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TESTING SOME NON-PERTURBATIVE QCD PREDICTIONS WITH DIRAC EXPERIMENT AT CERN

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Abstract
A theoretical presentation of the non-perturbative QCD predictions tested by DIRAC Experiment at CERN.
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1. Introduction

Quantum Chromodynamics (QCD) is the general accepted strong interaction theory. It has successfully been tested only in the perturbative region of high momentum transfer (Q>2GeV) or equally, at short relative distance $\Delta x \sim \hbar/Q$ ($\Delta x<0.1$fm). Here, the constituent quarks behave as weakly interacting, nearly massless particles and the strong coupling "constant" is not constant at all, but evolves with Q^2 (see Fig. 1).

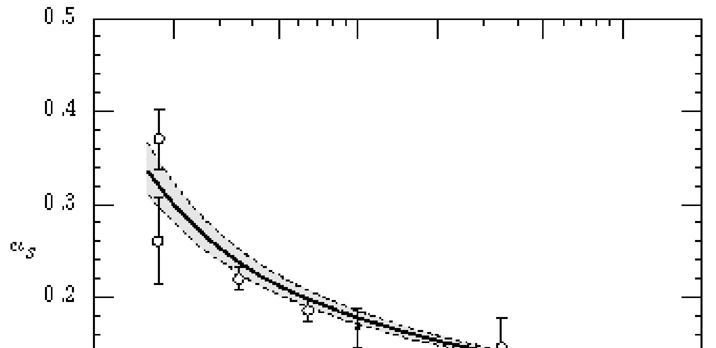

Figure 1. α_S strong coupling dependence on momentum transfer *Q*.

The reduction in α_S with increasing *Q* is called *asymptotic freedom.* It entails that perturbation theory is valid at large *Q* in QCD ($\Lambda_{QCD} \sim 200$ MeV), i.e. at short relative distances. The QCD in the perturbative region, as any gauge theory with massless fermions, presents *chiral symmetry*.

The non-perturbative QCD phenomena (*quark confinement*, *quark condensate*, *chiral symmetry breaking*, *bound state structure*, *phase transitions*) manifest on large relative distance Δx, or equivalently at low momentum transfer (low energy) say Q<100MeV, where the quark interaction cannot be treated perturbatively as an expansion in strong coupling α_S. Here, the *asymptotic freedom* is absent, and *quark confinement* takes place. Also here, in the non-perturbative region, the *chiral symmetry* of QCD must be spontaneously broken.

The *Chiral Perturbation Theory* (ChPT) [1,2] is the QCD candidate theory for low energy processes. It exploits the mechanism of *spontaneous breakdown of chiral symmetry*, or, in other words, the existence of a *quark condensate*. The ChPT as non-perturbative candidate of QCD, is replacing the QCD quark degrees of freedom with the pion ones (see Fig.2).

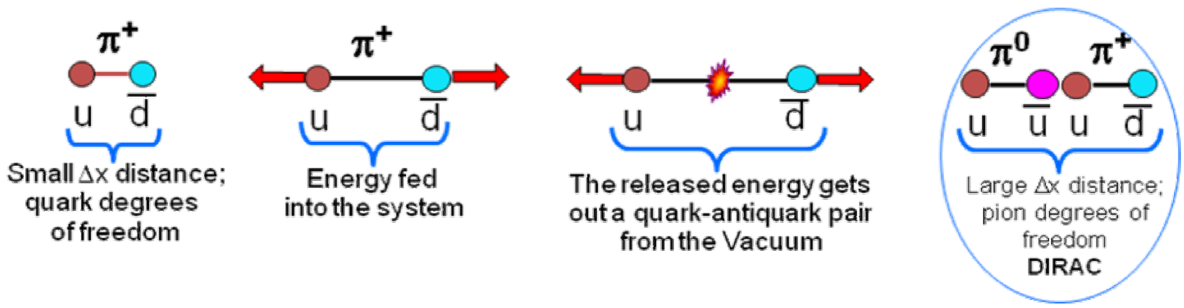

Figure 2. QCD processes at small and large distance.

