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AHARONOV-BOHM OSCILLATIONS IN CONCENTRIC QUANTUM RING SYSTEMS

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Abstract
We consider a system of concentric rings with a variable M number of sites in each ring. The influence of the magnetic field is also taken into account. The system is described using the attractive Hubbard Hamiltonian for electron-hole pairs. We study the evolution of Aharonov-Bohm oscillations when rings are added. The case of the small distance between rings can be approximated with the continuum case. When the distance between the rings is large enough, we can find that the AB oscillations decrease very rapidly once we increase the circumference of the ring.

1. Introduction

Remarkable progress in the field of controlled production of nanostructured systems such as quantum rings, allow the study of quantum transport properties through artificially fabricated nanostructures [1-3]. Many interesting quantum effects can be also found in coupled nanostructures where the electronic transport is affected by the phenomena of quantum interference. Quantum rings offers the possibility of trapping magnetic flux in their interior, showing new and interesting quantum phenomena, such as the Aharonov-Bohm effects [4]. When the ring width becomes large the Aharonov-Bohm effects are suppressed.

In this work, we consider a system consisting of nano-rings with an electron and hole threaded by a perpendicular magnetic flux. We study in a controlled way how the excitonic Aharonov-Bohm oscillations evolve when additional rings are added. We describe the electron-hole system on the ring by Hubbard model.

In the first case we consider a single ring, with a finite number of sites. We study the 1D model and the analytical expressions are obtained and compared to the 1D continuum approach.

In the case of two rings we compare the Aharonov-Bohm oscillations with those obtained in the 1D case.

2. Model and formulation

An interesting nanodevice is a system of N rings and M sites in each ring, threaded by magnetic field perpendicular to the lattice as shown in Fig.1.

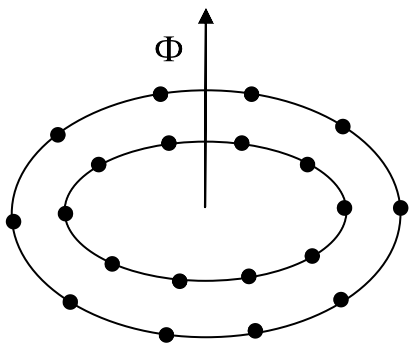

Figure 1. Schematic view of 2 rings and M sites in each ring.

In our work we consider up to two annular rings with a single electron and a single hole. The interaction potential is short-ranged and the system is described by fermionic Hubbard model. The 1D Hubbard model with the neighbor hopping was solved exactly by Lieb and Wu [5]. The case involving magnetic flux was also considered [6-9]. We suppose that only the possibility of nearest– neighbor hopping exists.

In the general case the full system can be described by the Hamiltonian:

$$\hat{H} = -\gamma\sum_{n=1}^{N}\sum_{m=1}^{M} a_{n,m}^{+}a_{n,m}b_{n,m}^{+}b_{n,m} - \sum_{n=1}^{N-1}\sum_{m=1}^{M} t_n^{\perp}[a_{n,m}^{+}a_{n+1,m} + a_{n+1,m}^{+}a_{n,m} + \mu(b_{n,m}^{+}b_{n+1,m}b_{n,m})] - \\ -\sum_{n=1}^{N}\sum_{m=1}^{M} t_n^{\|}[e^{2\pi i\varphi_n/M}(a_{n,m}^{+}a_{n,m+1} + \mu(b_{n,m+1}^{+}b_{n,m}) + e^{-2\pi i\varphi_n/M}(a_{n,m+1}^{+}a_{n,m} + \mu(b_{n,m}^{+}b_{n,m+1})] \tag{1}$$

where $a_{n,m}^{+}$, $a_{n,m}$ and $b_{n,m}^{+}$, $b_{n,m}$ are raising (lowering) operators for electrons and holes, satisfying the standard fermionic anti-commutation relations. The parameters $t_n^{\|}$ represent the hopping coefficient between neighboring sites along the n-th ring, and $t_n^{\perp}$ is the hopping coefficient between neighboring sites on the n-th and $n+1$-th ring.[10] The notation $\varphi_n = \Phi_n / \Phi_0$ is used, where Φ_0 is the flux quanta.

$$t_n^{\perp} = \frac{\varepsilon}{(r_{n+1} - r_n)^2}, \tag{2}$$

$$t_n^{\|} = \frac{\varepsilon}{4r_n^2 \sin(\pi / M)}. \tag{3}$$

The effective mass of the electron and hole has a fixed value, so the inter- and intra-ring hopping coefficients have an inverse-square law dependence [11] on the radius of the ring. As usually ε =1 and μ=0.2 [12]. Also r_n is the radius of the n-th ring and $d_n = 2r_n \sin(n/M)$ is the chord distance between neighboring lattice points of the *n*-th ring.

