# ON THE INVESTIGATION OF THE AFTER-EFFECT FUNCTION AND PRECESSIONAL DECAY TIME OF NANO-PARTICLE COLLOIDS 

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#### Abstract

Measurement of the field and frequency dependent magnetic complex susceptibility, $\chi$ $(\omega, \mathrm{H})=\chi^{\prime}(\omega, \mathrm{H})-\mathrm{i} \chi^{\prime \prime}(\omega, \mathrm{H})$, of magnetic fluids, has proven to be a reliable method for investigating a number of important properties of such fluids including ferromagnetic resonance and relaxation times. Also, because of the inverse Fourier transform relationship which exists between the after-effect function, $\mathrm{b}(\mathrm{t})$, and $\chi^{\prime \prime}(\omega), \mathrm{b}(\mathrm{t})$ may be obtained and used in determining a value of the precessional decay time, $\tau_{0}$. Here, results obtained from measurements on a sample of $\mathrm{Mn}_{0.1} \mathrm{Fe}_{0.9} \mathrm{Fe}_{2} \mathrm{O}_{4}$ particles suspended in Isopar M , by means of the transmission line technique in the $\mathrm{MHz}-\mathrm{GHz}$ range, are presented. Keywords: magnetic fluids, after-effect function, precessional decay time.


## 1. Introduction

Magnetic fluids are stable colloidal systems consisting of single-domain magnetic particles covered by a surfactant in order to prevent particle agglomeration and dispersed in a carrier liquid. The particles have radii ranging from approximately 2-10 nm and when in suspension their magnetic properties can be described by the paramagnetic theory of Langevin. The particles are considered to be in a state of uniform magnetization with a magnetic moment $m=M_{S} v$, where $M_{S}$ denotes the saturation magnetization and $v$ is the magnetic volume of the particle.

There are three characteristic times which govern the behaviour of a particle and two of these are associated with the relaxation of the particles magnetic moment, namely the Brownian relaxation time ( $\tau_{\mathrm{B}}$ ) and the Néel relaxation time $\left(\tau_{\mathrm{N}}\right)$. The third is the decay time associated with precession ( $\tau_{00}$ ) of the magnetic moment.

It is this latter component which is of particular interest in this work and it is shown how it may be determined by means of the unique relationship which exists between the aftereffect function, $\mathrm{b}(\mathrm{t})$, and the imaginary susceptibility component $\chi$ ' $(\omega)$.

## 2. Theory

$\chi(\omega)$, of an assembly of single domain particles can also be described in terms of its parallel, $\chi_{\|}(\omega)$, and perpendicular, $\chi_{\perp}(\omega)$, components, with [1]

$$
\begin{equation*}
\chi(\omega)=\frac{1}{3}\left(\chi_{\|}(\omega)+2 \chi_{\perp}(\omega)\right) . \tag{1}
\end{equation*}
$$

Over the frequency range considered here relaxation due to Brownian rotational diffusion of the particles will, in general, be ignored and thus the $\tau_{/ /}$component, will be considered to be dominated by the Néel relaxation mechanism with relaxation time $\tau_{N}$

The perpendicular or transverse susceptibility, $\chi_{\perp}(\omega)$, can have a resonant character, this effect being indicated by a transition in the value of $\quad \chi^{\prime}(\omega \square \square$ from $\mathrm{a}+v e$ to a $-v e$ quantity at an angular frequency, $\omega_{\text {res }}=2 \pi f_{\text {res }}$.

