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IRREDUCIBLE APPROACH TO THIRD-ORDER REDUCIBLE SECOND-CLASS CONSTRAINTS

O. Balus, C. Bizdadea, E. M. Cioroianu, S. O. Saliu, S. C. Sararu

Faculty of Physics, University of Craiova, 13 Al. I. Cuza Str., Craiova 200585, Romania

Abstract
An irreducible canonical approach to third-order reducible second-class constraints is given. The procedure is illustrated on gauge-fixed 4-forms.

1. Introduction

The canonical approach of the systems with reducible second-class constraints represents a difficult problem (not all the second-class constraint functions are independent), demanding a modification of the usual rules as the matrix of the Poisson brackets among constraints is no longer invertible.

In order to construct the Dirac bracket for such systems in a consistent manner we have the three options: to isolate a maximally set of independent constraint functions and then build the Dirac bracket in terms of this smaller set **Eroare! Fără sursă de referință.-Eroare! Fără sursă de referință.**, the second option is to construct the Dirac bracket in terms of a noninvertible matrix without separating the independent constraint functions **Eroare! Fără sursă de referință.-Eroare! Fără sursă de referință.** and the third possibility is to substitute the reducible second-class constraints by some equivalent irreducible ones [by an appropriate enlarging of the original phase-space] and further work with the Dirac bracket based on the irreducible constraints **Eroare! Fără sursă de referință.-Eroare! Fără sursă de referință.**. The procedure is illustrated on gauge-fixed 4-forms.

2. Third-stage reducible second-class constraints

Our starting point is a system with the phase-space locally parametrized by N canonical pairs $z^a = \left(q^i, p_i\right)$ subject to the third-stage reducible second-class constraints

$$\chi_{\alpha_0}\left(z^a\right) \approx 0,\ \alpha_0 = \overline{1, M_0}, \tag{1}$$

$$Z_{\alpha_1}^{\alpha_0}\chi_{\alpha_0} = 0, \alpha_1 = \overline{1, M_1}, \tag{2}$$

$$Z_{\alpha_2}^{\alpha_1} Z_{\alpha_1}^{\alpha_0} \approx 0, \alpha_2 = \overline{1, M_2}, \tag{3}$$

$$Z_{\alpha_3}^{\alpha_2} Z_{\alpha_2}^{\alpha_1} \approx 0, \alpha_3 = \overline{1, M_3}, \tag{4}$$

These constraints are purely second-class if any maximal, independent set of $M \equiv M_0 - M_1 + M_2 - M_3$ constraint functions χ_A, $A = \overline{1, M}$ among the χ_{α_0} is such that the matrix

$$C_{AB} = [\chi_A, \chi_B], \tag{5}$$

is invertible. In terms of such a set of independent constraints, the Dirac bracket takes the form

$$F, G]^* = [F, G] - [F, \chi_A] M^{AB} [\chi_B, G], \tag{6}$$

where $M^{AB} C_{BC} \approx \delta_C^A$. The split of the constraints may lead to the loss of important symmetries, so it should be avoided.

A second idea is to construct the Dirac bracket in terms of a noninvertible matrix without separating the independent constraint functions. In this sense, we denote the matrix of the Poisson brackets among the second-class constraint functions by

$$C_{\alpha_0\beta_0} = [\chi_{\alpha_0}, \chi_{\beta_0}]. \tag{7}$$

The matrix $C_{\alpha_0\beta_0}$ is not invertible because

$$Z_{\alpha_1}^{\alpha_0} C_{\alpha_0\beta_0} \approx 0. \tag{8}$$

If $A_{\alpha_0}^{\alpha_1}$ stand for some functions that satisfy

$$\operatorname{rank}\left(Z_{\alpha_1}^{\alpha_0} \overline{A}_{\alpha_0}^{\beta_1} \right) \equiv \operatorname{rank}\left(D_{\alpha_1}^{\beta_1} \right) = M_1 - M_2 + M_3, \tag{9}$$

then we can introduce another matrix **Eroare! Fără sursă de referință.** $M^{\alpha_0\beta_0}$ through the relations

$$C_{\alpha_0\gamma_0} M^{\gamma_0\beta_0} \approx D_{\alpha_0}^{\beta_0}, \tag{10}$$

with $M^{\alpha_0\beta_0} = -M^{\beta_0\alpha_0}$, such that the bracket

$$[F, G]^* = [F, G] - [F, \chi_{\alpha_0}] M^{\alpha_0\beta_0} [\chi_{\beta_0}, G], \tag{11}$$

defines the same Dirac bracket like (6) on the surface (1), where

$$D_{\alpha_0}^{\ \beta_0} = \delta_{\alpha_0}^{\ \beta_0} - \bar{A}_{\alpha_0}^{\ \beta_1} Z_{\beta_1}^{\ \beta_0}. \tag{12}$$

