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MEAN FIELD APPROXIMATION - PHASE TRANSITIONS

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The starting point of the mean-field approach to ferromagnetism is a suitable theory of paramagnetism yielding a magnetic equation of state of a paramagnet, $M(H,T)$, in explicit form,

$$M = M_0 f(x), \tag{1}$$

$$x = \frac{\mu H}{kT}, \tag{2}$$

where $f(x)$ is a known function. Its argument x is Langevin's dimensionless ratio of the magnetic and thermal energies. The quantity μ, called magnetic moment, is of the order of several Bohr magnetons. The mean-field approximation consists in turning from the description of paramagnetism to ferromagnetism by augmenting the external magnetic field H in Eq. (2) with Weiss's molecular field, proportional to the magnetisation:

$$H \to H + \gamma M \, . \tag{3}$$

The factor γ is known as the molecular field constant. The underlying microscopic mechanism is the exchange interaction.

It can be readily appreciated that a new magnetic equation of state has arisen. It can be expressed either in an implicit form,

$$M = M_0 f\left(\mu \frac{H + \gamma M}{kT} \right), \tag{4}$$

or parametrically, either as

$$\begin{cases} T = \dfrac{\mu H + \mu\gamma M_0 f(x)}{kx} \\ M = M_0 f(x) \end{cases} \tag{5}$$

or alternatively as

$$\begin{cases} H = \dfrac{kT}{\mu} x - \gamma M_0 f(x) \\ M = M_0 f(x) \end{cases} \tag{6}$$

The running parameter in the above expressions is Langevin's ratio, $0 < x < \infty$. Equations (5) and (6) are convenient for generating M-vs-T and M-vs-H plots.

For a detailed development of the theory the function $f(x)$ has to be specified. From general principles it is clear that $f(x)$ must be an odd function, $f(-x) = -f(x)$. This follows from the fact that both M and H change sign under time inversion. Furthermore, the magnetisation cannot grow without limit in high fields; there must be an upper bound to it. Usually $f(x)$ is defined so that $f(x)\to 1$ as $x\to\infty$. Then the prefactor M_0 in Eqs. (2, 4-6) has the meaning of saturation magnetisation.

The main two paradigms in magnetism are the localised and the itinerant (band) models. In the localised approach $f(x)$ is the Brillouin function,

$$B_J(x) = \frac{2J+1}{2J}\coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J}\coth\left(\frac{1}{2J}x\right). \tag{7}$$

The argument x is defined by Eqs. (2, 3) with

$$\mu = g_J J \mu_B \tag{8}$$

For itinerant magnets with a single rectangular band $f(x)$ is given by

$$f(x) = \frac{(1-\Lambda)x}{\ln\left(\dfrac{\sinh x}{\sinh \Lambda x}\right)}, \tag{9}$$

$$x = \frac{\mu_B H}{kT}. \tag{10}$$

Here Λ is a material constant defined by

$$\Lambda = 1 - \frac{1}{ID(E_F)}, \tag{11}$$

where I is the Stoner integral and $D(E_F)$ is the density of states at the Fermi level. According to the Stoner criterion, for ferromagnets $ID(E_F) > 1$; therefore, $0 < \Lambda < 1$. This quantity is a measure of the degree of localisation. For example, nickel can be regarded as a half-localised magnet, because it has $\Lambda \approx 0.5$. The limiting case $\Lambda \rightarrow 1$ corresponds to full localisation; Eq. (9) then becomes

$$f(x) = \tanh x \,. \tag{12}$$

This is a special case of $J = 1/2$ in Eq. (7), describing a system of atoms with one unpaired electron.

The main predictions of the mean-field theory:

- In the absence of a magnetic field the magnetisation is nil at $T > T_C$. At $T < T_C$ the stable solution is non-trivial and corresponds to spontaneous magnetisation. The Curie temperature is given by

$$T_C = \frac{\gamma \mu M_0 f'(0)}{k} \,. \tag{13}$$

- On approach to the Curie point from below the spontaneous magnetisation depends on temperature as follows:

$$\frac{M}{M_0} \approx f'(0)\sqrt{\frac{6f'(0)}{-f'''(0)}}\sqrt{\frac{T_C - T}{T_C}} \,. \tag{14}$$

- Well above the Curie point the susceptibility complies with the Curie-Weiss law,

$$\chi = \frac{M}{H} = \frac{C}{T - \theta} \,, \tag{15}$$

where

$$C = \frac{\mu M_0 f'(0)}{k} \tag{16}$$

and

$$\theta = \gamma C \,. \tag{17}$$

- At low temperature, $T \ll T_C$, the spontaneous magnetisation tends to saturation. The approach to saturation is exponentially rapid, $M_0 - M \propto \exp(-T_0/T)$, where $T_0 \sim T_C$. (Depending on the assumed $f(x)$, the asymptotic expression may also contain a power prefactor.) This prediction of the mean-field theory is at variance with experiment. The correct low-temperature behaviour, $M_0 - M \propto T^{3/2}$, is obtained in the spin-wave theory.

- The specific heat is predicted to have a finite discontinuity at $T = T_C$:

$$\Delta C = \frac{3\left[f'(0)\right]^2}{f'''(0)} Nk \,, \qquad (18)$$

where $N = M_0/\mu$ is the number of spins. Note that $f'''(0) < 0$, therefore, $\Delta C < 0$.

References

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