Analele Universității de Vest din Timişoara
Vol. LIV, 2010
Seria Fizică

SIMULATING THE ABELIAN CHERN-SIMONS THEORY ON THE MOYAL PLANE

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Abstract
We perform Monte Carlo simulation of the Noncommutative Abelian Chern-Simons theory on the Moyal (fuzzy) plane using the action proposed by Polychronakos and Nair. Using the simple metropolis algorithm, we see that the action and the specific heat of the system approaches zero for small values of the non-commutativity parameter θ on the Moyal plane but shows fluctuations with increasing value of the parameter. We compare this with the classical model.
Keywords: Noncommutative geometry, topological field theory, numerical simulation.

1. Introduction

Noncommutativity of spacetime was initially considered by Heisenberg as a possible mechanism to control the divergences in quantum field theory, though the first paper documented to discuss the quantisation of spacetime is due to Snyder [2]. However, the subsequent development of the highly successful renormalisation program pushed this idea into obscurity. Yet, over the last ten years or so, this idea has reappeared due to the development of the Noncommutative geometry due to Alain Connes [3] and also due to new results in string theory [4].

Noncommutative Geometry (NCG) can be thought of as a framework to understand geometry within an algebraic setting as one thinks of the manifold as a representation space of the algebra of the functions defined on the manifold. So, it might be interesting to study "quantum" geometry using this framework. As a preliminary step in this direction one is tempted to study topological field theories in this framework as they manages to encode the topology of the underlying spacetime, on which the theory is defined.

In this work, we report on our attempts to numerically simulate a simple topological field theory, namely, the noncommutative Abelian Chern-Simons theory [5] defined on a spacetime whose spatial section is the Moyal plane which is defined by the operator relation $[x,y] = i\,\theta$, θ being the parameter measuring the noncommutativity. In this framework, the coordinates x, y are then represented by NxN dimensional Hermitian matrices.

2. The Model

The basic object for defining a gauge theory on a noncommutative manifold happens to be the covariant derivative operator, which will also be represented by an NxN dimensional Hermitian matrix. In particular, the action for the noncommutative Abelian Chern-Simons model on a 3-manifold is given by (our convention is the same as in [5]):

$$S = 2\pi\theta\,\varepsilon^{abc}\,\mathrm{Tr}\left(\frac{2}{3}D_a[D_b, D_c] + \omega_{ab} D_c\right) \qquad (1)$$

where D_a, with a=1, 2, 3, are NxN Hermitian matrices and ω_{ab} is the symplectic form, which, on the Moyal plane, has non-vanishing components $\omega_{12} = -\,\omega_{21} = \frac{1}{\theta}$, with rest of the elements being equal to zero. This choice is dictated by the fact that the symplectic form will measure the noncommutativity of the spacetime derivatives and thus will be the matrix inverse of the commutator of the spacetime coordinates.

The resulting equation of motion for the system is given by

$$[D_a, D_b] = i\,\omega_{ab} \qquad (2)$$

which is basically the equation for flat connections, similar to the field equations for the commutative Chern-Simons theory.

3. Results and Discussions

To get a preliminary feeling for the non-perturbative theory, we have performed numerical simulation using the standard Metropolis algorithm impelemented in GNU-GCC, for the system described by (1) and have measured the action S and specific heat C_v of the system. The code had been verified by reproducing the standard results for the scalar fields. Note that we have employed the relation $C_v = \langle S^2 \rangle - \langle S \rangle^2$. We have also taken a sample size of 8192 and varied the noncommutativity parameter θ for various values of N ranging from 2 to 10. However, here we have only plotted the values N=4, 6, 8 in the accompanying figures, though the pattern is similar for other values of N. The results point to the fact that for small values of θ, both the action and the specific heat of the system is zero, which is similar to its commutative analogue whose on-shell action should be zero and the same for the specific heat. Though it might appear weird that the action is picking up both positive and negative values – this is not unnatural as the action contains only the odd powers of the covariant derivative of D. Also one should remember that the commutative version of the Chern-Simons action is a first order action and thus bound to suffer from these problems. This is verified by noting that the fluctuation in S occurs is both directions.

