

## ELECTRON-PHONON INTERACTION IN NANO CYLINDER AND SOME CONSEQUENCES

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### Abstract

The nano cylinder with macroscopic height and nano cross-section was analyzed and the currents parallel to cylindrical axis was examined. It was found that superconductivity temperature is dependent on distance between neighbor atoms and that the magnetic field of circular currents has high value, but not enough for applications for controlled fusion.

**Keywords:** phonon, nano-cylinder, superconductivity.

### 1. Introduction

Cylindrical nanostructures have very intensive production nowadays [1-2]. The most popular are carbon cylinders. The goal of our analysis is metallic cylindrical structures and their superconductive properties in cylinders with nano cross-section and infinite length. In the nearest neighbors approximation electron Hamiltonian of cylinder is a sum of two independent parts (one of electrons propagating along chains and the second related to the electrons propagating in discs):  $H = H_c + H_d$ .

### 2. Superconductivity in chains

The BCS approach starts from the Hamiltonian  $H_c = H_{ec} + H_{pc} + H_{epc}$ , where  $H_{ec}$  is electronic Hamiltonian,  $H_{pc}$  - phonon Hamiltonian and  $H_{epc}$  - Hamiltonian of electron-phonon interaction. In momentum representation we have, respectively:

$$H_{ec} = \sum_k E_k \alpha_k^\dagger \alpha_k ; \quad E_k = 4X \sin^2 \frac{ak}{2} ; \quad k \in \left( -\frac{\pi}{a}, \frac{\pi}{a} \right) \quad (1)$$

$$H_{pc} = \sum_k \varepsilon_q \left( b_q^+ b_q + \frac{1}{2} \right); \quad \varepsilon_q = \hbar \omega_q = 2\hbar \sqrt{\frac{C}{M}} \sin \frac{aq}{2}; \quad q \in \left( -\frac{\pi}{a}, \frac{\pi}{a} \right) \quad (2)$$

$$H_{epc} = \frac{1}{\sqrt{N}} \sum_{k,q} \left( J_{k,q} \alpha_k^+ \alpha_q b_{k-q} + J_{k,q}^* \alpha_q^+ \alpha_k b_{k-q}^* \right) \quad (3)$$

where  $\alpha$  is electron Fermi operator,  $b$  - phonon Bose operator,  $X$  - interaction constant between neighbor atoms,  $M$  - mass of atoms,  $C$  - Hooke's constant.

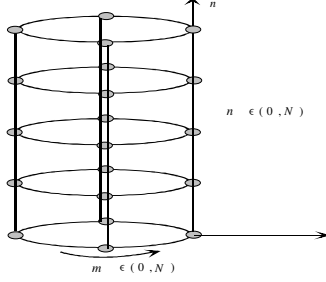


Figure 1. Distribution of electrons in nano cylinder of infinite length

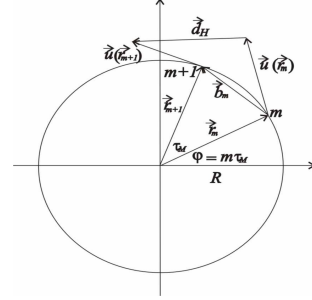


Figure 2. Distances between neighbours atoms in frozen and heated crystal

By unitary Fröhlich transformation [3], after averaging over phonon vacuum:

$$H = \sum_k \rho_k \alpha_k^+ \alpha_k - \frac{1}{N} \sum_{k,q} W_{k,q} \alpha_k^+ \alpha_q^+ \alpha_q \alpha_k; \quad (4)$$

$$\rho_k = X a^2 (k^2 - k_F^2); \quad W_{k,q} = \frac{2|J_{k-q}|}{E_k - E_q + \varepsilon_{k-q}}.$$

After the Bogolyubov transformation [4] in which  $\alpha_k$  and  $\alpha_{-k}$  are expressed in terms of Cooper pairs operators  $\xi_k$  and  $\eta_k$ , the Hamiltonian (6) reduces to

$$H = \sum_k \sqrt{\Delta_k^2 + \rho_q^2} (\xi_k^+ \xi_k + \eta_k^+ \eta_k). \quad (5)$$

Since the electron-electron interaction, proportional to  $W_{k,q}$ , is attractive in narrow domain  $k_g$  enveloping Fermi wave vector  $k_F$ , the gap  $\Delta_k$  is

$$\Delta_k = \frac{1}{2} \frac{\langle W \rangle}{N} \sum_q \frac{\Delta_q}{\sqrt{\Delta_k^2 + \rho_q^2}}; \quad k_F - k_g \leq k \leq k_F + k_g \quad (6)$$

wherefrom it follows

$$\Delta = \frac{2Xa^2k_Fk_g}{\sinh\left[\frac{5}{4}\frac{XC}{(ak_F)^3}\left(\frac{\partial X}{\partial a}\right)^{-2}\right]} \quad (7)$$

Assuming that  $X$  is Coulomb interaction,  $\Delta$  is estimated to be  $\Delta \approx 5k_B$  and the critical superconductive temperature for electrons about 5 K.