3. The model

We consider the canonical approach to gauge-fixed four-forms, described by the Lagrangian action

$$S_0^L\left[A_{\mu\nu\rho\lambda}\right] = -\int d^D x \frac{1}{2\cdot 5!} F_{\mu\nu\rho\lambda\sigma} F^{\mu\nu\rho\lambda\sigma}, \tag{13}$$

where

$$F_{\mu\nu\rho\lambda\sigma} = \partial_{[\mu} A_{\nu\rho\lambda\sigma]}, \tag{14}$$

and $D \geq 5$. Everywhere in this paper the notation $[\mu \ldots \nu]$ signifies complete antisymmetry with respect to the indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. The canonical analysis of this model leads to the first-class constraints

$$G^{(1)}_{i_1 i_2 i_3} \equiv \pi_{0 i_1 i_2 i_3} \approx 0, \tag{15}$$

$$\chi^{(1)}_{i_1 i_2 i_3} \equiv -4\partial^k \pi_{k i_1 i_2 i_3} \approx 0, \tag{16}$$

where the momentum $\pi_{\mu\nu\rho\lambda}$ are respectively conjugated to $A^{\mu\nu\rho\lambda}$. In order to fix the gauge, we have to choose a set of canonical gauge conditions. An appropriate set of such gauge conditions is given by

$$G^{(2) j_1 j_2 j_3} \equiv A^{0 j_1 j_2 j_3} \approx 0, \tag{17}$$

$$\chi^{(2) j_1 j_2 j_3} \equiv -\partial_k A^{k j_1 j_2 j_3}. \tag{18}$$

The relations (15)-(18) represent nothing but some third-stage reducible second-class constraints. It is simple to see that (15) and (17) generate a submatrix (of the matrix of the Poisson brackets among the constraint functions) of maximum rank, therefore they are not relevant by virtue of our approach. Thus in the following we examine only the constraints (16) and (18), which we organize as

$$\chi_{\alpha_0} \equiv \begin{pmatrix} \chi^{(1)}_{i_1 i_2 i_3} \\ \chi^{(2) j_1 j_2 j_3} \end{pmatrix} \approx 0. \tag{19}$$

The second-class constraint functions from (19) are third-stage reducible, with the first-, second- and third-stage reducibility functions given by

$$Z_{\alpha_1}^{\alpha_0} = \begin{pmatrix} \frac{1}{3}\delta_{k_1}^{[i_1}\delta_{k_2}^{i_2}\partial^{i_3]} & 0 \\ 0 & \delta_{j_1}^{l_1}\delta_{j_2}^{l_2}\partial_{j_3]} \end{pmatrix}, \tag{20}$$

$$Z_{\alpha_2}^{\alpha_1} = \begin{pmatrix} \frac{1}{2}\delta_{m_1}^{[k_1}\partial^{k_2]} & 0 \\ 0 & \delta_{l_1}^{n_1}\partial_{l_2]} \end{pmatrix}, \tag{21}$$

and respectively

$$Z_{\alpha_2}^{\alpha_1} = \begin{pmatrix} \partial^{m_1} & 0 \\ 0 & \partial_{n_1} \end{pmatrix}. \tag{22}$$

The matrix of the Poisson brackets among the constraints (19) is expressed by

$$C_{\alpha_0\beta_0} = \begin{pmatrix} 0 & \Delta D_{i_1 i_2 i_3}^{k_1 k_2 k_3} \\ -\Delta D_{l_1 l_2 l_3}^{j_1 j_2 j_3} & 0 \end{pmatrix}, \tag{23}$$

where

$$D_{i_1 i_2 i_3}^{j_1 j_2 j_3} = \frac{1}{3!}\left(\delta_{i_1}^{j_1}\delta_{i_2}^{j_2}\delta_{i_3]}^{j_3} - \frac{\delta_{i_1}^{k_1}\delta_{i_2}^{k_2}\partial_{i_3]}\delta_{k_1}^{[j_1}\delta_{k_2}^{j_2}\partial^{j_3]}}{2\Delta} \right), \tag{24}$$

and $\Delta = \partial^k \partial_k$.