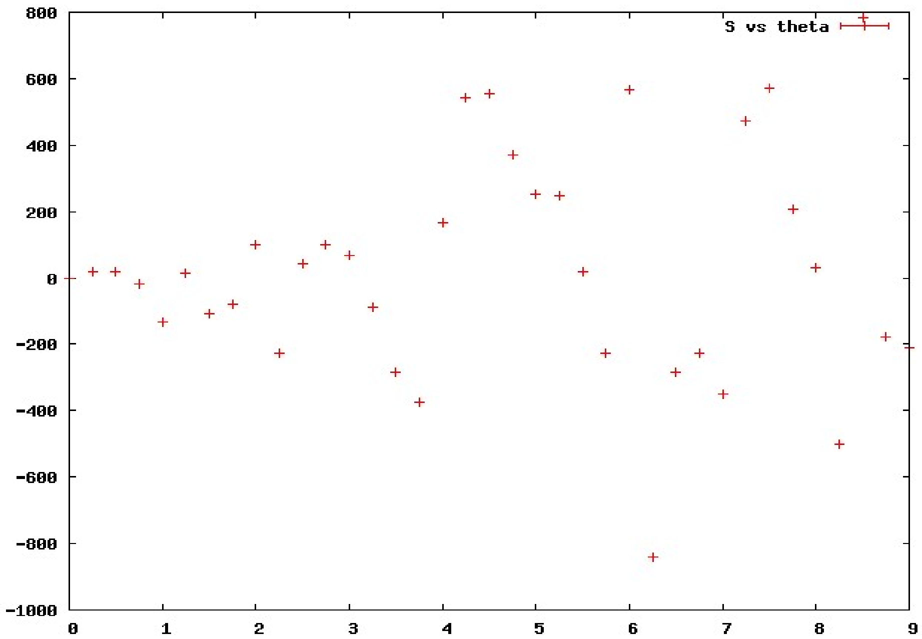

FIGURE 1: Action S vs. θ for N=4.

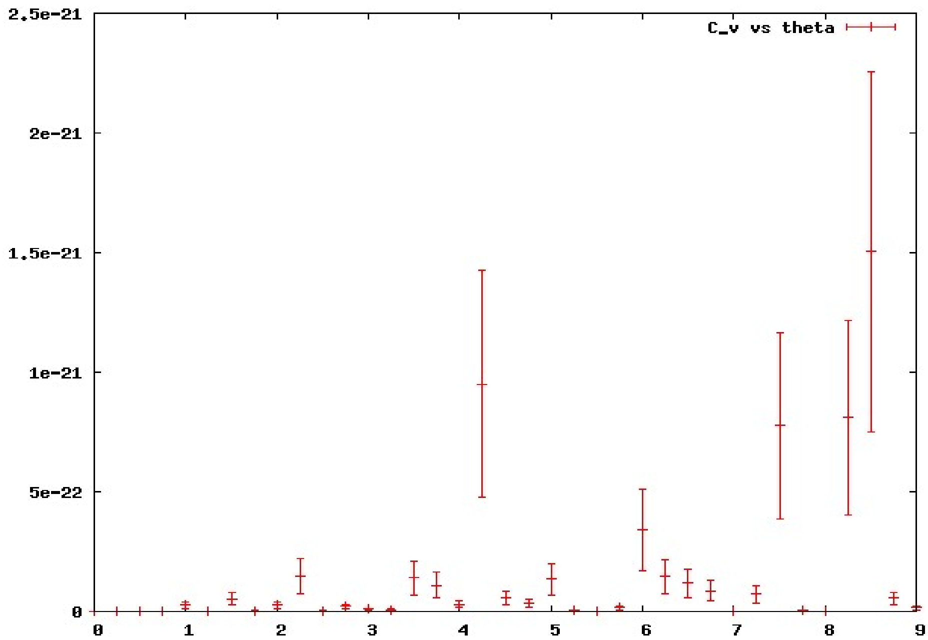

FIGURE 2. C_v vs θ for N=4.

