Analele Universităţii de Vest din Timişoara
Vol. LV, 2011
Seria Fizică

ANALYSIS OF A NONLINEAR ANELASTIC MODEL AND ITS EXPERIMENTAL IDENTIFICATION

Dan Viorel Stănescu, Gheorghe Drăgănescu, Liviu Bereteu

Department of mechanics, Polytechnic University of Timişoara, Bd. M. Viteazu, No. 1,300222 Timişoara, Romania

Article Info
Received: 31 December 2010
Accepted: 17 January 2011

Keywords:
nonlinear vibrations,
parameter identification,
He's variational method

Abstract
It is introduced a new nonlinear anelastic model in order to describe relaxation phenomena in polycrystalline solids. The model is studied with the aid of He's variational iteration method and compared with the numeric solution. In order to identify the parameters of the model it is proposed a method based on the wavelet multiresolution analysis.

1. Introduction

The knowledge of the dynamic properties of materials and structures is very important in the theoretical dynamics, in the experimental investigation and a series of applications to the mechanical systems.

The aim of this paper is to introduce a nonlinear rheological model adequate to describe phenomena of relaxation facilitated by vibration, which is experimentally known in steel structures. Previous investigations are published in [1,2]. Our model is a generalization of standard anelastic model. In our model we used the assumption that in the case of polycrystalline materials the speed of relaxation ($\sim 1/\tau_0$) is not constant. In our case the speed of relaxation depends in a linear way on the stress *σ(t)* applied on the grain boundaries. Results that $1/\tau_0$ was replaced by $\frac{1}{\tau_0} \to \frac{1}{\tau}(\alpha\sigma + \beta)$.

We consider that to the system is applied a stress *σ(t)*, and the strain response will be *ε(t)*. The relation between the stress and the strain is expressed by the constitutive equation:

$$\frac{d}{dt}(\sigma - E_r\varepsilon) + \frac{1}{\tau}(\alpha\sigma + \beta)(\sigma - E_r\varepsilon) = \delta E\,\dot{\varepsilon}, . \tag{1}$$

where τ is a relaxation parameter, α is a small parameter, β is a parameter around of unity, E_r and $E_r + \delta E$ are relaxed and not relaxed longitudinal elastic modulus.

The excitation strain $\varepsilon(t)$ consists in a constant term ε_0 applied and a harmonic excitation:

$$\varepsilon(t) = u(t)(\varepsilon_0 + A\cos(\omega t)) \tag{2}$$

where $u(t)$ is the Heavyside function, A the amplitude of the strain and ω the angular frequency.

The stress response of this model is investigated in analytical way with the aid of He's variational iteration method, and compared with the numerical solution.

Finally, it is proposed a parameter identification procedure, based on wavelet multiresolution analysis method.

2. Problem Formulation

Following we will study the model described by the equation (1), using the function $x = \sigma - E_r\varepsilon$. This equation may be written as:

$$\frac{dx}{dt} + \frac{\alpha}{\tau}(x^2 + E_r\varepsilon x) + \frac{\beta}{\tau}X = (E_r + \delta E)\dot{\varepsilon}, \tag{1'}$$

For this case, to describe the relaxation processes facilitated by external applied vibrations, we will consider the excitation represented by the normal specific strain $\varepsilon(t)$ given by the function (2).

Initial condition is for this case: $x(0) = \delta E\varepsilon_0$.

Solving this kind of equation is quite difficult. To realise this we will use a variational iteration method [3].

The method can be applied to equation in the following form:

$$Lx(t) + Nx(t) = g(t), \tag{3}$$

where L denotes a linear operator, N denotes a non-linear operator, and $g(t)$ is a known given function.

The variational iteration method is e method based on Lagrange multipliers, which offers the possibility to writte the solution using a correction functional as:

$$x_{n+1}(t) = x_n(t) + \int_0^t \lambda(\tau)(Lx_n(\tau) + N\bar{x}_n(\tau) - g(\tau))d\tau, \tag{4}$$

where $x_n(t)$ is an initial aproximation possibly containg unknowns, and $\lambda(t)$ is a Lagrange multiplier, and $\bar{x}_n$ is contained in a term which is imposed a restrained variation $\delta\bar{x}_n(t) = 0$. The Lagrange multiplier may be determined from the stationarity condition of the correction functional:

$$\delta x_{n+1}(t) = 0.$$

To solve the equation (1) we will take:

$$Lx = dx/dt + \beta\tau_0 x, \qquad Nx = \alpha\tau_0(x^2 + E_r\varepsilon), \quad g = (E_r + \delta E)\dot{\varepsilon},$$

and we will use the following notations:

$$\alpha_\tau = \alpha/\tau, \qquad \beta_\tau = \beta/\tau$$

Requiring the minimize condition for the correction functional, it results:

$$\dot{\lambda}(\tau) - \beta_\tau\lambda(\tau) = 0, \quad [1 + \lambda(\tau)]|_{\tau=0} = 0.$$