3.1 "Reducible" Dirac bracket

Now, we construct the Dirac bracket with respect to the constraints (19). In order to construct the matrices $D_{\alpha_0}^{\beta_0}$ (12), we take $\bar{A}_{\alpha_0}^{\beta_1}$

$$\bar{A}_{\alpha_0}^{\beta_1} = \begin{pmatrix} \frac{1}{4\Delta}\delta_{i_1}^{k_1}\delta_{i_2}^{k_2}\partial_{i_3]} & 0 \\ 0 & \frac{1}{12\Delta}\delta_{l_1}^{[j_1}\delta_{l_2}^{j_2}\partial^{j_3]} \end{pmatrix}. \tag{25}$$

Then, by means of (12) we find

$$D_{\alpha_0}^{\beta_0} = \begin{pmatrix} D_{i_1 i_2 i_3}^{k_1 k_2 k_3} & 0 \\ 0 & D_{l_1 l_2 l_3}^{j_1 j_2 j_3} \end{pmatrix}. \tag{26}$$

Using (23) and (26) it follows that (10) is fulfilled for

$$M^{\alpha_0\beta_0} = \begin{pmatrix} 0 & -\frac{1}{\Delta} D^{i_1 i_2 i_3}_{k_1 k_2 k_3} \\ \frac{1}{\Delta} D^{l_1 l_2 l_3}_{j_1 j_2 j_3} & 0 \end{pmatrix}. \tag{27}$$

With $M^{\alpha_0\beta_0}$ at the hand, we can construct the Dirac bracket by means of formula (11) . After some computation, we find that the only non-vanishing fundamental Dirac brackets are

$$\left[A^{i_1 i_2 i_3 i_4}(x), \pi_{j_1 j_2 j_3 j_4}(y)\right]^*_{x^0=y^0} = D^{i_1 i_2 i_3 i_4}_{j_1 j_2 j_3 j_4} \delta^{D-1}(\vec{x}-\vec{y}), \tag{28}$$

where

$$D^{i_1 i_2 i_3 i_4}_{j_1 j_2 j_3 j_4} = \frac{1}{4!}\left(\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}\delta^{i_4}_{j_4]} - \frac{\delta^{[i_1}_{k_1}\delta^{i_2}_{k_2}\delta^{i_3}_{k_3}\partial^{i_4]}\delta^{k_1}_{j_1}\delta^{k_2}_{j_2}\delta^{k_3}_{j_3}\partial_{j_4]}}{\Delta}\right), \tag{29}$$

In this way, the Dirac analysis (reducible) of this model is completed.

4. Irreducible analysis

In this section we reobtain the Dirac bracket (28) but in an irreducible manner.

4.1. Original phase-space approach

Initially, we investigate the problem of the construction of Dirac bracket for our model in the original phase-space in terms of an invertible matrix.

It can be proved that for systems with third-stage reducible second-class constraints the Dirac bracket can be written in terms of an invertible matrix.

Theorem 1 *There exists an invertible antisymmetric matrix* $\mu^{\gamma_0\delta_0}$ *such that the Dirac bracket* (11) *takes the form*

$$[F,G]^* = [F,G] - [F,\chi_{\alpha_0}]\mu^{\alpha_0\beta_0}[\chi_{\beta_0},G], \tag{30}$$

on the surface. (1).

In the case of our model the matrix $\mu^{\gamma_0\delta_0}$ takes the form

$$\mu^{\alpha_0\beta_0} = \begin{pmatrix} 0 & -\frac{1}{3!\Delta}\delta^{i_1}_{k_1}\delta^{i_2}_{k_2}\delta^{i_3}_{k_3]} \\ \frac{1}{3!\Delta}\delta^{[l_1}_{j_1}\delta^{l_2}_{j_2}\delta^{l_3]}_{j_3} & 0 \end{pmatrix}. \tag{31}$$

By computing the fundamental Dirac bracket with the help of (30), we reobtain precisely (28).

4.2.Extended phase-space approach

In the sequel we construct some equivalent irreducible second-class constraints associated with (1) such that the Dirac bracket constructed with respect to irreducible set coincides with the Dirac bracket corresponding to the reducible second-class model.