Our physical understanding of the situation is that <u>at low energies</u> one has only hadronic states. These can be thought of as consisting of quarks carrying a dynamically generated quark mass $m=m_u=m_d$ for two flavours, and form *quark condensates* constructed into *baryonic states* or *mesonic states* according to the Goldstone theorem. <u>At high energies</u> at which the *chiral symmetry* is restored and the constituent quarks take on their *current mass* value, deconfinement occurs simultaneously. The hadronic states dissolve, and one moves to a plasma containing only quark and gluonic degrees of freedom.

In order to test the existence of the *quark condensate*, the particularly significant symmetry breaking effect refers to the *S*-wave $\pi\pi$ *scattering length*. The values predicted with high accuracy (~2%) by ChPT for the isospin $I=0$ and $I=2$ *S*-wave $\pi\pi$ scattering lengths a_0 and a_2, can be confronted with the experimental results. By measuring pionium ($\pi^+\pi^-$ hadronic atom) lifetime, DIRAC [3] determines in a model-independent way the difference $|a_0-a_2|$ between isoscalar and isotensor *S*-wave $\pi\pi$ *scattering lengths*. Therefore such a measurement will be a sensitive check in understanding *chiral symmetry breaking* of QCD, giving an indication about the size of the *quark condensate* – an order parameter of QCD.

2. Chiral Representation, Chiral Symmetry and Chiral Perturbation Theory

Dirac equation is a relativistic equation with a two-component structure solutions to accommodate the spins of the particles and antiparticles. It must be linear in *p* operator $\forall(\partial/\partial x, \partial/\partial y, \partial/\partial z)$, to be relativistic covariant and linear in *E* operator $\partial/\partial t$, to avoid negative probabilities. The original form of the free particle Dirac equation in momentum space [1] is

$$\left(\vec{\alpha}\cdot\vec{p}+\beta m\right)\psi = E\psi \qquad \text{for the bispinor} \qquad \psi=\begin{pmatrix}\psi_A\\ \psi_B\end{pmatrix} \tag{1}$$

where α and β matrices in the standard (Pauli-Dirac) representation are

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}\\ \vec{\sigma} & 0\end{pmatrix} \qquad \beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix} \tag{2}$$

In the chiral (Weyl) representation they are

$$\vec{\alpha}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & -\vec{\sigma}\end{pmatrix} \qquad \beta=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix} \tag{3}$$

Now, explicitely the Dirac equation in *chiral representation* is

$$\begin{cases}\vec{\sigma}\cdot\vec{p}\psi_R + m\psi_L = E\psi_R\\ -\vec{\sigma}\cdot\vec{p}\psi_L + m\psi_R = E\psi_L\end{cases} \qquad \text{for the bispinor} \qquad \psi=\begin{pmatrix}\psi_R\\ \psi_L\end{pmatrix} \tag{4}$$

So, for m=0 particles, the Dirac equation in *chiral representation* splits into two decoupled equations for right (ψ_R) and left (ψ_L) helicity spinors.

Hadron world has the *isospin symmetry*, a consequence of the symmetry at the quark level. The fact that the neutron and proton have almost the same mass means, within quark model, that the masses of the *u* and *d* quarks should be equal. Indeed their *constituent masses* are both about 300 MeV. However, the mass that enters the QCD Lagrangian is the mass of the quark when it travels over very short distances, not across a whole hadron. Its mass is the so called *current mass*. For *u* and *d* quarks these are a few MeV. In this case,

$$m_u, m_d \ll \Lambda_{QCD} \approx 200 MeV \Longrightarrow m_u = m_d \approx 0 \tag{5}$$

Dynamics of a system is described by Lagrangian. So let's see the *chiral transformation* effect on a simplified two flavour QCD Lagrangian of massless non-interacting fermions:

$$L_{QCD} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \tag{6}$$

where γ^{μ} are the Dirac matrices. This Lagrangian is invariant (symmetrical) under both ***vector isospin*** transformation, expressed by the isospin Pauli matrices $\vec{\tau}$

$$\psi_V \to \psi'_V = \exp\left(-\frac{i}{2}\vec{\alpha}_V \cdot \vec{\tau}\right)\psi_V \approx \left(I - \frac{i}{2}\delta\vec{\alpha}_V \cdot \vec{\tau}\right)\psi_V \tag{7}$$

and ***axial-vector isospin*** transformation, which includes also the γ^5 matrix:

$$\psi_A \to \psi'_A = \exp\left(-\frac{i}{2}\gamma^5\vec{\alpha}_A \cdot \vec{\tau}\right)\psi_A \approx \left(I - \frac{i}{2}\gamma^5\delta\vec{\alpha}_A \cdot \vec{\tau}\right)\psi_A \tag{8}$$