If the polar angle $\theta$ is small, $\omega_{\text {res }}$, is given by [1],

$$
\begin{equation*}
\omega_{\mathrm{res}}=2 \pi \mathrm{f}_{\mathrm{res}}=\square \boldsymbol{\gamma} \square \mathrm{H}_{\mathrm{A}} \tag{2}
\end{equation*}
$$

$H_{A}=2 K / M_{S}$, where $K$ is the anisotropy constant in $J / m^{3}$ and $\gamma$ is the gyromagnetic ratio.
If a radio frequency field is applied perpendicular to $H_{A}$, the motion of the magnetic moment has a typical resonant character which is commonly described by means of the Landau and Lifshitz equations, namely,

$$
\begin{equation*}
\frac{\chi(\omega)}{\chi(0)}=\frac{\left(1+\alpha^{2}\right) \omega_{0}^{2}+\mathrm{i} \alpha \omega \omega_{0}}{\left(1+\alpha^{2}\right) \omega_{0}^{2}-\omega^{2}+2 \mathrm{i} \alpha \omega \omega_{0}} \tag{3}
\end{equation*}
$$

where $\alpha$ is a damping parameter $\gamma \square$ and the precessional decay time, $\tau_{0}=\left(\alpha \omega_{0}\right)^{-1}$.
The after-effect function, $b(t)$, represents the decay of magnetization after the sudden removal of an external polarizing magnetic field, and $\chi(\omega)$ and $b(t)$ are related by the expression [2],

$$
\begin{equation*}
b(t)=2 \operatorname{Re}\left\{F^{-1}\left[\frac{\chi^{\prime \prime}(\omega)}{\omega}\right]\right\} \tag{4}
\end{equation*}
$$

where $F^{-1}$ denotes the inverse Fourier transform.
Scaife [3] has shown that $b(t)$ for the Landau and Lifshitz equations, has the form,

$$
\begin{equation*}
\mathrm{b}(\mathrm{t})=\mathrm{b}(0) \exp \left(-\frac{\mathrm{t}}{\tau_{0}}\right) \cos \omega_{0} \mathrm{t} \tag{5}
\end{equation*}
$$

If the area under $b(t), \quad{ }_{0}^{\infty}(\mathrm{t}) \mathrm{dt}=\mathrm{B}$ say, then it follows that, $\mathrm{B}=\mathrm{b}(0)_{0}^{\infty} \mathrm{dt} \cos \omega_{0} \mathrm{texp}\left(-\mathrm{t} / \tau_{0}\right)$, resulting in a normalised value of [4],

$$
\begin{equation*}
\frac{\mathrm{B}}{\mathrm{~b}(0)}=\frac{\left(1 / \tau_{0}\right)}{\left(1 / \tau_{0}\right)^{2}+\omega_{0}^{2}}=\frac{\tau_{0}}{1+\omega_{0}^{2} \tau_{0}^{2}} \tag{6}
\end{equation*}
$$

from whence, by knowing $B / b(0)$ and $\omega_{0,} \tau_{\square}$ can be determined.

## 3. Measurements

Measurements reported here were made by means of the short-circuited, coaxial transmission line technique [5], [6] using a Hewlett Packard (HP) 50■ coaxial line incorporating a co-axial cell, in conjunction with an HP 8753C network analyser. To obtain polarised measurements the coaxial cell containing the ferrofluid sample, terminated in a standard HP short circuit load, was placed between the pole faces of an electromagnet, the axis of the cell being perpendicular to the biasing field. The biasing field, $H$, was altered between 0 and $104 \mathrm{kAm}^{-1}$. Automatic swept measurements of the input impedance of the line containing the sample were measured and from these measurements the complex components, $\chi^{\prime}(\omega \square \square) \square \square$ and $\square \chi^{\prime \prime}(\omega \square \square \square)$ were determined.

## 4. Results

Measurements are presented for a 104 Gauss fluid consisting of $\mathrm{Mn}_{0.1} \mathrm{Fe}_{0.9} \mathrm{Fe}_{2} \mathrm{O}_{4}$ particles suspended in Isopar M. Measurements were performed over the range 100 MHz to 6 GHz and for 10 values of H . This curves where then fitted [7] up to 10 GHz in order to facilitate a more accurate determination of the inverse Fourier transform of $\chi^{\prime \prime}(\omega)$ ),


Fig 1.Plot of $\chi^{\prime}$ and $\chi^{\prime \prime}$ against $f(\mathrm{~Hz})$.


