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# DIMENSIONAL ANALYSIS OF FREE DROP MOTION

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**Abstract**
Dimensional analysis and similarity theory have been applied widely to physico-chemical hydrodynamics and related subjects, especially to heat and mass transfer theory. In this paper we study the movements of a drop, initially at rest. Because the drop is motionless, we introduced a viscous velocity U and a new dimensional number Ch. This approach permits us to evaluate the deformation and breakup of a drop, under surfactants adsorption, in dimensionless form.
**Keywords:** drop deformation and breakup, surfactants adsorption, dimensionless number.

## 1. Introduction.

Generally**,** the boundary between two phases of matter is known as the interfacial zone or, the interface. It is that thin layer surrounding a geometric surface of separation, in which the physical properties differ much from those in either of the bulk phases. The thickness of this layer is ill-defined because the variations of physical properties across it are continuous. We shall consider it as infinitely thin; i.e., we regard it as a geometric surface. Since the thickness of interface is of the order of molecular dimensions, such an approximation is justified in treating the macroscopic movements of liquids. If the surface tension, $\sigma$, of the liquid interface changes from point to point, a tangential force will be exerted in addition to the pressure normal to the surface and its magnitude is determined by the surface tension gradient [1]. Surface active compounds (surfactants), present in even small quantities, have an important role in determining the hydrodynamic behavior of a two phase system. There are many examples where the presence of surfactant, have an important role. Probably the best known is the effect of surfactants on a liquid drop, immersed in a bulk liquid, initially at rest [2, 3].

The aim of this paper is to give a dimensionless description of the movements of a drop, initially at rest, introducing a viscous velocity, noted U. This approach permits to study the movements of a drop, initially at rest, using a dimensionless analysis. Further, we introduce a new dimensionless number Ch and calculate the Marangoni forces acting on the drop.

## 2. Method and samples

We shall consider a viscous liquid drop L' (density ρ') immersed in an immiscible bulk liquid L, (density ρ). If the two liquids have the same density $\rho' = \rho$, the drop is called free and is motionless or at zero gravity. The two liquids inside and outside the drop (see Fig.1) are considered Newtonians, incompressible and viscous having the viscosities μ and μ'. The surface between the two liquids is characterized by an interfacial tension, noted $\sigma_0$. On more physical and chemical aspects of the problem see our previous works [4, 5].

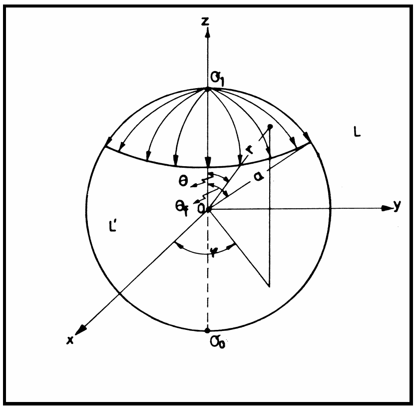


Fig. 1. The spreading of a surfactant on a free drop surface.

A small quantity of a surfactant (e.g. a droplet of $10^{-3}$-$10^{-2}$ cm$^3$, which is very small compared with the volume of the initial drop) is introduced in a point (called injection point) on the drop surface. The surfactant, because of its molecular structure, is spread and simultaneously adsorbed at the liquid-liquid interface and it is continuously swept along the meridians of the drop, by the convective transport. In the injection point the interfacial tension is instantaneously lowered to $\sigma_1$ ($\sigma_1 < \sigma_0$) value. Since the interfacial tension is a function of the surfactant concentration, a gradient of interfacial tension is established over the surface of the drop. Consequently, the Marangoni spreading of the surfactant takes place from low surface tension to high surface tension.

The equations, governing the flow considered quasisteadly (even steady in L') and axisymmetric [6], are the continuity equations for an incompressible fluid:

$$\nabla \cdot \vec{v} = 0\,, \tag{1}$$

$$\nabla \cdot \vec{v}' = 0\,, \tag{2}$$

where $\vec{v}$ and $\vec{v}$' are the velocities of the bulk liquid L and of the liquid L' within the drop, respectively, and the Navier- Stokes equations, for a steady flow, are given by:

$$(\vec{v} \cdot \nabla)\, \vec{v} = -\frac{1}{\rho} \nabla\, p + \frac{\mu}{\rho}\, \Delta \vec{v}\ , \tag{3}$$

