

## ABOUT THE DIFFUSION IN MAGNETIC STOCHASTIC TURBULENCE

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### Abstract

The test particles behavior in a stochastic anisotropic turbulent magnetic field was studied by the decorrelation trajectory method. Specific decorrelation trajectories in given subensemble and diffusion coefficients were analyzed for different values of the specific parameters of the magnetic turbulence.

### 1. Introduction

A central issue for fusion plasma at high temperature consists in the study of turbulence phenomena. The magnetic turbulence appears as a plausible candidate for determining the anomalous transport properties. In toroidal confinement geometry, such as in a tokamak any particle makes radial displacements, thus enhancing the radial transport. The study of transport in such stochastic magnetic fields is then of great importance for the fusion. The intensity of magnetic fluctuations is measured by the dimensionless magnetic Kubo number  $K_m$  and the magnetic shear measured by  $K_s$ . We show that in a stochastic turbulent magnetic field there exists an intrinsic decorrelation mechanism. The stochastic anisotropy is also taken into account and the corresponding stochastic parameter  $\Lambda$  is introduced. Because of the existence of the above mentioned specific parameters, a richer class of behaviors of the diffusion coefficients can be observed.

The paper is organized as follows. The Langevin equations for the guiding center for a stochastic stationary turbulent magnetic field, the framework of DCT method, the analytical expressions of the Lagrangian correlations and the diffusion tensor components are established and calculated.

## 2. The Langevin equations for the magnetic field

The magnetic field  $\mathbf{B}$  is assumed to have a strong constant background component  $B_0$  in the  $Z$ -direction and stochastic components in the perpendicular direction,  $b_X$  and  $b_Y$  with the dimensionless parameter  $\beta$  determining the strength of the perturbations and  $\mathbf{X} = (X, Y)$  the position. We consider sheared geometry for the confining magnetic field, i.e.

$$\mathbf{B}(\mathbf{X}; Z) = B_0 \left\{ \mathbf{e}_Z + \beta b_X(\mathbf{X}; Z) \mathbf{e}_X + \left[ \beta b_Y(\mathbf{X}; Z) + XL_s^{-1} \right] \mathbf{e}_Y \right\}. \quad (1)$$

The magnetic field fluctuations are described by the dimensionless functions  $b_i(\mathbf{X}, Z)$ ,  $i=(X, Y)$ , taken to be *Gaussian processes*. The dimensionless coordinates  $\{\mathbf{x}=(x, y), z\}$  are defined as:

$$x = X / \lambda_x, y = Y / \lambda_y, z = Z / \lambda_z.$$

The dimensionless Langevin equations for the anisotropic turbulent magnetic field lines are

$$\begin{aligned} \frac{dx(z)}{dz} &= K_m b_x(\mathbf{x}; z), \\ \frac{dy(z)}{dz} &= \Lambda K_m b_y(\mathbf{x}; z) + \Lambda K_s x(z). \end{aligned} \quad (2)$$

The parameters involved in the previous formulas are as follows:

- $K_m = \beta (\lambda_X/\lambda_Z)$  that is the magnetic Kubo number,  $\Lambda = \lambda_X/\lambda_Y$  that is the stochastic anisotropy parameter;
- $K_s = \lambda_Z/L_s$  that is the shear Kubo number.
- $\lambda_X, \lambda_Y, \lambda_Z$  are the correlation lengths.

The parameter values we used for slab sheared geometry are:  $\beta \approx 5 \cdot 10^{-3}$  for the strength of the magnetic perturbations, and  $\lambda_X \approx \lambda_Y = 10^{-2}$  m,  $\lambda_Z = 1$  m for the correlation lengths.

