} R_\mu$$

anticommutator

if R_μ conserved charge besides $T_{\mu\nu}$ (P^μ + angular momentum)

$\Rightarrow$ trivial theory with no scattering Coleman-Mandula

e.g. 4-point scattering for fixed center of mass s

depends only on the scattering angle θ

if extra charge $R^\mu \Rightarrow$ trivial

$\Rightarrow R_\mu = P_\mu$: supersymmetry = "$\sqrt{\text{translations}}$"

SUSY transformations

$$\begin{aligned}
\delta\phi &= \sqrt{2}\xi\psi \\
\delta\psi &= -i\sqrt{2}\left(\sigma^{\mu}\bar{\xi}\right)_{\alpha}\partial_{\mu}\phi - \sqrt{2}F\xi \\
\delta F &= -i\sqrt{2}\left(\partial_{\mu}\psi\right)\sigma^{\mu}\bar{\xi}
\end{aligned}$$

F: complex auxiliary field to close the SUSY algebra off-shell

- ϕ, F: complex
- chirality to scalars: $\phi \leftrightarrow$ left $\phi^{*} \leftrightarrow$ right

Exercise: $\delta_1\delta_2 - \delta_2\delta_1 = 2i\left(\xi_1\sigma^{\mu}\bar{\xi}_2 - \xi_2\sigma^{\mu}\bar{\xi}_1\right)\partial_{\mu}$ on all fields

$\nearrow$

$$\left\{Q_{\alpha}, \bar{Q}_{\dot{\alpha}}\right\} = -2\sigma^{\mu}_{\alpha\dot{\alpha}}P_{\mu}$$

SUSY algebra

SUSY algebra: ‘Graded’ Poincaré:

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu \qquad \{Q, Q\} = \{\bar{Q}, \bar{Q}\} = [P_\mu, Q] = 0$$

$$[Q_\alpha, M^{\mu\nu}] = i\underset{\nwarrow\,[\sigma^\mu,\sigma^\nu]}{\sigma^{\mu\nu}_{\alpha\beta}} Q_\beta \qquad M^{\mu\nu}\text{: Lorentz transformations}$$

generalization for many supercharges $\Rightarrow$ N extended SUSY

$$\left\{Q^i_\alpha, \bar{Q}^j_{\dot\alpha}\right\} = -2\sigma^\mu_{\alpha\dot\alpha} P_\mu \delta^{ij} \qquad \left\{Q^i_\alpha, Q^j_\beta\right\} = \epsilon_{\alpha\beta} Z^{ij} \qquad i, j = 1, \ldots N$$

Z^{ij}: antisymmetric central charge $[Z, Q] = 0$

SUSY representations

Q_α has spin 1/2 $\Rightarrow$ $Q|J\rangle = |J \pm 1/2\rangle$ J: spin

- same mass: $[Q, P_\mu] = 0$ $\Rightarrow$ $M_J = M_{J\pm 1/2}$
- same number of fermionic and bosonic degrees of freedom

supermultiplets characterized by their higher spin → massless:

- chiral: $\begin{pmatrix} \chi_\alpha \\ \phi \end{pmatrix}$ $\begin{matrix} \text{Weyl spinor} \\ \text{complex scalar} \end{matrix}$
- vector: $\begin{pmatrix} V_\mu \\ \lambda_\alpha \end{pmatrix}$ $\begin{matrix} \text{spin 1} \\ \text{Majorana spinor} \end{matrix}$
- gravity: $\begin{pmatrix} g_{\mu\nu} \\ \psi^\mu_\alpha \end{pmatrix}$ $\begin{matrix} \text{spin 2} \\ \text{Majorana spin 3/2} \end{matrix}$
- spin 3/2: $\begin{pmatrix} \psi^\mu_\alpha \\ V_\mu \end{pmatrix}$ $\begin{matrix} \text{spin 3/2} \\ \text{spin 1} \end{matrix}$ extended supergravites

Massive supermultiplets

- spin-1/2: - chiral with Majorana mass

  - 2 chiral with Dirac mass
- spin-1: 1 vector + 1 chiral

  spin-1 + Dirac spinor + real scalar

Superspace

$(P^\mu, Q, \bar{Q}) \quad \longrightarrow \quad \left(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}\right)$ $\qquad \theta, \bar{\theta}$: Grassmann variables