$$(\vec{v}'\cdot\nabla)\,\vec{v}' = -\frac{1}{\rho}\nabla\, p' + \frac{\mu'}{\rho}\,\Delta\vec{v}', \tag{4}$$

where p, p' are the pressures, outside and inside the drop. The equation of the interfacial flow [7, 8] is given by:

$$\Gamma\left(\vec{w}\cdot\nabla_S\right)\vec{w} = \vec{F} + \nabla_S\sigma + (\kappa+\varepsilon)\nabla_S\left(\nabla_S\cdot\vec{w}\right), \tag{5}$$

where $\vec{w} = \vec{v}_s$ is the interface velocity, $\vec{F} = \Gamma\vec{g} + \vec{T} - \vec{T}'$ is the external force acting on the drop surface, $\vec{T}$ and $\vec{T}'$ are the tractions exerted by the outer and inner liquid on the drop interface, $\Gamma$ is the surface density, $\kappa$ and $\varepsilon$ are the surface dilatational and shear viscosity respectively, and $\nabla_S$ is the surface gradient operator. Because the surface density is very small ($\Gamma \approx 10^{-7}$ g cm$^{-2}$) the inertial term in Eq. (5) can be neglected against the remainder terms. In order to find the distributions of the velocities $\vec{v}, \vec{v}'$ and of the pressures p, p', the system of Eqs (1)-(5) must be solved taking into account some appropriate boundary conditions [1, 4, 6].

The surfactant front position, in this radial flow, is noted by the angle $\theta_f$. The interfacial tension, $\sigma$, is considered a unique function of the angle $\theta$. Within the surfactant invaded region, ($0 \le \theta \le \theta_f$), for the variation of the interfacial tension with $\theta$, we take [4]

$$\sigma(\theta) = \frac{\sigma_0 - \sigma_1}{1-\cos\theta_f}\,(1-\cos\theta) + \sigma_1, \tag{6}$$

where $\sigma_0 = \sigma(\theta_f)$ and $\sigma_1 = \sigma(0)$.

Eqs. (1-5) with the appropriate boundary conditions lead to the distribution of the velocities $\vec{v}$ and of the pressures p,

$$v_r = \frac{(\sigma_0 - \sigma_1)\,a^3}{3(\mu+\mu'+2\kappa/3a)(1-\cos\theta_f)}\left(\frac{1}{r^3} - \frac{1}{a^2 r}\right)\cos\theta, \tag{7a}$$

$$v_\theta = \frac{(\sigma_0 - \sigma_1)a^3}{3(\mu+\mu'+2\kappa/3a)(1-\cos\theta_f)}\left(\frac{1}{2r^3} + \frac{1}{2a^2 r}\right)\sin\theta, \tag{7b}$$

$$p(r,\theta) = -\frac{\mu(\sigma_0 - \sigma_1)\,a}{3r^2(\mu+\mu'+2\kappa/3a)(1-\cos\theta_f)}\cos\theta, \tag{7c}$$

for the outside flow [ 9,10].

For the dimensional analysis, the equations that describe the phenomena must be expressed in dimensionless form and therefore all dimensional variables that appear in hydrodynamics equations must be expressed in terms of factors characteristic of these variables. In our case the radius (a) of the drop is a characteristic length. All linear dimensions can therefore be dimensionless ratios, of the form $\overline{r} = \frac{r}{a}$. In the same manner, the initial interface tension $\sigma_0$ may be considered as the characteristic dimension of the interfacial tension $\sigma$.

Because the drop is initially at rest, we don't possess a characteristic velocity U, so that we shall introduce one, expressed with the aid of some characteristic data of our problem

$$U = \frac{\mu}{\rho a}, \tag{8}$$

which permits to consider the Reynolds number for the outer flow equal to unity, Re = 1. This U velocity is called viscous velocity. For the flow inside the drop we'll obtain for the Reynolds number $Re' = \mu/\mu'$. With the values of the viscosities taken from [3, 4], the Reynolds number corresponding to the drop phase ranges between 1/80 and 1/40, and consequently the Reynolds number is less than unity $Re' < 1$. Introducing the ratio of the viscosities [11], $\lambda = \mu'/\mu$, we have $Re' = 1/\lambda$.