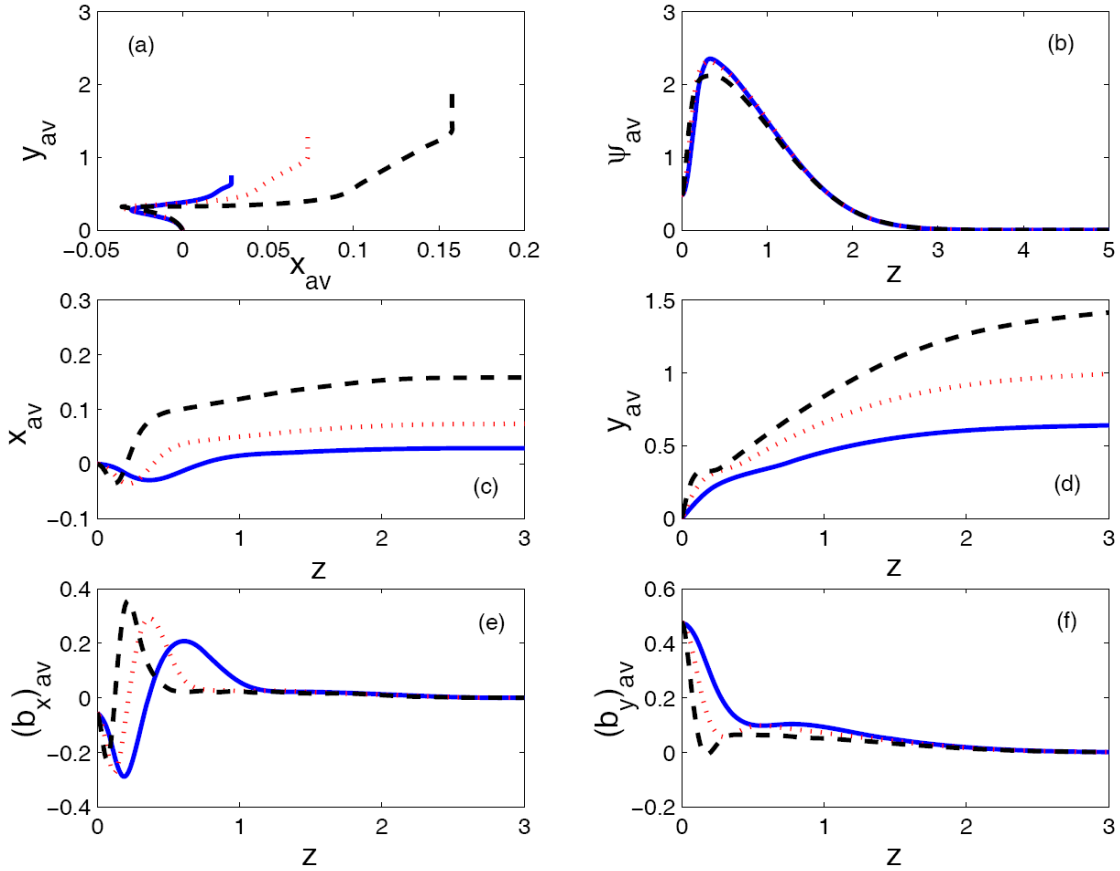
## 3. DCT results: global quantities

The global averages for the trajectories, magnetic potential, solutions of the system (2) and the stochastic components of the magnetic field are represented in Figure 1. The averages are practically the super positions of the information obtained by integrating the system (2) in the entire set of sub-ensembles specific to the DCT method [1, 3-5]. The average,  $(A)_{av}$  of an arbitrary quantity  $A$  is obtained for the whole set of realizations by summing up the contributions over the sub-ensembles:

$$(\mathbf{A}(z))_{\text{av}} = \int_{-\infty}^{+\infty} d\psi^0 \int_{-\infty}^{+\infty} db_x^0 \int_{-\infty}^{+\infty} db_y^0 P(\psi^0, b_x^0, b_y^0) A^S(x^s, y^s, z), \quad (3)$$

where  $P$  is the probability of the potential and of the stochastic components to have the values  $(\psi^0, b_x^0, b_y^0)$  at the moment  $z = 0$ .

The anisotropy parameter  $\Lambda$  is varied between 0.2 and 5.0 (through the variation of  $\lambda_Y$ ) and  $K_m$  is varied in the range  $[0, 6]$ . In our presentation  $\Lambda$  is considered to be equal to 5.0 (through the variation of  $\lambda_Y$ ),  $K_s = 2$  and  $K_m$  is: 0.5, 1 and 2.