Grassmann algebra: $\quad x_1, x_2, \ldots, x_n \;\to\; \theta_1, \theta_2, \ldots, \theta_n$ $\quad$ 1-component

anticommuting: $\qquad [x_i, x_j] = 0 \qquad \{\theta_i, \theta_j\} = 0 \quad \Rightarrow$

$\theta_i^2 = 0 \;\Rightarrow\; f(\theta) = f_0 + f_1\theta$

$$f(\theta_1, \ldots, \theta_n) = f_0 + f_i\theta_i + f_{ij}\,\theta_i\theta_j + \cdots + f_{1\cdots n}\,\theta_1\theta_2\cdots\theta_n$$

integration: analog of $\int_{-\infty}^{\infty} dx$

translation invariance: $\int d\theta f(\theta) = \int d\theta f(\theta + \varepsilon) \;\Rightarrow$

$\int d\theta = 0 \;\; \int d\theta\theta = 1 \;\; \int d\theta f(\theta) = f_1$ integration $\int d\theta \equiv$ derivation $\frac{d}{d\theta}$

$\int d\theta_1 \cdots d\theta_n f(\theta_1, \cdots, \theta_n) = f_{1\cdots n}$

super-covariant derivatives

SUSY transformation $\equiv$ translation in superspace

SUSY group element: $G(x,\theta,\bar{\theta}) = e^{i(\theta Q + \bar{\theta}\bar{Q} + x^{\mu}P_{\mu})}$

multiplication using $e^{A}e^{B} = e^{A+B+\frac{1}{2}[A,B]}$:

$$G(x,\theta,\bar{\theta})G(y,\eta,\bar{\eta}) = G(x^{\mu}+y^{\mu}+\underset{\overset{\nearrow}{\frac{1}{2}\theta\{Q,\bar{Q}\}\bar{\eta}}}{i\theta\sigma^{\mu}\bar{\eta}}-i\eta\sigma^{\mu}\bar{\theta},\theta+\eta,\bar{\theta}+\bar{\eta}) \;\Rightarrow$$

$$\begin{aligned} Q_{\alpha} &= \;\; i\left[\frac{\partial}{\partial\theta^{\alpha}} - i\left(\sigma^{\mu}\bar{\theta}\right)_{\alpha}\partial_{\mu}\right] \qquad P_{\mu} = i\partial_{\mu} \qquad \left\{Q_{\alpha},\bar{Q}_{\dot{\alpha}}\right\} = -2i\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu} \\ \bar{Q}_{\dot{\alpha}} &= -i\left[\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\left(\theta\sigma^{\mu}\right)_{\dot{\alpha}}\partial_{\mu}\right] \qquad\qquad\qquad\qquad \left\{Q_{\alpha},Q_{\beta}\right\} = 0 \end{aligned}$$

super-covariant derivatives

commute with SUSY transformations:

$$\{D, Q\} = \{D, \bar{Q}\} = \{\bar{D}, Q\} = \{\bar{D}, \bar{Q}\} = 0 \;\Rightarrow$$

$$\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i\left(\sigma^\mu\bar{\theta}\right)_\alpha \partial_\mu \\ \bar{D}_{\dot{\alpha}} &= \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\left(\theta\sigma^\mu\right)_{\dot{\alpha}} \partial_\mu \end{aligned}$$

$$\left\{D_\alpha, \bar{D}_{\dot{\alpha}}\right\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$$

$$\{D, D\} = D^3 = \bar{D}^3 = 0$$

Chiral superfield

$\Phi(x,\theta,\bar\theta)$ satisfying: $\bar D_{\dot\alpha}\Phi = 0 \;\Rightarrow\; \Phi(y,\theta) \qquad y = x + i\theta\sigma\bar\theta \quad (\bar D_{\dot\alpha} y = 0)$

$\Rightarrow$ chiral superspace $\{y^\mu, \theta_\alpha\}$

similar antichiral superfield $\Phi^\dagger(\bar y, \bar\theta) \qquad \bar y = x - i\theta\sigma\bar\theta$

expansion in components:

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) - \theta^2 F(y)$$

mass dimension $[\theta] = -1/2$

Exercise: From a translation in superspace derive

the SUSY transformations for the components (ϕ, ψ, F)

SUSY action of chiral multiplets

$$\mathcal{L}_K = \int d^2\theta d^2\bar{\theta}\, \Phi^\dagger \Phi = |\partial\phi|^2 + i\bar{\psi}\bar{\sigma}^\mu \partial_\mu \psi + |F|^2$$

- free complex scalar + Weyl fermion
- $F = 0$ by equations of motion

$$\mathcal{L}_W = \int d^2\theta W(\Phi) = F\frac{\partial W}{\partial \phi} - \psi\psi \frac{\partial^2 W}{(\partial\phi)^2} \qquad \psi\psi = \psi_\alpha \psi_\beta \epsilon^{\alpha\beta}$$

superpotential W : arbitrary analytic function

$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_W + \mathcal{L}_{W^\dagger} \quad \Rightarrow \quad F^* = -\frac{\partial W}{\partial \phi} \Rightarrow$

- scalar potential $\mathcal{V} = \left|\frac{\partial W}{\partial \phi}\right|^2$
- Yukawa interaction $\frac{\partial^2 W}{(\partial\phi)^2}\psi\psi + \mathrm{c.c.}$

renormalizability $\Rightarrow$ W at most cubic

$W(\Phi_i) = \frac{1}{2}M_{ij}\Phi_i\Phi_j + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k \quad \Rightarrow$

- $V_{\rm scalar} = \sum_i |M_{ij}\phi_j + \lambda_{ijk}\phi_j\phi_k|^2$
- $\mathcal{L}_{\rm Yukawa} = -M_{ij}\psi_i\psi_j - \lambda_{ijk}\phi_i\psi_j\psi_k$

generalization: $\mathcal{L} = \int d^4\theta K\left(\Phi, \Phi^\dagger\right) + \left\{\int d^2\theta W(\Phi) + \text{c.c.}\right\}$

Kähler potential K : arbitrary real function

$\Rightarrow \quad \mathcal{L}_K = g_{i\bar{j}}(\partial\phi^i)(\partial\bar{\phi}^{\bar{j}}) + \cdots$

Kähler metric $g_{i\bar{j}} = \frac{\partial}{\partial\phi^i}\frac{\partial}{\partial\bar{\phi}^{\bar{j}}}K(\phi, \bar{\phi})$ → Kähler manifold

Vector superfield

$V(x, \theta, \bar\theta)$: real

abelian gauge transformation: $\delta V = \Lambda + \Lambda^\dagger$ $\quad\Lambda$: chiral

super-gauge fixing $\Rightarrow$ Wess-Zumino gauge:

$$V = \theta\sigma^\mu \underset{\substack{\nearrow \\ \text{gauge field}}}{\bar\theta} A_\mu + i\bar\theta^2\theta \underset{\substack{\nwarrow \\ \text{gaugino}}}{\lambda} - i\theta^2\bar\theta\bar\lambda + \tfrac{1}{2}\bar\theta^2\theta^2 \underset{\substack{\nwarrow \\ \text{real auxiliary field}}}{\mathrm{D}}$$

$$\delta A^\mu = \bar\xi\bar\sigma^\mu\lambda + \bar\lambda\bar\sigma^\mu\xi \qquad\qquad \delta\lambda = \left(i\sigma^{\mu\nu}F_{\mu\nu} + \mathrm{D}\right)\xi$$

$$\delta\mathrm{D} = -i\bar\xi\bar\sigma^\mu\partial_\mu\lambda + i(\partial_\mu\bar\lambda)\bar\sigma^\mu\xi$$

chiral field strength: $\mathcal{W}_\alpha = -\frac{1}{4}\bar D^2 D_\alpha V$ $\quad\bar D\mathcal{W}_\alpha = 0$

$$\mathcal{W}_\alpha = -i\lambda_\alpha + \theta_\alpha\mathrm{D} + \tfrac{i}{4}\left(\theta\sigma^{\mu\nu}\right)_\alpha F_{\mu\nu} + \theta^2\left(\sigma^\mu\partial_\mu\bar\lambda\right)_\alpha$$