Firstly we introduce some new variables $\left(y_{\alpha_1}\right)_{\alpha_1=\overline{1,M_1}}$ and $\left(y_{\alpha_3}\right)_{\alpha_3=\overline{1,M_3}}$ with the Poisson brackets

$$\left[y_{\alpha_1}, y_{\beta_1}\right] = \omega_{\alpha_1\beta_1}, \qquad \left[y_{\alpha_3}, y_{\beta_3}\right] = \omega_{\alpha_3\beta_3}, \qquad \left[y_{\alpha_1}, y_{\alpha_3}\right] = 0, \tag{32}$$

where the elements $\omega_{\alpha_1\beta_1}$ define an invertible, antisymmetric matrix (similar for $\omega_{\alpha_3\beta_3}$), and consider the system subject to the reducible second-class constraints

$$\chi_{\alpha_0} \approx 0, \qquad y_{\alpha_1} \approx 0, \qquad y_{\alpha_3} \approx 0. \tag{33}$$

The Dirac bracket on the phase-space locally parametrized by $\left(z^a, y_{\alpha_1}, y_{\alpha_3}\right)$ corresponding to the above second-class constraints reads as

$$\begin{aligned} \left[F,G\right]^*\Big|_{z,y} &= \left[F,G\right] - \left[F,\chi_{\alpha_0}\right]\mu^{\alpha_0\beta_0}\left[\chi_{\beta_0},G\right] \\ &- \left[F,y_{\alpha_1}\right]\omega^{\alpha_1\beta_1}\left[y_{\beta_1},G\right] - \left[F,y_{\alpha_3}\right]\omega^{\alpha_3\beta_3}\left[y_{\beta_3},G\right], \end{aligned} \tag{34}$$

where the Poisson brackets from the right-hand side of (34) contain derivatives with respect to all z^a's, y_{α_1}'s and y_{α_3}'s. After some computation we infer that

$$\left[F,G\right]^*\Big|_{z,y} \approx \left[F,G\right]^*, \tag{35}$$

where $\left[F,G\right]^*$ is given by (30).

Under these considerations, we are able to formulate the following theorem.

Theorem 2 *There exists a set of constraints*

$$\tilde{\chi}_{\alpha_0} \equiv \chi_{\alpha_0} + A_{\alpha_0}^{\alpha_1} y_{\alpha_1} \approx 0, \tag{36}$$

$$\tilde{\chi}_{\alpha_2} \equiv Z_{\alpha_2}^{\alpha_1} y_{\alpha_1} + A_{\alpha_2}^{\alpha_3} y_{\alpha_3} \approx 0, \tag{37}$$

such that:

i) (36)-(37) is equivalent with (33) [this means that both sets describe the same surface in the enlarged phase-space]

$$\tilde{\chi}_{\alpha_0} \approx 0, \tilde{\chi}_{\alpha_2} \approx 0 \Leftrightarrow \chi_{\alpha_0} \approx 0, y_{\alpha_1} \approx 0, y_{\alpha_3} \approx 0; \tag{38}$$

ii) second-class behavior, i.e. the matrix

$$C_{\Delta\Delta'} = [\tilde{\chi}_{\Delta}, \tilde{\chi}_{\Delta'}], \tag{39}$$

is invertible, where

$$\tilde{\chi}_{\Delta} = \left(\tilde{\chi}_{\alpha_0}, \tilde{\chi}_{\alpha_2}\right), \tag{40}$$

iii)irreducibility.

The functions $A_{\alpha_0}^{\alpha_1}$ are defined by the relation

$$\bar{A}_{\alpha_0}^{\alpha_1} = A_{\alpha_0}^{\beta_1} \hat{e}_{\beta_1}^{\alpha_1}, \tag{41}$$

where $\hat{e}_{\beta_1}^{\alpha_1}$ are the elements of an invertible matrix. In the formula (37) $A_{\alpha_2}^{\alpha_3}$ are some functions that satisfy

$$\text{rank}\left(Z_{\alpha_3}^{\alpha_2} A_{\alpha_2}^{\beta_3}\right) \equiv \text{rank}\left(D_{\alpha_3}^{\beta_3}\right) = M_3, \tag{42}$$

The existence of such functions is guaranteed by the fact that the second-class constraints (1) are third-stage reducible (2)-(4).

