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LANDAU'S THEORY OF SECOND-ORDER PHASE TRANSITIONS AS APPLIED TO FERROMAGNETS

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Landau proceeds from a non-equilibrium thermodynamic potential $\Phi(M,H,T)$ [1]. When minimised with respect to the magnetisation, Φ delivers the usual (i.e. equilibrium) Gibbs energy $G(H,T)$:

$$\min_M \Phi(M,H,T) = G(H,T). \tag{1}$$

Both potentials can depend on further thermodynamic variables, e.g., on pressure. Φ is expanded in powers of M:

$$\Phi = \Phi_0 + \tfrac{1}{2}aM^2 + \tfrac{1}{4}bM^4 + \tfrac{1}{6}cM^6 + \ldots - MH . \tag{2}$$

Only even powers of M enter in this expansion since Φ must be invariant under the time inversion ($M \to -M$, $H \to -H$). The coefficients Φ_0, a, b, c… may depend on temperature (pressure etc.), but not on M or H.

At thermal equilibrium, $\partial\Phi/\partial M = 0$, one has

$$aM + bM^3 + cM^5 + \ldots = H . \tag{3}$$

This is, in implicit form, the magnetic equation of state.

Now let H=0. There is always a trivial solution to Eq. (3), M=0. Of interest, however, is a non-trivial solution, which corresponds to a spontaneously magnetised state. The transition between the two states (with M=0 and $M \neq 0$) is a phase transition. According to Landau, it is necessary that the coefficient a change sign at the transition point T_C. In close vicinity to T_C one has

$a \approx \alpha\,(T_C - T)$, $b \approx$ const., $c \approx$ const. etc.

The phase transition is of second order if $b>0$ and of first order if $b<0$ (in the latter case c must be positive). Following Landau, we shall concentrate on the former case.

Let us assume in addition that $\alpha>0$. Then the ferromagnetic phase ($M \neq 0$, $H=0$) is stable at $T<T_C$, while the paramagnetic one ($M=0$) is stable at $T>T_C$. (If α were negative, the stability domains would have to be switched around; this is not impossible but rather rare.) Thus we arrive at

$$\alpha\left(T_C - T\right) = bM^2 + cM^4 + \ldots \qquad (4)$$

Hence follows the asymptotic behaviour of the spontaneous magnetisation at $T \to T_C$:

$$M = \left(\frac{\alpha}{b}\right)^{1/2} \left(T_C - T\right)^{1/2}, \text{ if } \quad b > 0, \qquad (5)$$

and

$$M = \left(\frac{\alpha}{c}\right)^{1/4} \left(T_C - T\right)^{1/4}, \text{ if } \quad b = 0. \qquad (6)$$

The latter is a special case corresponding to a so-called tricritical point. When M is finite rather than infinitesimal, the choice between Eqs. (5) and (6) is decided by the relation between b and cM^2. In most ferromagnets, $0 < b \ll cM_0^2$, so the behaviour at finite M is not described by either (5) or (6) but rather is something in between, $M \propto (T_C - T)^{1/3}$. The latter is not a true power law, it slowly goes over to $M \propto (T_C - T)^{1/2}$, as M tends to zero.

Further predictions of Landau's theory: Specific heat

In the vicinity of the Curie point the specific heat is given by

$$C = -T\Phi_0''\left(T\right) - \tfrac{1}{2}\alpha T \frac{\partial (M^2)}{\partial T}. \qquad (7)$$

On approach to T_C from below M^2 is approximately proportional to $(T_C - T)^{2/3}$ and

$C \propto (T_C - T)^{-1/3}$,

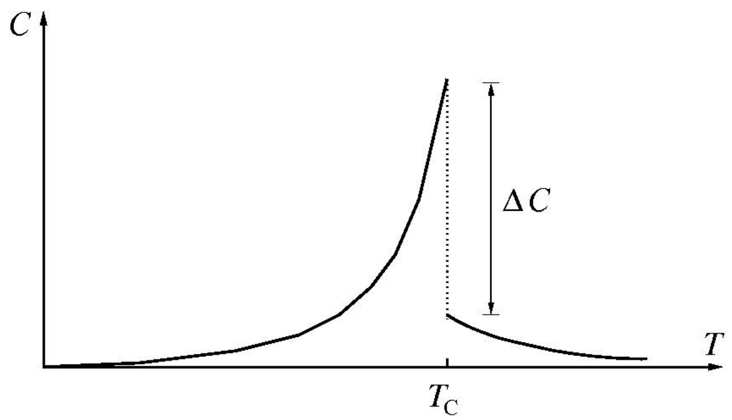
i.e. the specific heat seems to diverge. However, this is not a genuine divergence; infinitesimally close to T_C M^2 becomes $(\alpha/b)(T_C - T)$ and C tends to a finite limit,

$$C = -T\Phi_0''(T) - \tfrac{1}{2}\alpha T \frac{\partial (M^2)}{\partial T}. \qquad (8)$$

In the paramagnetic region ($T > T_C$, $M \equiv 0$) Eq. (7) turns into

$$C = -T\Phi_0''(T), \qquad (9)$$

i.e. $C \neq 0$ (due to the short-range order).



At the Curie point the specific heat has a discontinuity,

$$\Delta C = \frac{\alpha^2 T_C}{2b}. \qquad (10)$$

In summary, for the zero-field specific heat the theory predicts a sharp but finite peak and a discontinuous jump at T_C.

Inverse susceptibility

From Eq. (3) one obtains

$$\frac{1}{\chi} = \frac{H}{M} = a + bM^2 + cM^4 + ...$$

The inverse initial susceptibility ($H \to 0$, $M \to 0$) is just $a(T)$. According to the above, near T_C it can be presented as follows

$$\frac{1}{\chi} = a(T) = \alpha(T - T_C). \qquad (11)$$

Eq. (11) means merely that $a(T)$ vanishes at $T=T_C$ and is differentiable at that point. This should not be confused with the Curie-Weiss law (linearity of $1/\chi$ across a broad temperature interval).

On the whole, Landau's theory provides a correct description of the observed behaviour of ferromagnets near T_C. It is, however, a purely phenomenological theory. It contains no tools for the evaluation of any of the relevant parameters.

References

1. L.D. Landau and E.M. Lifshitz, Electrodynamique des milieux continus (Mir, Moscow, 1990).