SUSY gauge actions

- Kinetic terms chiral: $\mathcal{L}_{\text{gauge}} = \frac{1}{4}\int d^2\theta\, \mathcal{W}^2 + \text{h.c.}$

$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{i}{2}\lambda\sigma^\mu \overleftrightarrow{\partial}_\mu \bar{\lambda} + \frac{1}{2}\mathrm{D}^2$$

- Covariant derivatives in matter action

gauge transformation $\delta\Phi_q = q\,\Lambda\Phi_q$ q: charge $\Rightarrow$

$$\Phi_q^\dagger\Phi_q \quad\longrightarrow\quad \Phi_q^\dagger e^{-qV}\Phi_q$$

additional contribution to the scalar potential: $\frac{1}{2g^2}\mathrm{D}^2 - q\left|\phi_q\right|^2\mathrm{D}$ $\Rightarrow$

$$\mathrm{D} = g^2\sum_i q_i\left|\phi_i\right|^2 \qquad \mathcal{V}_{\text{gauge}} = \frac{g^2}{2}\left(\sum_i q_i\left|\phi_i\right|^2\right)^2$$

- Non abelian case: $V \to V_a\, t^a$ $\leftarrow$ gauge group generators

$$\Rightarrow \quad \mathrm{Tr}\,\mathcal{W}^2 \qquad q_i\left|\phi_i\right|^2 \to \bar{\phi}_i t^a\phi_i$$

- generalization: $\mathcal{L}_{\text{gauge}} = \frac{1}{4}\int d^2\theta f(\Phi)\,\mathcal{W}^2 + \text{h.c.}$ f: analytic

R-symmetry

Chiral rotation of superspace coordinates: $\theta \to e^{i\omega}\theta$ R-charge $= 1$

$\bar{\theta} \to e^{-i\omega}\bar{\theta}$ R-charge $= -1$

$\Rightarrow$ superfield components: $\Delta Q_R \left|_{\text{bosons}-\text{fermions}}\right. = \pm 1$

e.g. R-charges: $\Phi_q = \phi_q + \sqrt{2}\theta_1\psi_{q-1} - \theta_1^2 F_{q-2}$

Chiral measure $d^2\theta$ has charge -2 $\int d^2\theta\,\theta^2 = 1$ $\Rightarrow$

- gauge superfield $\mathcal{W}$: R-charge $+1 =$ for gauginos
- superpotential W: charge $+2 \Rightarrow$ constraints on charges of Φ's

Renormalization properties

- Non renormalization of the superpotential

- Wave function renormalization of matter fields

  heuristic proof: promote Yukawa coupling λ to background

  chiral superfield of R-charge $+2$

  $\Rightarrow$ loop corrections analytic in $\lambda \to$ violate R-charge

  in wave functions posible because $\lambda\lambda^\dagger$ dependence allowed

- gauge kinetic terms only one loop

  promote $1/g^2$ to a chiral superfield S and use

  perturbative invariance under imaginary shift $S \to S + ic$

- Non perturbative corrections break R-symmetry to a discrete group

multiplet of currents

supersymmetry invariance $\Rrightarrow$ conserved supercurrent $S_{\mu\alpha}$

$$\delta_s\mathcal{L} = \partial^\mu\left(K_\mu\xi\right) \Rrightarrow S_\mu = S_\mu^{(N)} - K_\mu$$

$$\text{Noether current} = \sum_\phi \frac{\delta\mathcal{L}}{\delta\partial^\mu\phi}\delta_s\phi$$

exercise: Show that $S_{\mu\alpha} = \sqrt{2}(\partial_\nu\bar{\phi})(\sigma^\nu\bar{\sigma}_\mu\psi)_\alpha + i\sqrt{2}\,\overline{\partial_\phi W}(\sigma_\mu\bar{\psi}^c)_\alpha$