3. Results and discussions

3. 1. Case for a single ring

In this case the Hamiltonian matrix is:

$$H_1^{(k)}(\gamma, q) = -\begin{pmatrix} \gamma & q^* & 0 & . & . & q \\ q & 0 & q^* & 0 & . & 0 \\ 0 & q & 0 & q^* & . & . \\ . & . & . & . & . & . \\ . & . & . & q & 0 & q^* \\ q^* & . & . & . & . & 0 \end{pmatrix} \tag{4}$$

with

$$q = t^{\|} e^{2\pi i \varphi / M} \left(\mu + e^{-ik}\right) \tag{5}$$

Considering the wave function of the form [13]:

$$|\psi_T\rangle = c_0 \sum_{j=1}^{M} (\hat{T}/\tau)^{j-1} \begin{vmatrix} 10...0 \\ 10...0 \end{matrix}\right\rangle + c_1 \sum_{j=1}^{M} (\hat{T}/\tau)^{j-1} \begin{vmatrix} 10...0 \\ 01...0 \end{matrix}\right\rangle + ... + c_{M-1} \sum_{j=1}^{M} (\hat{T}/\tau)^{j-1} \begin{vmatrix} 10...0 \\ 00...1 \end{matrix}\right\rangle, \tag{6}$$

the coefficients are:

$$c_n = \frac{c_0 e^{in\theta}}{\sinh[M\nu]} \left\{ \sinh\left[(M-n)\nu\right] + e^{-iM\theta} \sinh(n\nu) \right\} \tag{7}$$

where:

$$\theta = \arg(q) \tag{8}$$

and

$$\cosh(\nu) = -E/2|q| \tag{9}$$

The secular equation is:

$$\frac{\tanh(M\nu)}{\sinh(\nu)} = \frac{2|q|}{\gamma}\left[1 - \frac{\cos(M\theta)}{\cosh(M\nu)}\right] \tag{10}$$

and the bound state solutions exist if

$$\gamma > \frac{2|q|}{M}(1 - \cos M\theta) \tag{11}$$

Figure 2 shows the oscillations of the ground state energy versus the magnetic field. The Aharonov-Bohm oscillations decrease rapidly with increasing ring radius and the number of the sites on the ring. The maximum amplitude of the Aharonov-Bohm oscillations corresponds to integer and half-integer multiples of flux quanta, and the period is $T = \Phi/\Phi_0$.

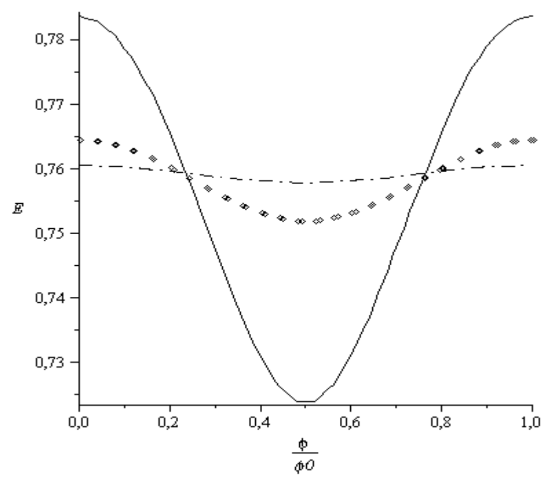

Figure 2. The Aharonov-Bohm oscillations of exciton energy corresponding to three values of the single ring site number, for γ=2, d=1, ε=1, μ=0.2, and k=0. M=5 (continuous line), M=7 (point line) and M=9 (dashed line)