Using these conditions, we can establish the Lagrange multiplier:

$$\lambda(\tau) = -\exp[\beta_\tau(\tau - t)]$$

We will use as first aproximation of the solution:

$$x_0(t) = c_0 e^{-\beta_\tau t} + \frac{\delta E\omega A(-\omega\cos(\omega t) + \beta_\tau \sin(\omega t))}{\beta_\tau^{\,2} + \omega^2}, \tag{5}$$

which represents the solution of the linear part of equation, where c_0 is an integration constant. We must underline that the method has a extremely powerfull convergence.After computing we will find first iteration of the solution:

$$x_1(t) = c_0 e^{-\beta_\tau t} + \frac{\delta E\omega A(-\omega\cos(\omega t) + \beta_\tau \sin(\omega t))}{\beta_\tau^{\,2} + \omega^2} +$$

$$+ \frac{\alpha_\tau}{2(\beta_\tau^{\,4} + 2\beta_\tau^{\,2}\omega^2 + \omega^4)(\beta_\tau^{\,2} + 4\omega^2)\beta_\tau\omega}$$

$$\times \{[8c_0^{\,2}\omega^7 + 2c_0^{\,2}\beta_\tau^{\,6}\omega + 12c_0^{\,2}\beta_\tau^{\,4}\omega^3 + 18c_0^{\,2}\beta_\tau^{\,2}\omega^5]e^{-2\beta_\tau t}$$

$$
\begin{aligned}
&\Big[-16c_0\omega^5\delta E A\beta_\tau{}^2+6A^2E_r\delta E\omega^6\beta_\tau+6\beta_\tau{}^3A^2E_r\delta E\omega^4-18\beta_\tau{}^3E_rAc_0\omega^4\\
&-8AE_rc_0\omega^6\beta_\tau-12\beta_\tau{}^5AE_rc_0\omega^2-4c_0\beta_\tau{}^6\delta E A\omega-20c_0\beta_\tau{}^4\delta E A\omega^3-2c_0{}^2\beta_\tau{}^6\omega\\
&-12c_0{}^2\beta_\tau{}^4\omega^3-18c_0{}^2\beta_\tau{}^2\omega^5-8c_0{}^2\omega^7+4\delta E^2\omega^7A^2-2\beta_\tau{}^7AE_rc_0+10\delta E^2\omega^5A^2\beta_\tau{}^2\\
&+2\beta_\tau{}^7AE_rc_0\cos(\omega t)+12\beta_\tau{}^5AE_rc_0\cos(\omega t)\omega^2+20c_0\beta_\tau{}^4\delta E A\cos(\omega t)\omega^3\\
&+4c_0\beta_\tau{}^5\delta E\omega^2A\sin(\omega t)+20c_0\beta_\tau{}^3\delta E\omega^4A\sin(\omega t)+18\beta_\tau{}^3E_rAc_0\omega^4\cos(\omega t)\\
&+16c_0\omega^5\delta E A\beta_\tau{}^2\cos(\omega t)+16c_0\omega^6\delta E A\sin(\omega t)\beta_\tau+8AE_rc_0\omega^6\cos(\omega t)\beta_\tau\\
&+4c_0\beta_\tau{}^6\delta E A\cos(\omega t)\omega)\Big]e^{-\beta_\tau t}\\
&-\beta_\tau{}^3E_rA^2\delta E\omega^4\cos(2\omega t)+\beta_\tau{}^5A^2E_r\delta E\omega^2\cos(2\omega t)+3\beta_\tau{}^4A^2E_r\delta E\omega^3\sin(2\omega t)\\
&+3\beta_\tau{}^2A^2E_r\delta E\omega^5\sin(2\omega t)-2A^2E_r\delta E\omega^6\beta_\tau\cos(2\omega t)-\beta_\tau{}^5A^2E_r\delta E\omega^2\\
&-5\beta_\tau{}^3A^2E_r\delta E\omega^4+\delta E^2\omega^3A^2\beta_\tau{}^4\cos(2\omega t)+4\delta E^2\omega^4A^2\beta_\tau{}^3\sin(2\omega t)\\
&-4A^2E_r\delta E\omega^6\beta_\tau-5\delta E^2\omega^5A^2\beta_\tau{}^2\cos(2\omega t)-2\delta E^2\omega^6A^2\beta_\tau\sin(2\omega t)\\
&-\delta E^2\omega^3A^2\beta_\tau{}^4-5\delta E^2\omega^5A^2\beta_\tau{}^2-4\delta E^2\omega^7A^2\Big\}
\end{aligned}
\tag{6}
$$

Actually, the calculus corresponding to find the first iteration are done in Maple.

