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IS THERE A CRITICAL POINT IN THE EXACT SOLUTION OF A 3-3-1 MODEL WITH RIGHT-HANDED NEUTRINOS?

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Abstract. The boson mass spectrum of a 3-3-1 model with right-handed neutrinos is investigated by tuning a unique free parameter within the method of exactly solving gauge models with high symmetries. The whole phenomenology of the Standard Model is naturally recovered, while the neutrino sector can be properly constructed. A very strange coincidence of the masses, both in the neutral and charged boson sectors, can occur at a not very high breaking scale. This could suggest – if radiative corrections are proven to be zero, or at least negligible – that in the vicinity of the critical point a new symmetry must act so that the new bosons screen the old ones.
Keywords: 3-3-1 models, boson mass spectrum, critical point.

1. Introduction

In this brief report we would like to emphasize the fact that in the 3-3-1 model – based on the gauge symmetry group $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ - with right-handed neutrinos (hereafter 331RHN), a critical point could well occur at a not very high breaking scale. Although the Particle Data [1] suggest that the neutral boson Z' of any extension of the Standard Model (SM) has to be heavier than the neutral boson of the SM, the particular case when their masses are identical should not be ignored in our opinion. A very suitable manner to investigate this issue by introducing a proper parameter set in the scalar sector of the theory is supplied by the exact algebraical approach for solving gauge models with high symmetries proposed several years ago by Cotăescu [2] and developed by the author in a series of recent papers [3] – [7] on the 331RHN model. For some interesting details of the phenomenology in such models and the specific spontaneous symmetry breakdown (SSB), the reader is referred also to Refs. [8] – [21], where the traditional manner to deal with gauge models is exploited.

The results of our theoretical method regarding the exact boson mass spectrum – shown in the following – are amazing, while they look plausible even from the phenomenological viewpoint. There seems to exist a critical point in the model where all the neutral bosons of the theory Z, Z' and Y gain the same mass whereas, for the same value of the free parameter, the charged bosons $W^{\pm}$ and $X^{\pm}$ are indistinguishable on the mass reason too.

The paper is organized as follows. Section 2 briefly reviews the main results of the theoretical method employed to solve the particular 331RHN model, so that the boson mass spectrum - depending on the unique remaining free parameter "a" – is given. Section 3 analyses the circumstances under which the critical point can occur and sketches the phenomenological implications of the particular value of the parameter "a" ensuring this behavior. Last section is devoted to our conclusions.

2. Boson mass spectrum

The particle content of the 331RHN model under consideration here is the same with the one explicitly shown in Eqs. (20) – (23) in Ref. [7]. In short, there are three lepton generations (triplets) and three quark generations (triplets), each lepton triplet consisting of the well-known left-handed pair of the SM (charged lepton – neutrino) plus the corresponding right-handed neutrino on the third position. Moreover, each lepton generation obeys the same representation with respect to the gauge symmetry group of the model, while in the case of the three quark families we note that two of them transform differently from the third one with respect to this group. Moreover, on the third position in each left-handed quark triplet an exotic quark is placed. All the charged right-handed fermion fields (let them be leptons or quarks) represented by a singlet with respect to the 3-3-1 group are necessary in order to couple their left-handed partners in Yukawa mass generating terms.

With those representations, the 3-3-1 model of interest here stands anomaly free, as one can easily checkout by using little algebra. These representations can be achieved starting with the general method [2] by just choosing an appropriate set of parameters. They are, in our particular case,

$$e, \theta_W, \nu_0 = 0, \nu_1 = 0, \nu_2 = 1. \qquad 1)$$

Their values are established either by experimental arguments (for the elementary electric charge e and the Weinberg angle θ_W) or by internal reasons of the general method (the three necessary versors ν_i). Along with the above parameters, one must add some new ones - as they determine the Higgs sector of the model and thus the SSB - grouped in a parameter matrix which reads

$$\eta^2 = (1-\eta_0^2)Diag\left[1-a, \frac{1}{2}(a+b), \frac{1}{2}(a-b)\right] \qquad 2)$$