$\delta_s S_\mu \sim (\sigma^\nu\bar{\xi})\,T_{\mu\nu} + \dots \Rrightarrow$ multiplet of currents $(S_\mu, T_{\mu\nu}, \dots)$

$$\delta_s S_\mu = i\bar{\xi}^{\dot{\alpha}}\left\{\bar{Q}_{\dot{\alpha}}, S_\mu\right\} \Rrightarrow$$

$$\int d^3x\,\delta_s S_{0\alpha} = i\bar{\xi}^{\dot{\alpha}}\left\{\bar{Q}_{\dot{\alpha}}, Q_\alpha\right\} = -2i(\sigma^\nu\bar{\xi})_\alpha P_\nu = \int d^3x\left[-2i(\sigma^\nu\bar{\xi})_\alpha\right]T_{0\nu}$$

Similarly: R-current j_μ^R ↔ trace T_μ^μ

The Ferrara-Zumino supermultiplet

Real superfield $\mathcal{J}_{\alpha\dot{\alpha}} = (j^R_\mu, S_{\mu\beta}, T_{\mu\nu})$

$$\bar{D}^{\dot{\alpha}}\mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X \quad \bar{D}_{\dot{\alpha}}X = 0$$

$X = 0 \leftrightarrow$ superconformal invariance: $\partial^\mu j^R_\mu = \gamma^\mu S_\mu = T^\mu_\mu = 0$

8 bosonic + 8 fermionic components

$$j^R_\mu : 4 - 1 = 3\,,\ T_{\mu\nu} : 10 - 4 - 1 = 5\,,\ S_\mu : 16 - 4 - 4 = 8$$

$X \neq 0$: 12 bosonic + 12 fermionic components

$$\text{bosonic} \quad j^R_\mu : 4\,,\ T_{\mu\nu} : 10 - 4 = 6\,,\ X : 2$$

example: Wess-Zumino model

$$\mathcal{J}_{\alpha\dot{\alpha}} = 2g_{i\bar{j}}(D_\alpha\Phi^i)(\bar{D}_{\dot{\alpha}}\bar{\Phi}^{\bar{j}}) - \frac{2}{3}\left[D_\alpha, \bar{D}_{\dot{\alpha}}\right]K \quad , \quad X = 4W - \frac{1}{3}\bar{D}^2K$$

Extended supersymetry

N supercharges $Q^i \quad i = 1, \ldots N$

- multiplets with spin$\leq 1 \Rightarrow N \leq 4 \leftarrow$ maximal global SUSY
- multiplets with spin$\leq 2 \Rightarrow N \leq 8 \leftarrow$ maximal supergravity
- dimension of massless reps $= (2) \times 2^N$
- dimension of massive reps $= (2) \times 2^{N+1}$

  in the presence of central charge $Z \neq 0 \Rightarrow$ short reps $\equiv$ massless

$N = 2$ massless multiplets $\equiv N = 1$ massive

- vector multiplet $=$ vector $+$ chiral of $N = 1$

  1 vector $+$ 1 Dirac spinor $+$ 1 complex scalar

- hypermultiplet $= 2$ chiral of $N = 1$

  2 complex scalars $+$ 1 Dirac spinor

$N = 4$ massless multiplet: vector = vector + hyper of $N = 2$

1 vector + 4 two-component spinors + 6 real scalars

superfield off-shell formalism only for $N = 2$ vector multiplets

chiral $N = 2$ superspace $\equiv$ double of $N = 1$ superspace: $\theta, \tilde{\theta}$

gauge chiral multiplet$\big|_{N=2}\,\mathcal{A} = (\text{vector } \mathcal{W} + \text{chiral } A)_{N=1}$

$$\mathcal{A}(y,\theta,\tilde{\theta}) = A(y,\theta) + i\sqrt{2}\tilde{\theta}\mathcal{W}(y,\theta) - \tfrac{1}{4}\tilde{\theta}^2\overline{DDA}(y,\theta)$$

$$\mathcal{L}^{N=2}_{Maxwell} = -\frac{1}{8}\int d^2\theta\, d^2\tilde{\theta}\,\mathcal{A}^2 + h.c. = \int d^2\theta\left(\frac{1}{2}\mathcal{W}^2 - \frac{1}{4}A\overline{DDA}\right) + h.c.$$

generalization: analytic prepotential $f(\mathcal{A})$

$$-\frac{1}{8}\int d^2\theta\, d^2\tilde{\theta}\, f(\mathcal{A}) + h.c. = \frac{1}{4}\int d^2\theta\left[f''(A)\mathcal{W}^2 - \frac{1}{2}f'(A)\overline{DDA}\right] + h.c.$$