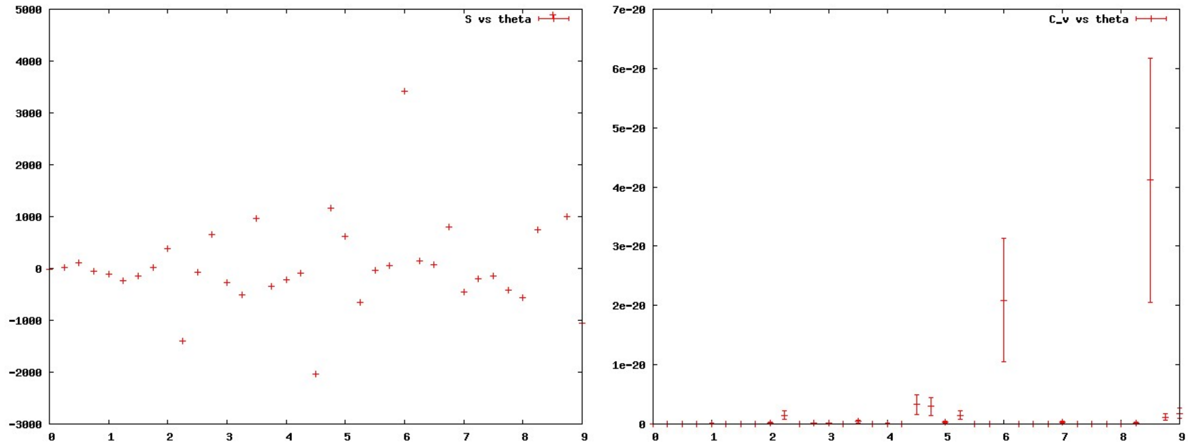

FIGURE 3. The Action S vs θ and the specific heat Cv vs θ graph for N=6.

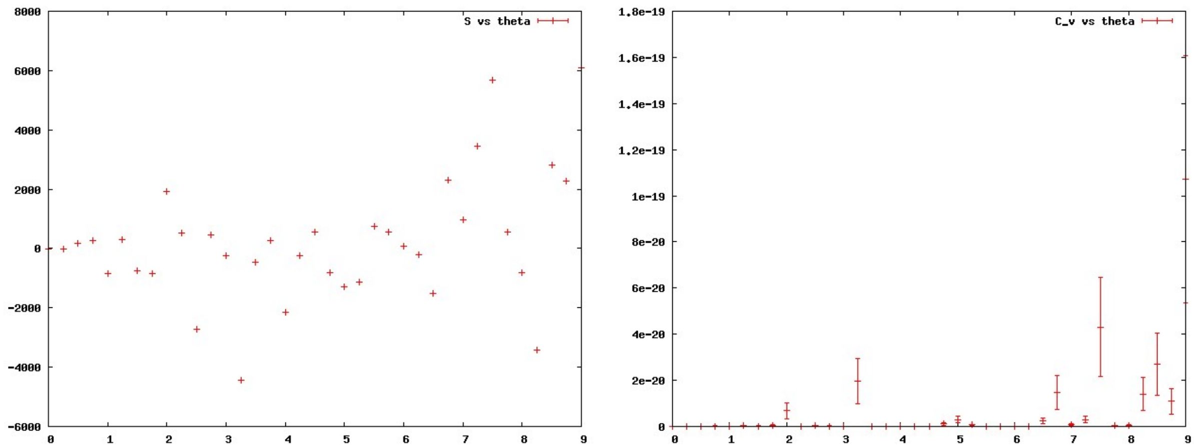

FIGURE 4. The Action S vs θ and the specific heat Cv vs θ graph for N=8.

It is interesting to check whether the uncertainty in the values of the action has to do with the noncommutativity of the underlying geometry. The increment of the fluctuation in both the action and the specific heat increasing theta seems to point to this fact. Whether this is indeed the case can be clarified by performing similar simulation on the compact geometries like the fuzzy torus and also simulating topologically massive gauge theories. We hope to report on these in the near future.

Acknowledgements

The financial aid programme of BRAC University, Dhaka, Bangladesh and generous support of the European Union's Erasmus Mundus Mobility with Asia (EMMA) exchange programme supported this work. NKB wishes to thank Lucian Blaga University for hospitality where a part of this work was done. Both of us wish to express our sincere gratitude to Prof V. P. Nair, City University, New York, for his insightful comments on the results. We also thank Prof. D. Chicea, Physics dept., Lucian Blaga University of Sibiu for his constant encouragement and the conference organisers for allowing us to present our results.

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