3. 2. Case for 2 rings

In this case the Hamiltonian matrix is:

$$H_2^{(k)} = \begin{pmatrix} H_{11} & H_{12} & H_{13} & 0 \\ H_{12}^* & H_{22} & 0 & H_{13} \\ H_{13}^* & 0 & H_{33} & H_{12} \\ 0 & H_{13}^* & H_{12}^* & H_{44} \end{pmatrix}, \tag{12}$$

where

$$
\begin{aligned}
H_{11} &= H_1^{(k)}(\gamma, q_1), \\
H_{12} &= -\mu t_1^{\perp} 1, \\
H_{13} &= -t_1^{\perp} 1, \\
H_{22} &= H_1^{(k)}(0, q_{12}), \\
H_{33} &= H_1^{(k)}(0, q_{21}), \\
H_{44} &= H_1^{(k)}(\gamma, q_2)
\end{aligned} \tag{13}
$$

with

$$
\begin{aligned}
q_1 &= t_1^{\|} e^{2\pi i \varphi_1 / M} \left(\mu + e^{-ik}\right) \\
q_2 &= t_2^{\|} e^{2\pi i \varphi_2 / M} \left(\mu + e^{-ik}\right) \\
q_{12} &= t_1^{\|} e^{2\pi i \varphi_1 / M} e^{-ik} + t_2^{\|} e^{2\pi i \varphi_2 / M} \mu \\
q_{21} &= t_2^{\|} e^{2\pi i \varphi_2 / M} e^{-ik} + t_1^{\|} e^{2\pi i \varphi_1 / M} \mu
\end{aligned} \tag{14}
$$

In Figure 3 we plot the ground state energy as a function of flux measured in flux quanta units. We observe the presence of slow and fast oscillations. The fast oscillations have the period of $T_f = \Phi_1/\Phi_0$ and the slow ones of $T_s = M/(\Phi_2/\Phi_1 - 1)$.

In the case of small M the presence of both oscillations can be observed. When the distance between the rings is large the fast oscillations are weakly modulated by slow ones. This is explained by the small probability of inter-ring hopping and of the weak influence of the second ring. Decreasing the distance between the rings shows the increasing amplitude of the slow oscillations compared to the fast ones.

When M is large enough the presence of the oscillation with the period Φ_1/Φ_0 is completely absent. This finding is consistent with the single ring case. The slow oscillation is always present and has the period indicated. When the distance between the rings is small and the probability of inter-ring hopping is relatively large we can identify an oscillation due to this term in the Hamiltonian.