## 2. Superconductivity in discs

The disc has cyclical invariance of all their physical characteristics.

$$\Psi_m = \Psi_{m+M+1}. \quad (8)$$

where  $M+1$  - number of atoms in disc. Applying Eq. (11) to plain wave  $e^{ikm}$ , we have:

$k = \frac{2\pi\lambda}{M+1}$ ;  $\lambda = 0, \pm 1, \pm 2, \dots$ . This result enables the standard sum representation of Kronecker's

symbol and the electronic Hamiltonian can be written as:

$$H_{ed} = \sum_{\mu=0}^M E_{\mu} \beta_{\mu}^{\dagger} \beta_{\mu} ; \quad E_{\mu} = 4Y \sin^2 \frac{\pi\mu}{M+1} \quad (9)$$

while the phonon Hamiltonian is the following:

$$H_{pd} = \sum_{\lambda=0}^M \varepsilon_{\lambda} \left( b_{\lambda}^{\dagger} b_{\lambda} + \frac{1}{2} \right) ; \quad \varepsilon_{\lambda} = \hbar\omega_{\lambda} = 2\hbar\sqrt{\frac{C_c}{M}} \sin \frac{\pi\lambda}{2(M+1)} \quad (10)$$

where  $Y$  is the interaction between neighboring atoms.

The problem of electron-phonon interaction is more complicated. On Fig. 2 are given displacements  $\bar{\theta}$  of heated disc. Using cosine theorem and polar coordinates and after the Fröhlich transformation and averaging over phonon vacuum, the Hamiltonian can be written in the form:

$$H = \frac{1}{2} \sum_{\mu=0}^M \tilde{E}_{\mu} \left( \beta_{\mu}^{\dagger} \beta_{\mu} + \beta_{-\mu}^{\dagger} \beta_{-\mu} \right) - \frac{1}{M+1} \sum_{\mu, \nu=1}^M \frac{|D_{\mu, \nu}|^2}{\tilde{E}_{\nu} - \tilde{E}_{\mu} + \varepsilon_{\mu, \nu}} \beta_{\mu}^{\dagger} \beta_{-\nu}^{\dagger} \beta_{-\nu} \beta_{\mu} \quad (11)$$

where

$$D_{\mu, \nu} = 2i \frac{\partial Y}{\partial b_M} \sin \tau_M \sqrt{\frac{\hbar}{M}} \frac{\mu - \nu}{\sqrt{\omega_{\mu - \nu}}} \sin^2 \frac{\tau_M \nu}{2}. \quad (12)$$

By applying Bogolyubov's transformation ( $\beta_s$  and  $\beta_{-s}$  can be expressed through operators  $f$  and  $g$  of Cooper's pairs), Hamiltonian can be written as:

$$H = \sum_s \sqrt{\Delta + \rho_s} (f_s^+ f_s + g_s^+ g_s). \quad (13)$$

The number of electrons is small and therefore the introduction of narrow domain enveloping Fermi wave vector is senseless. Instead of this, domain has to be reduced to Fermi wave vector, only. So, the equation of the type (9) reduces to

$$\Delta_k = \frac{1}{M+1} \left\langle \frac{|D_{\mu,\nu}|^2}{\tilde{E}_\nu - \tilde{E}_\mu + \epsilon_{\mu,\nu}} \right\rangle. \quad (14)$$

For three atoms in disc, i.e. for  $M = 2$ , this quantity reduces to

$$\Delta = 0.175 \frac{1}{C_d} \left( \frac{\partial Y}{\partial b_M} \right)^2 \quad (15)$$

Assuming the Coulomb interaction between atoms, we obtain that  $\Delta \approx 100k_B$ . For this value of  $\Delta$  the calculated magnetic field produced by disc current is  $B = 2400$  T. It is understandable since superconductive currents produce very high magnetic fields. So, instead of conclusion, we can say that practical importance of cylindrical superconductors is the production of very high magnetic fields.

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