# ON THE DETERMINATION OF DYNAMIC REACTION FORCES 

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#### Abstract

Generally, the dynamic reaction forces have a special importance for a wide range of rotors that are used in the rotary machine building. This is the reason for which the determination of the dynamic reaction forces in the rotor bearings presents interest. The possibility of comparing the calculated quantities to the experimental results is limited. That is why in the paper it is approached a simple case which leads to simple and easily interpretable conclusions.


Keywords: reaction force, plane plate, dynamic modeling.

## 1. Introduction



FIGURE 1. Calculus Schema.

It is considered a rigid body under the form of a plane plate that rotates around of an axle in its plane. The theoretical study comprises the writing of dynamic equations [1]-[3] of the rigid body with fixed axle, corresponding to the calculus schema in fig. 1. In the schema, the plate performs a uniform rotation motion, under the action of a motor torque. The equations of the rigid body with rotation fixed axle are used to determine the reaction forces in the two bearing of rotor. The bearing reaction force components depend on the plate shape and they are functions of the body position.

The coordinate system $O x y z$, joint to the plate, is chosen so that $O y$ to be the rotation axle $O O^{\prime}$, and the plate to be placed in the plane $O x y$. Similarly, in the fixed coordinate system $O x_{1} y_{1} z_{1}$, it is supposed that $O y_{1}$ is the fixed horizontal rotation axle, and the axle $O z_{1}$ is vertical.

The $x_{G}$ and $y_{G}$ coordinates of the gravity center $G$ and also the plate moments of inertia $J_{x}$, $J_{y}, J_{x y}$ are supposed as known. The inertia force reduction torsor in the point $O_{1}$ has the components

$$
\begin{equation*}
\bar{R}_{I}=-m \bar{a}_{G}=m x_{G}\left(\omega^{2} \bar{i}+\varepsilon \bar{k}\right) \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{M}_{I}=J_{x y} \varepsilon \bar{i}-J_{x} \bar{\varepsilon} \bar{j}-J_{x y} \omega^{2} \bar{k} . \tag{2}
\end{equation*}
$$

On the plate acts the gravity force $m \bar{g}=m g(\bar{i} \sin \theta-\bar{k} \cos \theta)$ whose moment in relation to $O_{1}$ is

$$
\begin{equation*}
\bar{r}_{G} \times(m \bar{g})=m g\left(-y_{G} \bar{i} \cos \theta+x_{G} \bar{j} \cos \theta-y_{G} \bar{k} \sin \theta\right) . \tag{3}
\end{equation*}
$$

In the two bearings $O$ and $O^{\prime}\left(O O^{\prime}=l\right)$, it must be introduced the reaction forces $\bar{R}(X, Y, Z)$ and $\bar{R}^{\prime}\left(X^{\prime}, Y^{\prime}, Z^{\prime}\right)$. The moment in relation to $O$ of the first force is null, and of the second one is

$$
\begin{equation*}
\overline{O O} \bar{\prime}^{\prime} \bar{R}^{\prime}=l\left(Z^{\prime} \bar{i}-X^{\prime} \bar{k}\right) \tag{4}
\end{equation*}
$$

The uniform rotation motion ( $\varepsilon=0$ ) is realized under the action of the $\bar{M}$ motor torque, but this one does not influence the reaction forces. As consequence, the equations of dynamic of rigid body with fixed axle, given by D'Alembert principle are projected on the axles $O x$ and $O z$,

$$
\begin{equation*}
m g \sin \theta+X+X^{\prime}+m \omega^{2} x_{G}=0,-m g \cos \theta+Z+Z^{\prime}=0 \tag{5}
\end{equation*}
$$

and for the moments,

$$
\begin{equation*}
-m g y_{G} \cos \theta+l Z^{\prime}=0,-m g y_{G} \sin \theta-l X^{\prime}-J_{x y} \omega^{2}=0 . \tag{6}
\end{equation*}
$$

From this equation system, the components of $O$ reaction force result

$$
\begin{equation*}
X=\left(\frac{J_{x y}}{l}-m x_{G}\right) \omega^{2}-m g\left(1-\frac{y_{G}}{l}\right) \sin \theta, Z=m g\left(1-\frac{y_{G}}{l}\right) \cos \theta, \tag{7}
\end{equation*}
$$

and for the one in $O^{\prime}$ :

$$
\begin{equation*}
X^{\prime}=-\frac{l}{l}\left(J_{x y} \omega^{2}+m g y_{G} \sin \theta\right), Z^{\prime}=m g \frac{y_{G}}{l} \cos \theta \tag{8}
\end{equation*}
$$

## 2. Reaction force calculus

It must determinate the reaction force projections on the fixed axles $O x_{1} z_{1}$, the mobile axles $O x z$ being rotated by the angle $\theta$, in relation to the first ones. It results that the projections on the fixed axles of the reaction force in $O$ can also be found with the help of the transformation formulae

$$
\begin{equation*}
X_{1}=X \cos \theta+Z \sin \theta, Z_{l}=Z \cos \theta-X \sin \theta \tag{9}
\end{equation*}
$$

and the one in $O^{\prime}$ :

$$
\begin{equation*}
X_{l}^{\prime}=X^{\prime} \cos \theta+Z^{\prime} \sin \theta, Z_{l}^{\prime}=Z^{\prime} \cos \theta-X^{\prime} \sin \theta \tag{10}
\end{equation*}
$$

After the calculus effecting, these components become

$$
\begin{equation*}
X_{l}=\left(\frac{J_{x y}}{l}-m x_{G}\right) \omega^{2} \cos \theta, Z_{l}=m g\left(1-\frac{y_{G}}{l}\right)-\left(\frac{J_{x y}}{l}-m x_{G}\right) \omega^{2} \sin \theta \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
X_{l}^{\prime}=-\frac{J_{x y}}{l} \omega^{2} \cos \theta, Z_{l}^{\prime}=\frac{J_{x y}}{l} \omega^{2} \sin \theta+m g \frac{y_{G}}{l} . \tag{12}
\end{equation*}
$$

If initially it is supposed that the mobile axles are superposed on the fixed ones, then the angle between axles can be taken $\theta=\omega t$.

## 3. Calculus technique

In order to determine the variation of dynamic reaction force components, it was elaborated, in MathCAD, a program for the calculus and drawing the graphics, fig. 2.


FIGURE 2. Reaction Force Component Variations.

The input data are the $x_{G}$ and $y_{G}$ coordinates of plate gravity center, the moment of inertia $J_{x y}$, and also the angular velocity $\omega$. For example, it was considered a triangular plate, which
rotates around the cathetus of $l=0.1 \mathrm{~m}$ length, with the angular velocity $\omega=125 \mathrm{rad} / \mathrm{sec}$. The other cathetus is 0.06 m length, so that it results $m=0.244 \mathrm{~kg}, x_{G}=1.9810^{-2} \mathrm{~m}, y_{G}=4.8910^{-2} \mathrm{~m}$ and $J_{x y}=1.9410^{-4} \mathrm{~kg} \cdot \mathrm{~m}^{2}$.

## 4. Conclusions

In order to experimentally verify the theoretical calculus, it was construct a device composed by the driving electric motor, the rigid body under the form of an eccentric plate which rotates in dynamometer bearings and the measurement block. The analytically calculated values of the reaction forces were compared to the experimental ones and there were found enough small errors so that the validity of calculus was confirmed.

## References

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