The matrix $C_{\Delta\Delta'}$ takes the concrete form

$$C_{\Delta\Delta'} = \begin{pmatrix} \mu_{\alpha_0\beta_0} & A_{\alpha_0}^{\alpha_1} \omega_{\alpha_1\beta_1} Z_{\beta_2}^{\beta_1} \\ Z_{\alpha_2}^{\alpha_1} \omega_{\alpha_1\beta_1} A_{\beta_0}^{\beta_1} & Z_{\alpha_2}^{\alpha_1} \omega_{\alpha_1\beta_1} Z_{\beta_2}^{\beta_1} + A_{\alpha_2}^{\alpha_3} \omega_{\alpha_3\beta_3} A_{\beta_2}^{\beta_3} \end{pmatrix}, \tag{43}$$

where $\Delta = (\alpha_0, \alpha_2)$ indexes the line and $\Delta' = (\beta_0, \beta_2)$ the column and its inverse reads as

$$C^{\Delta'\Delta''} = \begin{pmatrix} \mu^{\beta_0\rho_0} & Z_{\gamma_1}^{\beta_0} \hat{e}_{\sigma_1}^{\gamma_1} \omega^{\sigma_1\lambda_1} \bar{A}_{\lambda_1}^{\rho_2} \\ \bar{A}_{\sigma_1}^{\beta_2} \omega^{\sigma_1\lambda_1} \hat{e}_{\lambda_1}^{\gamma_1} Z_{\gamma_1}^{\rho_0} & \psi^{\beta_2\rho_2} \end{pmatrix}, \tag{44}$$

where we used the notation

$$\psi^{\beta_2\rho_2} = \bar{A}_{\sigma_1}^{\beta_2} \omega^{\sigma_1\lambda_1} \bar{A}_{\lambda_1}^{\rho_2} + Z_{\sigma_3}^{\beta_2} \bar{D}_{\lambda_3}^{\sigma_3} \omega^{\lambda_3\tau_3} \bar{D}_{\tau_3}^{\gamma_3} Z_{\gamma_3}^{\rho_2} \tag{45}$$

By means of result (44), the Dirac bracket associated with the irreducible second-class constraints (36)-(37)

$$[F,G]^{*}\Big|_{\text{ired}} = [F,G] - [F, \tilde{\chi}_{\Delta}] C^{\Delta\Delta'} [\tilde{\chi}_{\Delta'}, G], \tag{46}$$

takes the concrete form

$$
\begin{aligned}
[F,G]^*\Big|_{\text{ired}} = {} & [F,G]-[F,\tilde{\chi}_{\alpha_0}]\mu^{\alpha_0\beta_0}[\tilde{\chi}_{\beta_0},G] \\
& -[F,\tilde{\chi}_{\alpha_0}]Z_{\gamma_1}^{\ \alpha_0}\hat{e}_{\sigma_1}^{\gamma_1}\omega^{\sigma_1\lambda_1}\bar{A}_{\lambda_1}^{\ \beta_2}[\tilde{\chi}_{\beta_2},G] \\
& -[F,\tilde{\chi}_{\alpha_2}]\bar{A}_{\sigma_1}^{\ \alpha_2}\omega^{\sigma_1\lambda_1}\hat{e}_{\lambda_1}^{\gamma_1}Z_{\gamma_1}^{\ \beta_0}[\tilde{\chi}_{\beta_0},G] \\
& -[F,\tilde{\chi}_{\alpha_2}]\Big(\bar{A}_{\sigma_1}^{\ \alpha_2}\omega^{\sigma_1\lambda_1}\bar{A}_{\lambda_1}^{\ \beta_2} \\
& +Z_{\sigma_3}^{\ \alpha_2}\bar{D}_{\lambda_3}^{\ \sigma_3}\omega^{\lambda_3\tau_3}\bar{D}_{\tau_3}^{\ \gamma_3}Z_{\gamma_3}^{\ \beta_2}\Big)[\tilde{\chi}_{\beta_2},G]
\end{aligned}
\tag{47}
$$

The matrix $\bar{D}_{\beta_3}^{\ \sigma_3}$ is the inverse of $D_{\alpha_3}^{\ \beta_3}$.