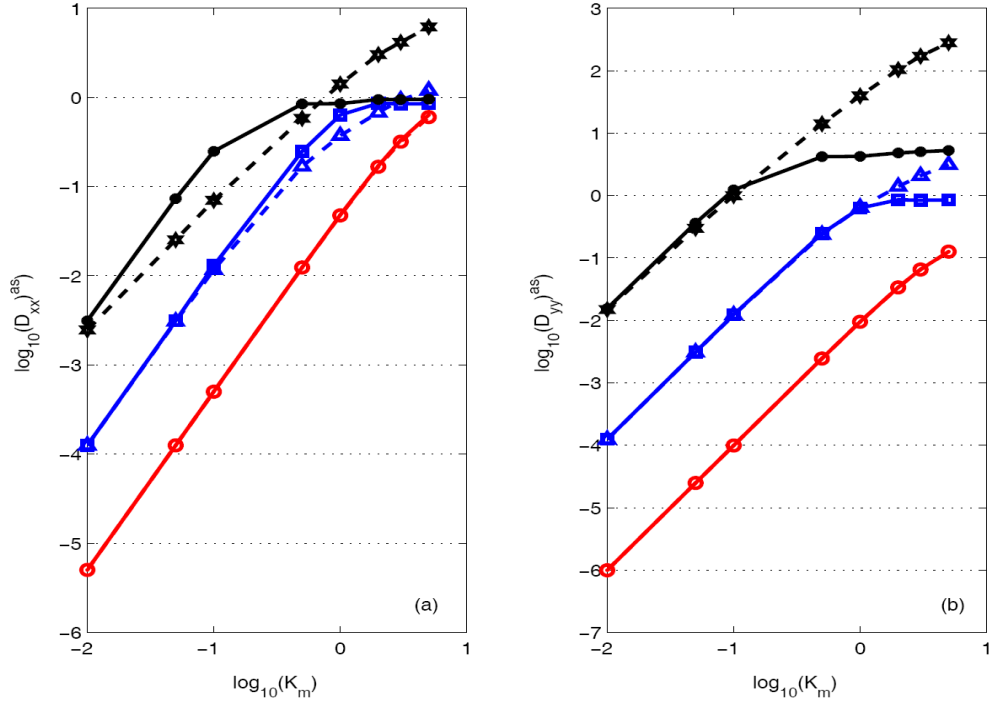


**Figure 1.** The representation of the global DCT quantities for three values of the magnetic Kubo number. (1) solid:  $K_m = 0.5$ ; (2) dot:  $K_m = 1$ ; (3) dash:  $K_m = 2$ . The other parameters are  $K_s = 2$  and  $\Lambda = 5$ . In (a): The mean DCT trajectory; in (b): The average magnetic potential; in (c) and (d): The average coordinates; in (e) and (f): The averaged fluctuating magnetic field components.

The running diffusion coefficients for the magnetic field are determined as:

$$D_{xx}(z) = \frac{\langle [x(z) - x(0)]^2 \rangle}{2z}, \quad D_{yy}(z) = \frac{\langle [y(z) - y(0)]^2 \rangle}{2z}. \quad (4)$$

In the next figure is given the Log-Log representation of both the radial and poloidal asymptotic diagonal diffusion coefficients as functions of  $K_m$ .



**Figure 2.** Log-Log representation of (a) the radial and (b) the poloidal asymptotic diagonal diffusion coefficients as functions of  $K_m$ . From top to bottom in both figures: (1) solid-asterisk: x-anisotropy  $\Lambda = 5$  and no magnetic shear; (2) dash-star: x-anisotropy  $\Lambda = 5$  and shear  $K_s = 2$ ; (3) solid-square: isotropy  $\Lambda = 1$  no shear; (4) dash-triangle: isotropy  $\Lambda = 1$  and weak shear  $K_s = 2$ ; (5) solid-circle: y-anisotropy  $\Lambda = 0.2$  and no shear.

## Conclusions

Whatever the value of the shear parameter, an increase in the anisotropy parameter produces an increase in the trapping effect. The stationary mean coordinate is always much larger in the Oy (poloidal) than in Ox (radial) direction. The asymptotic position in the poloidal direction takes a longer time to be reached as the shear is increased. For the averaged magnetic potential: whatever the parameters  $\Lambda$  and  $K_m$ ,  $(\psi(z))_{av}$  of  $z$  increases up to a maximum value and then decreases to zero, the time  $z$  and value of the maximum depending on all the parameters (see Figure 1).

In Figure 2, all the curves show a linear portion for small  $K_m$  with a slope approximately equal to 2, which represents the quasilinear value. In the shearless cases, the asymptotic diffusion coefficient appears to be independent of  $K_m$  for values of  $\Lambda$  larger or equal to the isotropy value 1, up to a slope about 1 for small values of  $\Lambda$ . In the presence of shear, for which the trapping phenomenon is important, the asymptotic slopes do not go to low values. Furthermore, the slope

is larger for larger  $\Lambda$ . We conclude that the trapping is not strongly modified when going from isotropy to an anisotropy specific to the solar wind, i.e.  $\Lambda > 1$  like the one studied in [6].

In the shearless case and for large stochastic y-anisotropy  $\Lambda = 0.2$ , the trapping is reduced (see Figure 2).

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