

## THE INFLUENCE OF TIME DEPENDENT ELECTRIC AND MAGNETIC FIELDS ON THE DYNAMIC LOCALIZATION OF LATTICE ELECTRONS

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### Abstract

Applying the method of characteristics leads to wavefunctions and dynamic localization conditions for electrons on the one dimensional lattice under perpendicular time dependent electric and magnetic fields. This results in selected conditions concerning the number of magnetic flux quanta times  $\pi$ , as well as the quotients between the Bloch frequency and the ones characterizing competing fields. Such conditions proceed, in general, in terms of sums of two-factor products of Bessel functions of the first kind. Besides pure field limits and superpositions between uniform electric and time dependent magnetic fields, parity and periodicity effects have also been discussed.

**Keywords:** Mesoscopic and nanoscale systems; Superlattices; Localization effects.

### 1. Introduction

The dynamic localization (DL) of electrons moving on the one dimensional (1D) lattice under the influence of a longitudinal time dependent (TD) electric field like  $E(t) = E_0 f(t)$  has received much interest since its discovery some 20 years ago [1-8]. This effect concerns the periodic return of the electron to the initially occupied site [1]. Accordingly, the mean square displacement (MSD) should remain bounded in time. In most cases which are of interest in practice the modulation function is periodic with period  $T$  but superpositions of several ac-fields can also be considered. The DL referred to above should then occur under selected conditions concerning the number of magnetic flux quanta, as well as the quotients between the field frequencies and the Bloch frequency  $\omega_b = E_0 ea / \hbar$ , where  $a$  stands for the lattice spacing. Besides applications in several areas like high field and nonlinear effects [2,3], trapping in two level atoms [4], persistent THz emission [5,6], the generation of higher harmonics [7] or the absolute negative conductance, the DL has been finally observed in the linear optical absorption coefficient of quantum dot superlattices [9]. It has also been found that the collapse of quasienergy bands is able to reflect the occurrence of DL [10-13]. Recent developments such as the DL of wave packets in barriers with TD parameters [14], the appearance and disappearance of resonant peaks in  $I - V$  characteristics [15], or the influence

of higher order neighbors [16], are worthy of being mentioned, too. We have to realize that the influence of the uniform magnetic field on lattice electrons, such as exhibited by the celebrated Harper-equation [17-19], looks quite interesting in several respects. The same concerns several superpositions between electric and magnetic fields which have been studied during course [20-22]. However, a systematic study of wavefunctions and of DL effects produced by superpositions for which both electric and magnetic fields are TD seems of having been, to the best of our knowledge, overlooked. We shall then use this opportunity to discuss DL effects provided by the longitudinal TD electric field  $\vec{E}(t) = (E(t), 0, 0)$  working in conjunction with a transversal magnetic field like  $\vec{B}(t) = (0, 0, B(t))$ , where  $B(t) = B_0 g(t)$ . To this aim we can start either by incorporating the electric field into the time dependent Harper-Hamiltonian [18], or from the decoupled limit of two parallel chains in electric and magnetic fields [21]. Such systems can be converted one into another with the help of gauge transformations [23]. The former alternative is appropriate for Hall conductance studies. We shall then choose the latter alternative since it complies in a more suitable manner with the DL problem. This opens the way to the derivation of general DL conditions, but concrete realizations such as superpositions between uniform electric and time dependent magnetic fields, will also be discussed. Last but not at least we shall deal with parity and periodicity effects.

## 2. The derivation of the wavefunction

The Hamiltonian describing the electron on the 1D lattice under TD electric and magnetic fields specified above is given by [1,18,21]

$$H = \varepsilon_0 \sum_m |m\rangle \langle m| - eaE_0 f(t) \sum_m m |m\rangle \langle m| + \\ + V \sum_m \exp(-i\gamma/2) |m+1\rangle \langle m| + V \sum_m \sum_m \exp(i\gamma/2) |m\rangle \langle m+1| \quad (1)$$

