

## ON THE DYNAMICS OF MOTIONS OF A ROBOT FOR THE ELECTRONIC INDUSTRY

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### Abstract

In the paper, there are studied some aspects concerning the dynamics of motions of a robot, dedicated to the automatic implantation of electronic components in the printed circuit boards. The aim of study is to put in evidence the dynamic interactions between the motions of robot. It is also evaluated the influence of these interactions of the positioning precision of the manipulated electronic components.

**Keywords:** industrial robot, positioning precision, motion interaction.

### 1. Introduction

In the paper, there are studied some aspects concerning the dynamics of motions of a robot [2][3][5], dedicated to the automatic implantation of electronic components in the printed circuit boards. The aim of study is to put in evidence the dynamic interactions between the motions of robot. It is also evaluated the influence of these interactions of the positioning precision of the manipulated electronic components [1].

### 2. Dynamic Model

The dynamic model of the robot is presented in figure 1 [4][6]: 1 – manipulated electronic component; 2 – prehension device; 3 - horizontal translation module; 4 – vertical translation module; 5 – rotation module, around the vertical axis.

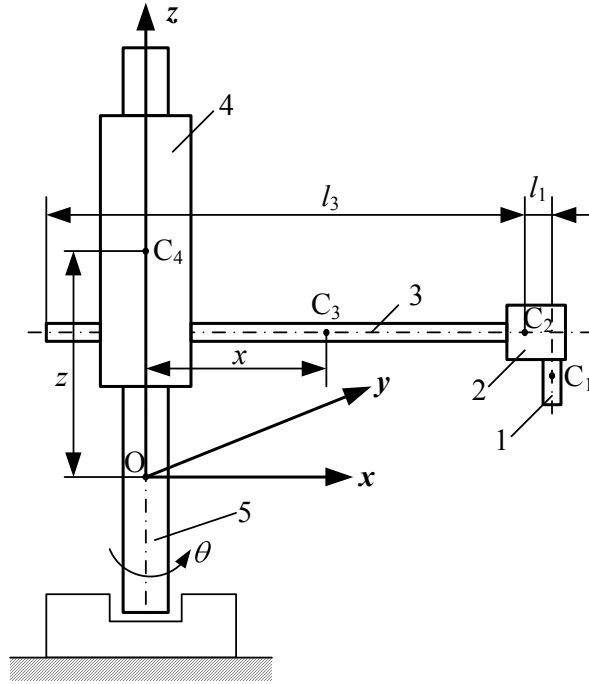
### 3. Differential Equations of Motion

To establish the differential equations of motions of robot, the Lagrange equations of second species are applied,  $\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) - \frac{\partial E}{\partial q_i} = Q_i, i = 1, 2, 3$ , where, as generalized coordinates, there are chosen:  $q_1 = x$  - abscissa of center of gravity  $C_3$  of the arm of horizontal translation,  $q_2 = z$  - applicate of center of gravity  $C_4$  of the module of vertical translation,  $q_3 = \theta$  - angle

of rotation of the rotation module, around the vertical axis  $Oz$ . Also, in the equations,  $E$  is the

total kinetic energy of robot,  $E = \frac{m_1 + m_2 + m_3}{2}(\dot{x}^2 + \dot{z}^2) + \frac{m_4}{2}\dot{z}^2 + \frac{1}{2}[J_{1C} + J_{2C} + J_{3C} + J_{4z} +$

$+ m_1(x + \frac{l_3}{2} + l_1)^2 + m_2(x + \frac{l_3}{2})^2 + m_3x^2]\dot{\theta}^2$ , where:  $m_1$  - mass of manipulated electronic



**Figure 1:** Dynamic model

component;  $J_{1C}$  - axial moment of inertia of manipulated electronic component;  $l_1$  - distance between the gravity centers  $C_1$  and  $C_2$  of the manipulated electronic component and the prehension device;  $l_3$  - length of the arm of horizontal translation;  $m_2$  and  $J_{2C}$  - mass and moment of inertia in relation to the vertical axis passing by  $C_2$  of prehension device;  $m_3$  and  $J_{3C}$  - mass and moment of inertia in relation to vertical axis passing by  $C_3$  of horizontal arm;  $m_4$  and  $J_{4z}$  - mass and moment of inertia in relation to the  $Oz$  axis of vertical translation module;  $J_{4z}$  - moment of inertia in relation to the  $Oz$  axis of rotation module.

For the generalized forces  $Q_i, i=1,2,3$ , there are obtained the expressions  $Q_1 = F_{Mx}$ ,  $Q_2 = F_{Mz} - G$ ,  $Q_3 = M_M$ , where:  $F_{Mx}$  - motive force for the horizontal translation;  $F_{Mz}$  - motive force for the vertical translation;  $G$  - total weight of manipulated electronic component and translation modules;  $M_M$  - moment of the motive torque for the rotation.