The composed $SU(2)_V \times SU(2)_A$ field transformation

$$\psi = \psi_V\psi_A \to \psi'_V\psi'_A = \exp\left[-\frac{i}{2}\left(\vec{\alpha}_V + \gamma^5\vec{\alpha}_A\right)\cdot\vec{\tau}\right]\psi \tag{9}$$

is also a symmetry transformation for massless fermions.
Associated with this $SU(2)_V \times SU(2)_A$ symmetry are the conserved vector V and axial-vector A currents:

$$V_i^{\mu} = \bar{\psi}\gamma^{\mu}\frac{\vec{\tau}_i}{2}\psi \qquad\qquad A_i^{\mu} = \bar{\psi}\gamma^{\mu}\gamma^5\frac{\vec{\tau}_i}{2}\psi \tag{10}$$

This invariance is referred to as *chiral symmetry* because the Lagrangian (6) can be decomposed into left- and right-handed parts,

$$L_{QCD} = i\bar{\psi}_L\gamma^{\mu}\partial_{\mu}\psi_L + i\bar{\psi}_R\gamma^{\mu}\partial_{\mu}\psi_R \tag{11}$$

which is invariant under *left* and *right isospin transformations*, with the new parameters

$$\vec{\alpha}_L = \vec{\alpha}_V - \vec{\alpha}_A \qquad\qquad \vec{\alpha}_R = \vec{\alpha}_V + \vec{\alpha}_A \tag{12}$$

The decoupled ψ_L and ψ_R fields are *helicity eigenstates*

$$\psi_L = \frac{1}{2}\left(1-\gamma^5\right)\psi \qquad\qquad \psi_R = \frac{1}{2}\left(1+\gamma^5\right)\psi \tag{13}$$

with conserved J_L and J_R currents

$$J_L^{\mu} = \frac{1}{2}\left(V^{\mu} - A^{\mu}\right) \qquad\qquad J_R^{\mu} = \frac{1}{2}\left(V^{\mu} + A^{\mu}\right) \tag{14}$$

The symmetry group can therefore be written as $SU(2)_L \times SU(2)_R$.

When fermions are massless, the left- and right-handed components decouple (4), and the QCD in perturbative region, presents Lagrangian with a bigger symmetry, the $SU(2)_L \times SU(2)_R$ *chiral symmetry*. Nevertheless, this is not reflected in hadron world. The observed hadron spectrum suggests that the ground state is asymmetric under $SU(2)_L \times SU(2)_R$ *chiral* *transformations* (spontaneous breakdown of chiral symmetry).

Symmetry of Lagrangian is not the same as the symmetry of eigenstates. A system is said to possess a symmetry that is *spontaneously broken* if the Lagrangian describing the dynamics of the system is invariant under these transformations, but the vacuum of the theory is NOT. Here the vacuum |0> is the state where the Hamiltonian expectation value <0|H|0> is minimum. In the particular case of QCD, the global symmetry – chiral symmetry, turns out to be *spontaneously broken*.

We saw, for $m_u=m_d=0$, Lagrangian (11) has the chiral symmetry $SU(2)_L \times SU(2)_R$ defined by the following transformations:

$$\psi_L \to \psi'_L = U_L \psi_L \qquad \psi_R \to \psi'_R = U_R \psi_R$$

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \psi_L = \frac{1}{2}\left(1-\gamma^5\right)\psi \qquad \psi_R = \frac{1}{2}\left(1+\gamma^5\right)\psi \tag{15}$$

$$U_L \in SU(2)_L \qquad U_R \in SU(2)_R$$

It turns out that the physical vacuum of QCD is not invariant under the full chiral $SU(2)_L \times SU(2)_R$ group, but just under the subgroup $SU(2)_V=SU(2)_{L+R}$ which is precisely the isospin group. The transformations given by the *axial subgroup*, $SU(2)_A$, do not leave the QCD vacuum invariant. Therefore, QCD with $m_u=m_d=0$ has a chiral symmetry which is *spontaneously broken* down to the isospin symmetry: $SU(2)_L\times SU(2)_R \rightarrow SU(2)_V$.