Fig2. Plot of fit to $\chi^{\prime \prime}$ against $f(\mathrm{~Hz})$.

Fig 1. shows a plot of the $\chi^{\prime}(\omega)$ and $\chi^{\prime \prime}(\omega)$ components of the sample and from these plots it can be seen that when $H=0, f_{\text {res }}=1.8 \mathrm{GHz}$, and when $H=104 \mathrm{kAm}^{-1}, f_{\text {res }}$ rises to 6 $G H z$. The $\chi^{\prime}{ }^{\prime}(\omega)$ component has a high frequency loss peak that occurs at a frequency of $f_{\max }$ $=1.1 \mathrm{GHz}$ at $H=0$. This shifts to 5.9 GHz at $H=104 \mathrm{kAm}^{-1}$. Thus the value of $f_{\max }$ approaches the value of $f_{\text {res }}$ as resonance becomes the dominant process.

Fig. 2 shows the $\chi^{\prime \prime}(\omega \square \square$ data which was transformed to obtain the $b(t)$ profiles shown in Fig. 3 and one can observe how, over the polarizing field range, $b(t)$ changes from an exponential type decay to an oscillatory one. This transition arises because with increasing $H$, the parallel relaxation component diminishes its contribution to the overall susceptibility and $b(t)$ becomes similar to that of the Landau Lifshitz form.


Fig 3.Plot of $b(t)$ against $t$ sec.


Fig 4.Plot of area under $b(t)$ against $H$.


Fig 5. Plot of $\tau_{1}$ and $\tau_{2}$ against H .

Computing the area, $B$, under the $b(t)$ curves, as shown in Fig. 4, and using the $f_{\text {res }}$ values, $\tau_{0}$ as a function of $H$, was determined by use of Eq.(6). The results are shown plotted in Fig.5. As can be observed from Fig.5, for the analyzed sample, Eq. (6) has real solutions only for polarizing fields larger than $50 \mathrm{kA} / \mathrm{m}$. This is due to the fact that the approximation used in computing the solutions of Eq.(6) (i.e. $\omega_{0}=2 \pi f_{\text {res }}$ ) is valid only for strong polarizing fields. One also notes that, Eq.(6) is a quadratic equation and has two solutions, $\tau_{1}$ and $\tau_{2}$. The correct values $\tau_{0}$ can be chosen by simply testing the values of $\tau_{1}$ and $\tau_{2}$ with the relation $\alpha=$ $\left(\tau_{0} \omega_{0}\right)^{-1}$, where $\alpha$ cannot be larger that one. Performing this test for all solutions of $\tau_{l}$ and $\tau_{2}$, we determine that the correct solutions of Eq.(6) are those of $\tau_{l}$ (see Fig.5) and at large values of $H$ a mean value of $\tau_{0}=1.510^{-10} \mathrm{~s}$ is obtained.

## 5. Conclusions

Using the frequency and polarizing field dependence of the complex magnetic susceptibility, $\left.\chi(\omega, H)=\chi^{\prime}(\omega, H)-i \square \chi^{\prime \prime} \square \square \omega, H\right)$, and the corresponding after-effect functions, the determination of the precessional decay time, $\tau_{0}$, under the strong polarizing fields has been presented. The significance of the area, $B$, under the after-effect functions has been highlighted and it has been shown that $\tau_{0}$ can be determined from the expression $\frac{\mathrm{B}}{\mathrm{b}(0)}=\frac{\tau_{0}}{1+\omega_{\mathrm{res}}^{2} \tau_{0}^{2}}$; this method being based on the analysis of the after effect function of a magnetic fluid at resonance.

The determined value of $\tau_{0}=1.5 \cdot 10^{-10} s$ lies within the generally accepted range for $\tau_{0}$.

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