For the moment, "a" and "b" are arbitrary non-vanishing real parameters that satisfy the trace condition: $Tr(\eta^2) = (1-\eta_0^2)$ imposed by the general method. At the same time $\eta_0, a \in [0,1)$

holds. All the details of the Higgs sector and its involvement in the SSB of the model can be found in Ref. [2]. For our purpose here it is important to mention that these parameters will determine – after the SSB in one step up to the universal residual $U(1)_{em}$ symmetry - a non-degenerate, in principle, boson mass. The exact expressions of the boson masses are given by the Eqs. (53) - (55) in Ref. [2], namely

$$M_i^j = \frac{1}{2} g\langle\phi\rangle\sqrt{\left(\eta^{(i)}\right)^2 + \left(\eta^{(j)}\right)^2} \qquad 3)$$

for the non-diagonal gauge bosons which usually are charged but, as one can easily observe in the 3-3-1 model under consideration here, one of them must be neutral, and

$$\left(M^2\right)_{ij} = \langle\phi\rangle^2 Tr\left(B_i B_j\right) \qquad 4)$$

with

$$B_i = g\left[D_i + \nu_i (D\nu)\frac{1-\cos\theta}{\cos\theta}\right] \qquad 5)$$

for the diagonal bosons of the model. The angle θ is the rotation angle around the versor ν orthogonal to the electromagnetic direction in the parameter space [2]. The versor sum condition $\nu_i \nu^i = 1$ holds. Since the electro-weak sector of the model is described by the chiral gauge group $SU(3)_L \otimes U(1)_N$, the two diagonal generators D_1 and D_2 (Ds - stands for the Hermitian diagonal generators of the Cartan subalgebra) in the fundamental representation of $SU(3)$ are: $D_1 = T_3$ and $D_2 = T_8$ - connected to the Gell-Mann matrices in the manner $T_a = \lambda_a / 2$ - and $D_0 = I$ for the new chiral hypercharge N. In the 3-3-1 model under consideration here, the relation between θ angle in the general method [2] and the Weinberg angle θ_W from SM was established [3] and it is

$$\sin\theta = \frac{2}{\sqrt{3}}\sin\theta_W \qquad 6)$$

One can express now, by using Eqs. (3) – (5), the masses of the non-diagonal (*i.e.* charged) bosons. They are – according to the parameter order in the matrix η - respectively, $m_W^2 = m^2 a$, $m_X^2 = m^2\left[1 + \frac{1}{2}(a+b)\right]$ and $m_X^2 = m^2\left[1 + \frac{1}{2}(a-b)\right]$. Throughout this paper $m^2 = g^2\langle\phi\rangle^2\left(1-\eta_0^2\right)/4$ is considered.

Evidently, W is the old charged boson of the SM which links positions 2 - 3 in the fermion triplet, namely the left-handed neutrino to its charged lepton partner and, respectively, the *up*-type traditional quarks to *down*-type traditional quarks. The neutral Y boson couples the left-handed neutrino to the right-handed one, and the traditional *up*-type (*down*-type) quarks to the *exotic up*–type (*exotic down*-type) quarks, that is positions 1 - 2 in the fermion triplets are involved. The remaining X boson is responsable for the charged current between positions 1 - 3 in triplets. The pure neutral bosons (diagonal ones) acquire their masses by diagonalizing the resulting matrix:

$$M^2 = m^2 \begin{pmatrix} 1-\frac{1}{2}a+\frac{1}{2}b & -\frac{1}{\sqrt{3-4s^2}}\left(1-\frac{3}{2}a-\frac{1}{2}b\right) \\ -\frac{1}{\sqrt{3-4s^2}}\left(1-\frac{3}{2}a-\frac{1}{2}b\right) & \frac{1}{3-4s^2}\left(1+\frac{3}{2}a-\frac{3}{2}b\right) \end{pmatrix} \qquad 7)$$

In the above expression the notation $s=\sin\theta_W$ has been made for simplicity.

One of the two diagonal bosons has to be identical to the neutral boson Z from SM. Therefore, the latter should be an eigenvector of this mass matrix, corresponding to the eigenvalue $m_Z^2 = m_W^2/\cos^2\theta_W$ firmly established in the SM. That is, one solves the eigenvector problem: $M^2|Z\rangle = \frac{m^2 a}{1-s^2}|Z\rangle$ which simply calls for computing $Det\left|M^2 - m^2 a/\left(1-s^2\right)\right| = 0$. This leads straightforwardly to the important constraint upon the parameters, namely $b = a\tan^2\theta_W$, which reduces their number to only one. Consequently, η becomes a promising one-parameter diagonal matrix describing the scalar sector $\eta^2 = (1-\eta_0^2)Diag\left[1-a, a/2\cos^2\theta_W, a\left(1-\tan^2\theta_W\right)/2\right]$. Under these new and appealing circumstances, the boson mass spectrum yields:

$$m_W^2 = m^2 a \qquad 8)$$

$$m_X^2 = m^2\left(1-\frac{a}{\cos^2\theta_W}\right) \qquad 9)$$

$$m_Y^2 = m^2\left[1-\frac{a}{2}\left(1-\tan^2\theta_W\right)\right] \qquad 10)$$

$$m_Z^2 = m^2\frac{a}{\cos^2\theta_W} \qquad 11)$$

$$m_{Z'}^2 = m^2\left[1+\frac{1}{3-4\sin^2\theta_W} - a\left(1+\frac{\tan^2\theta_W}{3-4\sin^2\theta_W}\right)\right] \qquad 12)$$

as $Tr\left(M^2\right) = m_Z^2 + m_{Z'}^2$ holds.

3. Critical point

When inspecting the boson mass spectrum in Eqs. (8) – (12), one can enforce certain conditions on the parameter "a" as to obtain realistic values, in accordance with the available experimental data [1] suggesting that they have to be in the TeV region. This state of affairs requires, in our method, a very high breaking scale (in the GUT region or even higher) [3, 4], since neutrino phenomenology (tiny masses and mixing angles) must also be taken into consideration as indisputable experimental evidence and incorporated in the model. As a consequence the parameter "a" runs in the vicinity of zero, which makes the results less likely. A proper see-saw mechanism was successfully exploited [5] by inserting a supplemental small parameter in the η matrix in order to keep consistency with acceptable orders of magnitude both for the neutrino mass spectrum and the breaking scale.

However, an unexplored yet opportunity is offered by our method, mentioned for the first time by Cotăescu [22] within the well-known Pisano-Pleitez-Frampton 3-3-1 model [23, 24] and then worked out in a published paper [25] devoted to the same model. As long as the exact masses of the new bosons have not been experimentally determined to date, one is entitled to ask if there is no screening between them and the old ones from the SM. What kind of consequences has such a hypothesis? The resulting parameter, since the requirement $m_Z^2 = m_{Z'}^2$ is enforced in Eqs. (11) – (12)

$$a = \frac{2\cos^2\theta_W}{3 - 2\sin^2\theta_W} \qquad 13)$$

reaches the value $a \cong 0.6$, since we dealt with $\sin\theta_W \cong 0.223$.

Furthermore, what are the values gained by the masses of the remaining bosons? Embedding Eq.(13) in (8) - (13), one obtains the amazing results

$$m_W^2 = m_X^2 = m^2 \frac{2\cos^2\theta_W}{3 - 2\sin^2\theta_W} \qquad 14)$$

$$m_Z^2 = m_{Z'}^2 = m_Y^2 = m^2 \frac{2}{3 - 2\sin^2\theta_W} \qquad 15)$$

These are the well-known values predicted by SM, namely 91.2GeV for the neutral bosons and 80.4GeV for the charged ones. Assuming that in the SM $m_W = g\langle\phi\rangle_{SM}/2$, and $\langle\phi\rangle_{SM} \cong 250\,\text{GeV}$, one can estimate the required breaking scale of the 3-3-1 model of interest here. That is $\langle\phi\rangle \geq \langle\phi\rangle_{SM}/\sqrt{a}$, which leads to $\langle\phi\rangle \geq 320$ GeV.

4. Conclusions

We have proven that the exact algebraical approach for solving gauge models with high symmetries offers - when applied to a 3-3-1 model with right-handed neutrinos - an interesting result: degenerate masses in the boson sector for a particular critical value assigned to the remaining free parameter "a". It occurs at the breaking scale $\langle \phi \rangle \geq 320$ GeV and the whole SM content is naturally recovered (as it was shown in Refs. [3] [7]). Since this amazing outcome exactly holds at tree level, one could presume that radiative corrections do not significantly alter it. But this assumption can be firmly stated only after a detailed analysis [26] is done regarding the way the oblique parameters S, T, U can influence the results at tree level. However, this task exceeds the scope of this work. Notwithstanding, one can consider this strange coincidence – $m_W = m_X$ and $m_Z = m_{Z'} = m_Y$ simultaneously - as more than a simple mathematical fit. It suggests a possible deeper identity between the bosons for a particular critical value of the free parameter, hidden by a possible new symmetry. This hypothesis should not be ruled out *a priori*, since more accurate results regarding the decays of the new bosons and high-energy scatterings involving their couplings to fermions have to be more exactly investigated in future experiments at LHC.

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