From this first iteration we observe that the solution is containing besides the relaxation components like $\exp(-\beta_\tau t)=\exp(-\beta/\tau_0 t)$ also some componens like $\exp(-2\beta_\tau t)=\exp(-2\beta/\tau_0 t)$. We observe the appearance of harmonic and damped harmonic terms with pulse ω and 2ω. For the next iteration we made the calculus, but the expression for $x_2(t)$ is extremely elaborate so is not showed here. Is significant to tell that the expression contains exponentially relaxation components like $\exp(-3\beta_\tau t)=\exp(-3\beta/\tau_0 t)$.

Results the conclusion that even this model is not appropriate, applying external vibration the relaxing process is hurried.

Also we will study the model described by the equation (1) obtaining the numerical solution based on Runge-Kutta method.

For this case also, to describe the relaxation processes facilitated by external applied vibrations, we will consider the excitation represented by the normal specific strain $\varepsilon(t)$ given by the function (2). We will use also the initial condition:

$$x(0) = \delta E \varepsilon_0.$$

We will use the following values of the parameters:

$$E_r = 2000\,N/mm^2, \qquad \delta E = 2000\,N/mm^2, \ \tau_0 = 10s, \qquad \varepsilon_0 = 0.02, \qquad \beta = 1.$$

The working solution of the probleme is done in MathCAD.

The time interval on which is searched the solution was $f \in [0, T]$, where $T = 20s$, the solution being sampled in $n = 301$ points with a sample rate $\Delta t = T/(n-1)$.

The following cases was studied:

I. The case of liniar model with $\alpha = 0$, situation a) of the free response, without vibrations $A = 0$ and b) response facilitated by vibrations having $A = \varepsilon_0/8$, and frequency $\nu = 2.4Hz$. The plot $\sigma(t)$ is represented in Fig. 1 in black for $A = 0$ and in red for $A = \varepsilon_0/8$.

II. The case of non-linear model with $\alpha = 0.03$, situation a) of free response, without vibrations $A = 0$ and b) response facilitated by vibrations with $A = \varepsilon_0/8$, and frequency $\nu = 2.4Hz$. The plot $\sigma(t)$ is represented in Fig. 2 in black for $A = 0$ and in red for $A = \varepsilon_0/8$.

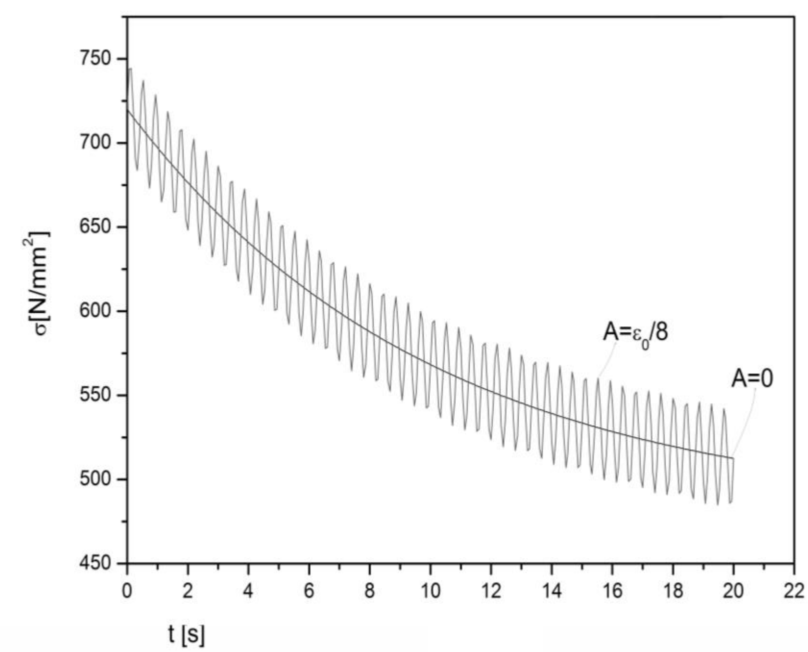

Figure 1. The stress relaxation for linear model for $A = 0$ and $A = \varepsilon_0/8$.