where  $m$  is an integer ranging from  $-\infty$  to  $\infty$  One has

$$\gamma = \gamma(t) = 2\pi\Phi / \Phi_0 = 2\pi\beta_0 g(t) \quad (2)$$

$\Phi = B_0 a^2 g(t)$  and  $\Phi_0 = hc/e$ , respectively. So, there is  $\beta_0$ , which stands for the magnetic commensurability parameter. The constant on-site energy is denoted by  $\varepsilon_0 = \hbar\omega_0$ , whereas  $V = \hbar U$  is responsible for the nearest neighbor hopping parameter. One deals, of course, with an orthonormalized Wannier-basis for which  $\langle m | m' \rangle = \delta_{m,m'}$ . We then have to account, as usual, for the TD single particle amplitude via

$$\langle \Psi(t) | = \sum_m C_m(t) \exp(-i\omega_0 t) |m\rangle \quad (3)$$

which leads in turn to the TD second-order discrete Schrödinger equation

$$i \frac{\partial}{\partial t} C_m(t) = U \exp(-i\gamma/2) C_{m-1}(t) + U \exp(i\gamma/2) C_{m+1}(t) - m\omega_B f(t) C_m(t) \quad (4)$$

The next step is to apply the discrete Fourier-transform

$$C_m(t) = \frac{1}{2\pi} \int_0^{2\pi} dk \exp(ik) \tilde{C}_k(t) \quad (5)$$

where  $k$  denotes the dimensionless wave number. Then (4) becomes

$$\left[ \frac{\partial}{\partial t} + 2iU \cos\left(k + \frac{\gamma}{2}\right) + \omega_B \tilde{f}(t) \frac{\partial}{\partial k} \right] \tilde{C}_k(t) = 0 \quad (6)$$

$$\text{where} \quad \tilde{f}(t) = f(t) - \frac{1}{2\omega_B} \frac{d\gamma(t)}{dt} = f(t) - \frac{\pi\beta_0}{\omega_B} \frac{dg(t)}{dt} \quad (7)$$

Now we have to remember that (4) and (6) have been discussed before when  $\gamma = 0$  by resorting to the method of characteristics [1]. What then remains is to generalize these latter results towards incorporating  $\gamma(t)$  such as given by (2). This results in the solution

$$\tilde{C}(t) = \exp\left[-2iU \int_0^t dt' \cos \Omega_k(t, t')\right] \quad (8)$$

as it can be easily verified by direct computation. This time one has

$$\Omega_k(t, t') = \cos\left[k + \frac{\gamma}{2} - \omega_B (\tilde{\eta}(t) - \tilde{\eta}(t'))\right] \quad (9)$$

$$\text{where} \quad \tilde{\eta}(t) = \int_0^t dt' \tilde{f}(t') = \eta(t) - \frac{1}{2\omega_B} (\gamma(t) - \gamma(0)) \quad (10)$$

$$\text{and} \quad \eta(t) = \int_0^t dt' f(t') \quad (11)$$

which is well known in the description of electric field problems. Accordingly, one gets faced with the normalized wavefunction

$$C_m(t) = \exp(-im\tilde{\varphi}) J_m\left(2U |\tilde{Z}(t)|\right) \quad (12)$$

by virtue of the well known properties of Bessel functions [24], where

$$\tilde{\varphi} = \tilde{\varphi}(t) = \frac{\pi + \gamma(t)}{2} - \arg \tilde{Z}(t) \quad (13)$$

$$\text{and } \tilde{Z}(t) = \int_0^t dt' \exp(-i\omega_B \tilde{\eta}(t')) \quad (14)$$

which deserve further attention.

### 3. Dynamic localization effects

Using (12) produces the MSD

$$\langle m^2 \rangle = \sum_m m^2 |C_m(t)|^2 = 2U^2 |\tilde{Z}(t)|^2 \quad (15)$$

which generalizes apparently (2.7) in [1] in terms of the substitution  $\eta(t) \rightarrow \tilde{\eta}(t)$ . Choosing as an example usual modulations like

$$f(t) = \cos(\omega_1 t) \quad (16)$$

$$\text{and } g(t) = \sin(\omega_2 t) \quad (17)$$

yields the characteristic function

$$\tilde{Z}(t) = \int_0^t dt' \exp\left(-i \frac{\omega_B}{\omega_1} \sin(\omega_1 t') + i\pi\beta_0 \sin(\omega_2 t')\right) \quad (18)$$