In the presence of quark masses ($m_u \neq 0,\ m_d \neq 0$) there is an additional quark term *breaking chiral symmetry explicitly*:

$$L_{QCD}(q,g) = L_{sym} + L_{break-sym} \tag{16}$$

At high energies one expects to restore chiral symmetry. That is, unlike the ground state, the higher states posses the same symmetry as Lagrangian.

At low energies, as long as the symmetry breaking is small, one would expect that its effect can be described in a perturbative approach. This is carried out in a systematic fashion in the framework of C*hiral Perturbation Theory*.

Low energy hadronic processes are dominated by pions and thus all observables can be expressed as an expansion in pion masses and momenta. It allows writing down an effective Lagrangian of the same form as before, but replacing the *q* degrees of freedom by π ones:

$$L_{eff}(\pi) = L_{sym} + L_{break-sym} \tag{17}$$

In the framework of *Chiral Perturbation Theory* [4], by performing an expansion in momentum (p) and mass, precise prediction for low energy hadronic (pionic) processes can be obtained from $L_{eff}(\pi)$.

The most precise predictions of ChPT [4] have been achieved for the *s*-wave *scattering lengths* (in m_π^{-1} units):

$$a_0 = 0.220 \pm 0.005, \qquad a_2 = -0.0444 \pm 0.0010, \qquad a_0 - a_2 = 0.265 \pm 0.004 \tag{19}$$

These values will be compared with those determined in the decay of the pionium ($\pi^+\pi^-$ atom)

3. DIRAC Experiment at CERN

This experiment has determined the pionium lifetime by measuring its breakup (ionization) probability. Pionium atoms, produced in the target, may either annihilate, or breakup after interaction with target atoms into $\pi^+\pi^-$ pairs (atomic pairs). These pairs are experimentally observable by their characteristic low relative momentum (Q ≤ 4 MeV/c). As long as the pionium travels a long path (long lifetime) it is a large probability to breakup, and conversely. There is a direct connection between pionium breakup probability and its the lifetime.

The first measurement of the $\pi^+\pi^-$ atom lifetime [5] shows the value

$$\tau = \frac{1}{\Gamma_0} = \left[2.91^{+0.49}_{-0.62}\right] \times 10^{-15}\, s \tag{20}$$

With the relation for the decay width Γ_0 as a function of the difference between isoscalar a_0 and isotensor a_2 scattering length [6]

$$\Gamma_0(\pi^+\pi^-) = \underbrace{\frac{16\pi}{9}\sqrt{\frac{2\Delta m_\pi}{m_\pi}}}_{\Delta m_\pi \text{ isospin symmetry}} \underbrace{\left|a_0 - a_2\right|^2}_{\textbf{strong} \text{ decay}} \underbrace{\left|\psi_c(0)\right|^2}_{\textbf{electromagnetic} \text{ binding}} \tag{21}$$

it is obtained

$$\left|a_0 - a_2\right| = 0.264^{+0.033}_{-0.020}\, m_\pi^{-1} \tag{22}$$

which is a confirmation of the ChPT predictions of [4]: $a_0 - a_2 = 0.265 \pm 0.004$.

4. Conclusions

The Chiral Perturbation Theory predictions on the $\pi^+\pi^-$ scattering lengths have been tested successfully with DIRAC Experiment by measuring the lifetime of the $\pi^+\pi^-$ hadronic atom.

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References

1. F.Martin, A.D. Halzen, *Quarks and Leptons: An Introductory Course in Modern Particle Physics*, John Wiley & Sons (1984).
2. I J R Aitchison and A J G Hey, GAUGE THEORIES IN PARTICLE PHYSICS, Taylor & Francis Group, New York, 2003
3. B.Adeva, … M. Pentia, et al., DIRAC proposal, CERN/SPSLC 95-1,SPSLC/P 284 (1995)
4. G.Colangelo, J.Gasser, H.Leutwyler, *Nucl. Phys.* B603 (2001) 125
5. B.Adeva, … M. Pentia, et al, *Phys.Lett.*B619 (2005) 50
6. J.Gasser, et al. *Phys.Rev.* D64 (2001) 016008; A.Gashi, et al. *Nucl. Phys.*A699 (2002) 732