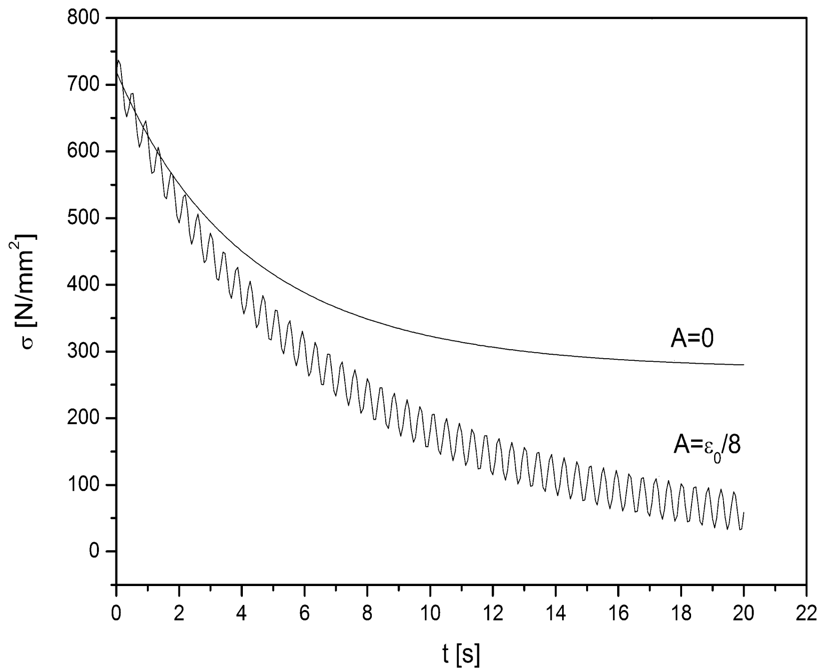

Figure 2. The stress relaxation for nonlinear model for $A=0$ and $A=\varepsilon_0/8$

The parameters of this model can be identified from experimental data using wavelet multiresolution series expansion [4]. A signal x(t) can be expanded as:

$$x(t)=\beta_0\Phi_0+\beta_1\Psi_1+...+\beta_n\Psi_n+..., \qquad (7)$$

where $\Psi(t)$ is a mother wavelet function, $\Phi(t)$ is the corresponding scale function and:

$$\beta_0=\int_{-\infty}^{\infty}x(t)\Phi(t)dt, \quad \beta_{2^n+l}=2n\int_{-\infty}^{\infty}x(t)\Psi(2^n t-l)dt. \qquad (8)$$

The equation (1') can be written in the formal form:

$$a\dot{x}+bx^2+c\varepsilon x+dx=e\dot{\varepsilon},$$

The measured quantities $\dot{x}(t), x^2(t), \varepsilon(t)x(t), x(t), \dot{\varepsilon}(t),$ can be expanded according (7) and (8) as $\dot{x}_k, x^2{}_k, (\varepsilon x)_k, x_k, \dot{\varepsilon}_k$, where k = 0, 1, 2, …, n. Results the equation:

$$\begin{bmatrix} \dot{x}_0 & x_0^2 & (\varepsilon x)_0 & x_0 \\ \dot{x}_1 & x_1^2 & (\varepsilon x)_1 & x_1 \\ ... & ... & ... & ... \\ \dot{x}_n & x_n^2 & (\varepsilon x)_n & x_n \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}=\begin{bmatrix} e\dot{\varepsilon}_0 \\ e\dot{\varepsilon}_1 \\ ... \\ e\dot{\varepsilon}_n \end{bmatrix},$$

of type [A][X]=[B] where A is a (n×4) matrix, B is a (4×1) matrix and is a (n×1) matrix. The parameters a, b, c, d of this over-determined equation can be written as:

$$[X] = ([A]^t [A])^{-1} [A]^t [B]$$

where "t" represents the transpose matrix and "-1" the corresponding inverse.

Conclusions

From fig. 1 we observe that in the case of linear model the relaxation process in the presence of vibrations is perfectly superposed to the free process. Results that applying vibrations does not increase the speed of relaxation process.

From fig. 2 we observe that in the case of non-linear model the relaxation process in the presence of vibrations does not fit the free case, the non-oscillatory component of the curve falling faster than the free response. Results that applying vibrations speeds-up the relaxation process in the case of non-linear model.

Acknowledgements

This work was partially supported by the strategic grant POSDRU 6/1.5/S/13, (2008) of the Ministry of Labor, Family and Social Protection, Romania, co-financed by the European Social Fund - Investing in People.

References

1. G. E. Drăgănescu, A. Ercuţa, L. Bereteu, Journal of Physics: Conference Series, **96** (2008) 012178.
2. Gh. E. Drăgănescu, A. Ercuţa, *Journal of Optoelectronics and Advanced Materials*, **5** (2003) 301.
3. J. H. He, *Int. J. Mod. Phys.*, **B 20** (2006) 1141.
4. D. E. Newland, An Introduction to Random Vibrations and Wavelet Analysis, Dover Publ. Mineola and NY, 1993.