which can be rewritten equivalently as

$$\tilde{Z} = \sum_m \sum_n \exp(i\Omega_{m,n} t) \frac{\sin(\Omega_{m,n} t)}{\Omega_{m,n}} J_n\left(\frac{\omega_B}{\omega_1}\right) J_m(\pi\beta_0) \quad (19)$$

by virtue of expansions characterizing generating functions of Bessel functions [24], where

$$\Omega_{m,n} = \frac{1}{2}(m\omega_2 - n\omega_1). \quad (20)$$

$$\text{The point is to decompose } \tilde{Z}(t) \text{ as } \tilde{Z}(t) = Q_1 t + Q_2(t) \quad (21)$$

in which  $Q_2(t)$  oscillates with time. Having discriminated the linear term in  $t$  then produces

$$\text{the DL condition [1,13]} \quad Q_1 = 0$$

(22)

in which case the MSD remains bounded in time. On the other hand (19) shows that the discrimination of the linear term one looks for proceeds in terms of selected  $m$  and  $n$  values for which  $\Omega_{m,n} \rightarrow 0$ . To this aim let us assume that the frequencies  $\omega_1$  and  $\omega_2$  are commensurate. This amounts to deal with selected quotients like

$$\frac{n}{m} = \frac{\omega_2}{\omega_1} = \frac{P}{Q} \quad (23)$$

in which  $P$  and  $Q$  are mutually prime integers. We then have to realize that the DL condition characterizing specifically the present superposition of TD electric and magnetic fields is given in terms of sums of two-factor products of Bessel functions like

$$Q_1 = Q_1 \left( \frac{\omega_B}{\omega_1}, \pi\beta_0 \right) = J_0 \left( \frac{\omega_B}{\omega_1} \right) J_0 (\pi\beta_0) + \sum_{l=1}^{\infty} p_l J_{Pl} \left( \frac{\omega_B}{\omega_1} \right) J_{Ql} (\pi\beta_0) = 0, \quad (24)$$

in which

$$p_l = 1 + (-1)^{l(P+Q)} \quad (25)$$

and  $P = Q\omega_2 / \omega_1$ . Using, however,  $\tilde{g}(t) = \cos(\omega_2 t)$  instead of (17) produces the DL condition

$$\bar{Q}_1 \left( \frac{\omega_B}{\omega_1}, \pi\beta_0 \right) = \left[ J_0 \left( \frac{\omega_B}{\omega_1} \right) J_0 (\pi\beta_0) + \sum_{l=1}^{\infty} p_l \exp \left( i \frac{\pi}{2} Ql \right) [1 + \exp(i\pi l P)] J_{Pl} \left( \frac{\omega_B}{\omega_1} \right) J_{Ql} (\pi\beta_0) \right] = 0 \quad (26)$$

which proceeds up to a phase factor like  $\exp(-i\pi\beta_0)$ . Such results indicate that contributions provided by electric and magnetic fields can be placed on the same footing.

One remarks that both (24) and (26) are sensitive to the parity of  $P + Q$ . So (24) becomes

$$Q_1 = Q_1^{(-)} \left( \frac{\omega_B}{\omega_1}, \pi\beta_0 \right) = J_0 \left( \frac{\omega_B}{\omega_1} \right) J_0 (\pi\beta_0) + 2 \sum_{l=1}^{\infty} J_{2Pl} \left( \frac{\omega_B}{\omega_1} \right) J_{2Ql} (\pi\beta_0) = 0, \quad (27)$$

when  $P + Q$  is an odd integer, but

$$Q_1 = Q_1^{(+)} \left( \frac{\omega_B}{\omega_1}, \pi\beta_0 \right) = J_0 \left( \frac{\omega_B}{\omega_1} \right) J_0 (\pi\beta_0) + 2 \sum_{l=1}^{\infty} J_{Pl} \left( \frac{\omega_B}{\omega_1} \right) J_{Ql} (\pi\beta_0) = 0, \quad (28)$$

if  $P + Q$  is even. Such conditions are rather complex, so that when dealing with applications we have to resort to numerical studies.

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