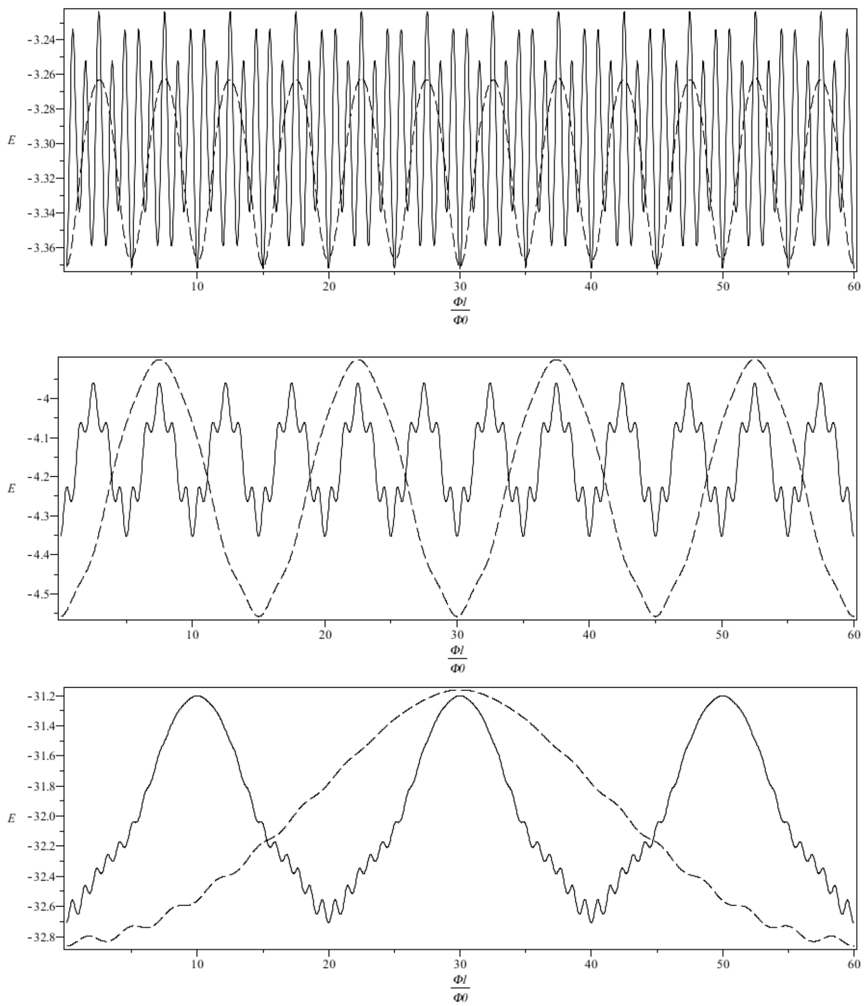

Figure 3. Ground state energy oscillations as a function of Φ_1/Φ_0 for Δr=1.5 and $\Phi_2/\Phi_1 = 4$ (top), Δr=0.8 and $\Phi_2/\Phi_1 = 2$ (centre) and Δr=0.2 and $\Phi_2/\Phi_1 = 1.25$ (bottom). The parameters are: γ=2, ε=1, μ=0.2, *k*=0. Continuous lines correspond to *M*=5 and dashed lines to *M*=15

Conclusions

In this paper we studied the Aharonov-Bohm oscillations in 2D ring structures described by an attractive fermionic Hubbard model in the presence of magnetic field. The existence of bound states of electrons and holes identical to spin-singlets when using the standard electronic interpretation is well known [5], and has been previously also been studied in the framework of quantum breathers [13]. In these cases, if the anharmonicity is strong, there is an extended bloch state with two or more strongly correlated particles [14].

In the case of a single ring we obtained analytical formulae for the energy spectrum of the electron. The Aharonov-Bohm oscillations show the usual periodicity pattern. The decay of the oscillations with increasing the number of the sites (and the circumference of the ring) can be observed [15, 16]. In the case of 2 rings, the magnetic flux dependence of the ground state energy shows fast and slow oscillations [17, 18]. The amplitude of two oscillations is inversely proportional with the distance between the rings. The period of the slow oscillations is equal to $T_f = \Phi_1 / \Phi_0$ and that of the fast one is $T_s = M / (\Phi_2 / \Phi_1 - 1)$. In the case of large number of sites per ring the oscillation due to inter-ring hopping can also be identified.

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References

1. S. Datta, *Transport in Mesoscopic Systems*, Cambridge University Press, (1995)
2. T. Chakraborty, *Quantum dots*, Elsevier Science, North Holland, (1999)
3. A. Lorke, R. J. Luyken, A. O. Govorov, J. P. Kothaus, J. M. Garcia and P. M. Petroff, Phys. Rev. Lett. **84** (2000) 2223
4. Y. Aharonov, D. Bohm, Phys. Rev. **115** (1959) 485
5. E. H. Lieb and F. Y. Wu, Phys. Rev. Lett. **20** (1968) 1445
6. B. S. Shastry and B. Sutherland, Phys. Rev. Lett. **65** (1990) 243
7. B. Sutherland and B. S. Shastry, Phys. Rev. Lett. **65** (1990) 1833
8. E. Papp, C. Micu, *Low-dim. nanoscale syst. on discrete spaces*, W. Sci., Singapore, (2007)
9. E. Papp, C. Micu, L. Aur, D. Racolta, Physica E **36** (2007) 178
10. F. Palmero, J. Dorignac, J. C. Eilbeck, R.A. Roemer, Phys. Rev. B **72** (2005) 075343
11. K. Maschke, T. Meier, P. Thomas and S. W Koch, Eur. Phys. J. B **19** (2001) 599
12. H. Hu, J. L. Zhu, D. J. Li and J. J. Xiong, Phys. Rev. B **63** (2001) 195307
13. A.C. Scott, Nonlinear Science (2nd. ed.), OUP, Oxford (2003)
14. J. Dorignac, J. C. Eilbeck, M. Salerno and A. C. Scott, Phys. Rev. Lett. **93** (2004) 025504
15. F. Göhmann and V. E. Korepin, Phys. Lett. A **260** (1999) 516
16. P. A. Orellana, M. Pacheco, Phys. Rev. B, **71** (2005) 235330
17. D. Racolta, Rom. Journ. Phys., **54**, 7–8 (2009) 659
18. A. V. Maslov and D. S Citrin, Phys. Rev. B **67** (2003) 121304(R)