With these scale references we have the following dimensionless quantities

$$\overline{r} = \frac{r}{a}, \quad \overline{v} = \frac{v}{U}, \quad \overline{p} = \frac{p}{\rho U^2}, \quad \overline{\sigma} = \frac{\sigma}{\sigma_0}, \quad \overline{F} = \frac{F}{\rho a U^2}, \tag{9}$$

where $\overline{v}$ is the dimensionless velocity, $\overline{p}$ the dimensionless pressure, $\overline{\sigma}$ the dimensionless interface tension and $\overline{F}$ the dimensionless force. With these dimensionless parameters we obtain for the velocity in the radial direction, $v_r$, angular direction, $v_\theta$, and pressure p in dimensionless form, for the outside flow:

$$\overline{v}_r = \frac{Ch\,(1-\beta)}{3(1+\lambda)(1-\cos\theta_f)}\left(\frac{1}{\overline{r}^3} - \frac{1}{\overline{r}}\right)\cos\theta, \tag{10a}$$

$$\overline{v}_\theta = \frac{Ch\,(1-\beta)}{3(1+\lambda)(1-\cos\theta_f)}\left(\frac{1}{2\overline{r}^3} + \frac{1}{2\overline{r}}\right)\sin\theta, \tag{10b}$$

$$\overline{p} = -\frac{Ch\,(1-\beta)}{3(1+\lambda)(1-\cos\theta_f)}\frac{\cos\theta}{\overline{r}^2}. \tag{10c}$$

where $\beta$ is the interfacial tensions ratio $\beta = \sigma_1/\sigma_0$. Here, we have introduced Ch, a new dimensionless number given by:

$$Ch = \frac{\sigma_0 \rho a}{\mu^2}. \tag{11}$$

Further, we calculate the Marangoni forces exerted on the free drop, due to Marangoni flow. The surface flow give rise to a stream of liquid directed to the drop along the Oz axis.

This stream arises as a consequence of the continual replacement of that liquid layer which has been displayed by the surface flow, like a ventilation effect [3]. We immediately find that as the flow occurs with the driving by viscosity of the outer liquid L, forces of hydrodynamic pressure will act on the drop L'. The resultant of the forces exerted by the fluid on the drop $F_M$, due to the symmetry of the Marangoni flow, is oriented along the Oz axis and may be calculated from the general expression of force [6] by integration on the covered drop surface:

$$F_M = \iint_S [(p_{rr})_{r=a}\cos\theta - (p_{r\theta})_{r=a}\sin\theta]\,ds \ , \tag{12}$$

where S is the surface covered with surfactant, ds is the surface element, and $p_{rr}$ , $p_{r\theta}$ are the normal and tangential components, of the viscous stress tensor [6]. We have, after integration, the Marangoni force in dimensionless form:

$$\overline{F}_M(\theta_f) = \frac{2\pi\,Ch\,(1-\beta)}{3(1+\lambda)}(1-2\cos\theta_f - 2\cos^2\theta_f)\,. \tag{13}$$

Eq. (13) represents the dimensionless Marangoni force acting on the drop surface along the Oz axis. It can be seen that this force depends on the $\theta_f$ angle, namely, on the extent to which the drop surface is covered by the surfactant, the interfacial tension ratio, β, the ratio of the viscosities, λ, and the dimensionless number Ch. From Eq. (13), it is observed that the force acting on the drop depends direct proportionally on the number Ch. The action of the Marangoni force $\overline{F}_M$ on the drop, like a hammer, is maximum at the injection point of the surfactant, for which $\theta_f = 0$, and it is given by:

$$\overline{F}_M(0) = \frac{2\pi Ch(1-\beta)}{1+\lambda} \quad , \tag{14}$$

where for physical meanings, we have taken the absolute value of the Marangoni force. It depends on Ch number, viscosities ratio, λ, and surface tension ratio, β.

## 4. Conclusion

A dimensionless hydrodynamic model has been developed for a liquid drop immersed in bulk liquid, initially at rest, known as a free drop problem. Because, there is not a reference velocity we have introduced a viscous velocity which permits to calculate the Marangoni forces in the dimensionless form. We have introduced also a new dimensionless number Ch in good relation with the characteristic numbers of capillary hydrodynamics. The motion of the free drop is caused by the addition of a surface active compound, which will generate a Marangoni real flow, and a Marangoni force will appear $\overline{F}_M(\theta_f)$, which is responsible for the drop deformations (hammer effect) and for the drop translation movements (lifting effect), [10, 12].

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