Theorem 3 *The Dirac bracket with respect to the irreducible second-class constraints,* (47)*, coincides with that of the intermediate system* (33)

$$
[F,G]^*\Big|_{\text{ired}} \approx [F,G]^*\Big|_{z,y}. \tag{48}
$$

5. "Irreducible" Dirac bracket

In order to construct the irreducible second-class constraints for our model we introduce the new variables y_{α_1} and y_{α_3}

$$
y_{\alpha_1} = \begin{pmatrix} P_{i_1 i_2} \\ B^{j_1 j_2} \end{pmatrix}, \qquad y_{\alpha_3} = \begin{pmatrix} p \\ \varphi \end{pmatrix}; \tag{49}
$$

and take

$$
\omega_{\alpha_1\beta_1} = \begin{pmatrix} 0 & -\frac{1}{2}\delta_{[i_1}^{k_1}\delta_{i_2]}^{k_2} \\ \frac{1}{2}\delta_{[l_1}^{j_1}\delta_{l_2]}^{j_2} & 0 \end{pmatrix}, \qquad \omega_{\alpha_3\beta_3} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{50}
$$

In the analyzed model the functions $A_{\alpha_0}^{\ \alpha_1}$ and $A_{\alpha_2}^{\ \alpha_3}$ are given by

$$
A_{\alpha_0}^{\ \alpha_1} = \begin{pmatrix} -\frac{1}{2}\delta_{[i_1}^{k_1}\delta_{i_2}^{k_2}\partial_{i_3]} & 0 \\ 0 & -\frac{1}{6}\delta_{l_1}^{[j_1}\delta_{l_2}^{j_2}\partial^{j_3]} \end{pmatrix}, \tag{51}
$$

$$
A_{\alpha_2}^{\ \alpha_3} = \begin{pmatrix} \partial_{i_1} & 0 \\ 0 & \partial^{j_1} \end{pmatrix}. \tag{52}
$$

Then, the equivalent irreducible second-class constraints are expressed by

$$
\tilde{\chi}^{(1)}_{i_1 i_2 i_3} \equiv -4\partial^k \pi_{k i_1 i_2 i_3} - \partial_{[i_1} P_{i_2 i_3]} \approx 0, \tag{53}
$$

$$\tilde{\chi}^{(2)j_1j_2j_4} \equiv -\partial_l A^{lj_1j_2j_3} - \frac{1}{3}\partial^{j_1} B^{j_2j_3]}, \tag{54}$$

$$\overset{(1)}{\tilde{\chi}}_{i_1} \equiv -\partial^k P_{ki_1} + \partial_{i_1} p, \tag{55}$$

$$\tilde{\chi}^{(2)j_1} \equiv -2\partial_l B^{lj_1} + \partial^{j_1}\varphi. \tag{56}$$

Now, we construct the Dirac bracket with respect to the irreducible second-class constraints (53)-(56). In order to construct the elements of the matrix $C^{\Delta'\Delta''}$, we choose $\hat{e}_{\beta_1}^{\alpha_1}$ and $\bar{A}_{\beta_1}^{\beta_2}$ like

$$\hat{e}_{\beta_1}^{\alpha_1} = \begin{pmatrix} -\frac{1}{4\Delta}\delta_{k_1}^{i_1}\delta_{k_2]}^{i_2} & 0 \\ 0 & -\frac{1}{4\Delta}\delta_{k_1}^{i_1}\delta_{k_2]}^{i_2} \end{pmatrix}, \tag{57}$$

$$\bar{A}_{\beta_1}^{\beta_2} = \begin{pmatrix} \frac{1}{\Delta}\delta_{i_1}^{k_1}\partial_{i_2]} & 0 \\ 0 & \frac{1}{2\Delta}\delta_{l_1}^{[j_1}\partial^{j_2]} \end{pmatrix}. \tag{58}$$

The matrix $\bar{D}_{\beta_3}^{\sigma_3}$ reads as

$$\bar{D}_{\beta_3}^{\sigma_3} = \begin{pmatrix} \frac{1}{\Delta} & 0 \\ 0 & \frac{1}{\Delta} \end{pmatrix}. \tag{59}$$

If we compute the Dirac bracket among the original field/momenta on behalf of (47), we reobtain the same fundamental non-vanishing Dirac brackets like in the reducible situation, namely, (28).

6. Conclusion

In this paper we have presented some equivalent approaches for the problem of the derivation of the Dirac bracket for a system with third-order reducible second-class constraints. Our strategy includes three main steps. First, we construct the Dirac bracket in terms of a noninvertible matrix $M^{\alpha_0\beta_0}$. Second, we derive the Dirac bracket based on an invertible matrix $\mu^{\alpha_0\beta_0}$. Third, we substitute the original second-class constraints by some equivalent irreducible ones in an enlarged phase-space and the Dirac bracket in this case is equivalent with those in the above mentioned approaches. The general procedure was exemplified on gauge-fixed 4-forms.

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