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TRANSPORT PROPERTIES THROUGH A QUANTUM WIRE ATTACHED TO QUANTUM DOTS

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Abstract
We consider a quantum wire coupled to a chain of quantum dots. The stationary states are described by the expansion coefficients providing the probability amplitude to find the electron in the site j of quantum wire. The conductance is related to the transmission coefficient t at the Fermi energy by the one-channel Landauer formula at zero temperature.
Keywords: quantum wire, quantum dots, conductance.

1. Introduction

Transport properties in nanoscale systems have been studied rather intensively in connection with recent progress in nanofabrication of quantum devices [1,2]. Among others, a quantum dot (*QD*) has played an important role to reveal correlation effects in nanoscale systems. In resonant tunneling regime, the electronic transport through *QD* array becomes sensitive to precise matching of the electron wavefunctions. A linear *QD* array can be seen as a one dimensional chain of sites. This type of chain coupled to the continuum states shows an even-odd parity effect in the conductance when the Fermi energy is localized in the center of the energy band [3,4]. The aim of this work is to study the transport properties of an alternative configuration of a side coupled *QD* array attached to a perfect quantum wire (*QW*). In this case the *QD* array acts as scattering center for transmission through the *QW* [5-7].

2. Model and Formulation

The nanodevice one deals with is a side coupled to a chain of quantum dots. The array consists of N quantum dots (QD) connected in a series by tunnel coupling. Such systems are modeled by using a noninteracting Anderson tunneling Hamiltonian that can be written as [8]

$$H = H_{QW} + H_{QD} + H_{QD-QW}\,, \tag{1}$$

where H_{QW} is responsible for the quantum wire, H_{QD} describe the chain of N quantum dots, while H_{QD-QW} stands for the tunneling interaction between the quantum wire and the quantum dots. One has

$$H_{QW} = V \sum_{j=-\infty}^{+\infty} (c_j^+ c_{j+1} + H.c.) \ , \tag{2}$$

where the operator c_j^+ creates an electron at site j and where V denotes the hopping parameter in the QW. Dot electrons are accounted for via

$$H_{QD} = \sum_{l=1}^{N} \varepsilon_l d_l^+ d_l + \sum_{l=1}^{N-1} \left(V_l d_l^+ d_{l+1} + H.c. \right), \tag{3}$$

where V_l is a real parameter denoting the tunneling coupling between the l-th and $(l+1)$-th quantum dots, ε_l is the energy level of the dot l, and

$$H_{QD-QW} = V_0 \left(d_1^+ c_0 + c_0^+ d_1 \right) . \tag{4}$$

The tunneling interaction concerns only the electrons located at $j = 0$ and $l = 1$, respectively. H_{QW} corresponds to the free-particle Hamiltonian on a lattice with spacing d and whose eigenfunctions are expressed as Bloch solutions like

$$|k\rangle = \sum_{j=-\infty}^{\infty} e^{ikdj} |j\rangle \ , \tag{5}$$

where $|k\rangle$ is the momentum eigenstate and $|j\rangle$ is a Wannier state localized at site j. The dispersion relation associated with these Bloch states reads $\varepsilon = 2v\cos(kd)$. (6)

The Hamiltonian supports an energy band from $-2v$ to $2v$ and the first Brillouin zone expands the interval $[-\pi/d, \pi/d]$. The stationary states of the entire Hamiltonian H can be expressed as

$$|\Psi_k\rangle = \sum_{j=-\infty}^{\infty} A_j c_j^+ |0\rangle + \sum_{l=1}^{N} B_l d_l^+ |0\rangle , \tag{7}$$

where A_j and B_l are expansion coefficients providing probability amplitudes needed. It is clear that $|j\rangle = c_j^+|0\rangle$ and $|l\rangle\rangle = d_l^+|0\rangle$, such that $\langle j | j'\rangle = \delta_{jj'}$ and $\langle\langle l | l'\rangle\rangle = \delta_{ll'}$. Accordingly

$$|\Psi_k\rangle = \sum_{j=-\infty}^{\infty} A_j |j\rangle + \sum_{l=1}^{N} B_l |l\rangle , \tag{8}$$

in which case the amplitudes A_j obey the following linear difference equations:

$$\varepsilon A_0 = v(A_{-1} + A_{+1}) + V_0 B_1$$

$$\varepsilon A_j = v(A_{j-1} + A_{j+1}) + V_0 B_1 \delta_{j0}$$

$$\varepsilon B_1 = \varepsilon_1 B_1 + V_{1,2} B_2 + V_0 A_0 \tag{9}$$

$$\varepsilon B_l = \varepsilon_l B_l + V_{l,l-1} B_{l-1} + V_{l,l+1} B_{l+1} \quad , \quad l \neq 1, N$$

$$\varepsilon B_N = \varepsilon_N B_N + V_{N,N-1} B_{N-1} .$$

The equation for A_0 can be cast in the form

$$\varepsilon A_0 = v(A_{-1} + A_1) + V_0^2 / Q_N A_0 , \tag{10}$$

whereas B_1 can be expressed in terms of A_0 as [9] where Q_N is a continued fraction.

$$B_1 = V_0 A_0 / Q_N \tag{11}$$

$$Q_N = \varepsilon - \varepsilon_1 - \cfrac{V_{1,2}^2}{\varepsilon - \varepsilon_2 - \ldots - \varepsilon - \varepsilon_{N-1} - \cfrac{V_{N-1,N}^2}{\varepsilon - \varepsilon_N}} . \tag{12}$$

For study the solutions of the equation (9) we assume that the electrons are described by a plan wave incident from the far left with unity amplitude and reflection amplitude r at the far right as well as by transmission amplitude t [8]. Such solutions can be written as

$$A_j = e^{ikdj} + re^{-ikdj} \quad , \quad j < 0 \tag{13}$$

$$A_j = te^{ikdj} \qquad , \quad j > 1 . \tag{14}$$

Extrapolating the above wavefunctions towards $j = 0$, one finds the matching condition

$$t - r = 1 , \tag{15}$$

which provides an appreciable simplification. Inserting $\varepsilon = \varepsilon(k)$ then gives the transmission amplitude

$$t = A_0(\varepsilon) = Q_N(\varepsilon) / \left(Q_N(\varepsilon) - \frac{iV_0^2}{\sqrt{4V^2 - \varepsilon^2}} \right) . \tag{16}$$

The level broadening Γ can be identified as $\Gamma = \Gamma(\varepsilon) = V_0^2 / \sqrt{4V^2 - \varepsilon^2}$. (17)

The conductance of the quantum wire at zero temperature is given by

$$G(\varepsilon) = \frac{2e^2}{h} \cdot \frac{Q_N^2}{Q_N^2 + T^2} , \tag{18}$$

by virtue of the one-channel Landauer-formula [10], where the transmission coefficient is

$$T_N(\varepsilon) = |t|^2 = |A_0|^2 = \frac{Q_N^2}{Q_N^2 + T^2} . \tag{19}$$

Resonance structure characterizing the energy dependence of $T_N(\varepsilon)$ can then be easily identified by looking for complex $\varepsilon = \varepsilon_c$ - roots for which $T_N(\varepsilon_c) = 1$ [8].

3. Results

The energy levels (zeroes of Q_N) depend only on the hopping in the QD array ($V_{N-1,N}$), while Γ is only a function of V_0^2 / v. Figure 1 shows the conductance (in units of $2e^2/h$) as a function of the Fermi energy (in units of the Γ) for $\varepsilon_i = 0$ ($i = 1,...,N$). There exists only one narrow antiresonance in the case of $N = 1$ QD, while bonding and antibonding antiresonances and one resonance are clearly revealed for N = 2. In addition, bonding and antibonding resonances as well as bonding and antibonding antiresonances arise when N =3 [11]. The system N=3 side-coupled QD shows particularly simple solutions when $V_{1,2} = V_{2,3} = V_c$

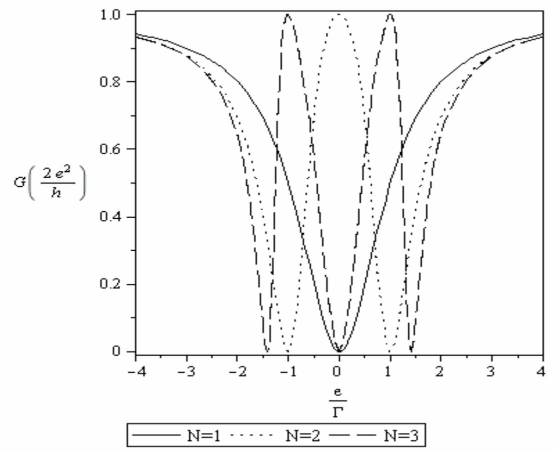

Fig. 1. Conductance, versus Fermi energy

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