Supersymmetry breaking

Spontaneous SUSY breaking

SUSY algebra in vacuum ($\vec{P} = 0, P^0 = H$): $\langle 0|\{Q, \bar{Q}\}|0\rangle = 2\langle 0|H|0\rangle$

exact SUSY ⇔ zero energy ; broken SUSY ⇔ positive energy

Indeed: $\mathcal{V}_{\text{scalar}} = \sum_{\substack{\text{chiral}\\ \text{superfields}}} |F|^2 + \frac{1}{2} \sum_{\substack{\text{vector}\\ \text{superfields}}} D^2$

$\Rightarrow$ broken SUSY: $\langle F\rangle$ or $\langle D\rangle \neq 0$

- SUSY can be broken only by a VEV of an auxiliary field

  scalar VEVs alone do not break SUSY if $\langle F\rangle = \langle D\rangle = 0$

  $\delta\psi = -\sqrt{2}\,\langle F\rangle\,\xi + \cdots \qquad \delta\lambda = \langle D\rangle\,\xi + \cdots$

F-breaking: O'Raifeartaigh model

$$W = -g\phi_0\phi_1^2 + m\phi_1\phi_2 + M^2\phi_0$$

$$F_0^* = \frac{\partial W}{\partial \phi_0} = -g\phi_1^2 + M^2 = 0 \quad \leftarrow \text{SUSY condition}$$

$$F_1^* = -2g\phi_0\phi_1 + m\phi_2 = 0 \qquad F_2^* = m\phi_1 = 0$$

$F_0 = F_2 = 0$: impossible $\Rightarrow$ supersymmetry breaking

Minimization of the scalar potential $\mathcal{V}$:

- $\phi_1 = \phi_2 = 0$, ϕ_0 arbitrary

  $F_0 = M^2,\ F_1 = F_2 = 0$ $\qquad \mathcal{V} = M^4$ $\qquad \frac{m^2}{2g} > M^2$

- $|\phi_1|^2 = \frac{M^2}{g} - \frac{m^2}{2g^2}$, $\phi_2 = \frac{2g}{m}\phi_0\phi_1$, ϕ_0 arbitrary

  $F_0 = \frac{m^2}{2g},\ F_2 = m\phi_1,\ F_1 = 0$ $\qquad \mathcal{V} = \frac{m^2}{g}M^2 - \frac{m^4}{4g^2}$ $\qquad \frac{m^2}{2g} < M^2$

D-breaking: Fayet-Iliopoulos model

$\xi \int d^4\theta V$ is SUSY and gauge invariant if V abelian $\Rightarrow$

$$\xi D \quad \rightarrow \quad D = \xi + e \sum_i q_i \left|\phi_i\right|^2 \qquad [\xi] = (\mathrm{mass})^2$$

Example: two massive chiral multiplets $\Phi_\pm$ with charges ± 1 under a $U(1)$

$$D = \xi + e\left(|\phi_+|^2 - |\phi_-|^2\right) = 0 \quad \leftarrow \text{SUSY condition}$$

$$W = m\phi_+\phi_- \quad \Rightarrow \quad F_+^* = m\phi_- = 0 \qquad F_-^* = m\phi_+ = 0$$

SUSY conditions incompatible $\Rightarrow$ supersymmetry breaking

Minimization of the scalar potential $\mathcal{V}$:

- $\phi_+ = \phi_- = 0 \quad F_+ = F_- = 0,\ D = \xi \quad \mathcal{V} = \frac{1}{2}\xi^2 \qquad \xi < \frac{m^2}{e}$
- $\phi_+ = 0,\ |\phi_-|^2 = \frac{e\xi - m^2}{e^2} \quad F_- = 0,\ D = \frac{m^2}{e},\ F_+ = m\phi_-$

$$\mathcal{V} = -\frac{m^4}{2e^2} + \frac{\xi m^2}{e} \qquad \text{otherwise}$$