Taking into account these expressions and introducing the notations

$$m_x = m_1 + m_2 + m_3, \quad m_z = m_1 + m_2 + m_3 + m_4, \quad l_x = \frac{m_1(l_3 + 2l_1) + m_2l_3}{2m_x},$$

$$J_z = \frac{J_{1C} + J_{2C} + J_{3C} + J_{4z} + J_{5z} + m_1\left(\frac{l_3}{2} + l_1\right)^2 + m_2\left(\frac{l_3}{2}\right)^2}{2}, \text{ the equations (1) become}$$

$$m_x \ddot{x} - m_x x \dot{\theta}^2 - \frac{m_x l_x}{2} \dot{\theta}^2 = F_{Mx}, \quad m_z \ddot{z} = F_{Mz} - G, \quad J_z \ddot{\theta} + 2m_x x \dot{x} \dot{\theta} + m_x x^2 \ddot{\theta} + m_x l_x \dot{x} \dot{\theta} + m_x l_x x \ddot{\theta} = M_M,$$

By analyzing the equations, it results that between the motion of the vertical translation module and the motions of the other two modules, there are not dynamic interactions, because its equation of motion (the second one in), is decoupled of the other ones. On the other hand, between the motions of the horizontal and rotation modules, there are important dynamic interactions. In order to put in evidence the influence of these interactions on the positioning precision of the manipulated electronic component, there are considered small perturbations  $\Delta x, \Delta z, \Delta \theta$ , around of its position,  $x = x_0 + \Delta x$ ,  $z = z_0 + \Delta z$ ,  $\theta = \theta_0 + \Delta \theta$ , where  $x_0, z_0, \theta_0$  are the cylindrical coordinates of the considered position in the considered working space of the robot. By introducing these expressions in the equations and neglecting the small infinities of second degree, it results the system of differential equations in perturbations,  $m_x \Delta \ddot{x} = \Delta F_{Mx}$ ,  $m_z \Delta \ddot{z} = \Delta F_{Mz}$ ,  $[J_z + m_x x_0(x_0 + l_x)] \Delta \ddot{\theta} = \Delta M_M$ .

#### 4. Analysis of Steady-State Regime of Motion of Robot

To analyze the steady-state motions of the robot, which are made with constant velocities  $v_{x0}$ ,  $v_{z0}$ ,  $\omega_0$ , there are considered their small perturbations  $\Delta \dot{x}$ ,  $\Delta \dot{z}$ ,  $\Delta \dot{\theta}$ :  $\dot{x} = v_{x0} + \Delta \dot{x}$ ,  $\dot{z} = v_{z0} + \Delta \dot{z}$ ,  $\dot{\theta} = \omega_0 + \Delta \dot{\theta}$ . By introducing these expressions in the equations and neglecting the small infinities of second degree, it results the system of differential equations in perturbations:  $m_x \Delta \ddot{x} - 2m_x \omega_0 (v_{x0} t + x^* + \frac{l_x}{2}) \Delta \dot{\theta} - m_x \omega_0^2 \Delta x = \Delta F_{Mx}$ ,  $m_z \Delta \ddot{z} = \Delta F_{Mz}$ ,  $[J_z + m_x (v_{x0} t + x^*) (v_{x0} t + x^* + l_x)] \Delta \ddot{\theta} + 2m_x v_{x0} (v_{x0} t + x^* + \frac{l_x}{2}) \Delta \dot{\theta} + 2m_x \omega_0 (v_{x0} t + x^* + \frac{l_x}{2}) \Delta \dot{x} + 2m_x v_{x0} \omega_0 \Delta x = \Delta M_M$ , where  $x^*$  is the abscissa of the initial position of the manipulated electronic component. The obtained equations are linear, with variable coefficients. These ones can be considered approximately constant if, on the duration  $t^*$  of the transitory process,

they are slowly variable. Thus, for the coefficient  $v_{x0}t + x^* + \frac{l_x}{2}$ , it results the condition  $v_{x0}t^* \leq \lambda(x^* + \frac{l_x}{2})$ , where  $\lambda = 0.1-0.2$ , and for the coefficient  $J_x + m_x(v_{x0}t + x^* + l_x)$ , it is obtained the condition  $v_{x0}t^* \leq -(x^* + \frac{l_x}{2}) + \sqrt{(x^* + \frac{l_x}{2})^2 + \lambda[\frac{J_z}{m_x} + x^*(x^* + l_x)]}$ . In all expressions above, the signs of the velocities  $v_{x0}$  and  $\omega_0$  determine the direction of the resultant moment of the complementary Coriolis forces of inertia.

## 5. Conclusions

By analyzing the differential equations in perturbations, it remarks that they are decoupled. The physical consequence is the fact that the positioning precision of the manipulated electronic component can be separately calculated, in relation to each coordinate and that it is not influenced by the interactions between the motions of the modules of robot.

## References

1. C. F. Beards, Engineering Vibration Analysis with Applications to Control Systems, Eduard Arnold Publishing, Aukland, (1999)
2. C. F. Beards, Structural Vibration: Analysis and Damping, Eduard Arnold Publishing, Sidney, (2001)
3. G. D. Birkhof, Dinamical Systems, American Mathematical Society Publishing, Providence, (2001)
4. F. Chaumette, P. Rives, B. Espian, Positioning of a Robot with Respect to an Object, IEEE Int. Conf. on Robotics and Automation, (2001), pp: 2248-2253
5. H.R. Harrison, T. Nettleton, Principles of Engineering Mechanics, Eduard Arnold Publishing, Melbourne, (1998)
6. Saedan M., Marcelli H., On the Control at an Industrial Robot, Proceed. of the IASTED Intern Conf. on Robotics and Applic, Florida, USA, (2005), pp: 152-157