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CONSISTENT INTERACTIONS BETWEEN A WEYL GRAVITON AND A MASSLESS Q-GRAVITINO

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Abstract
Consistent interactions in four space-time dimensions between a Weyl graviton and a massless Q-gravitino are investigated in the framework of BRST cohomology.

1. Introduction

There are many theoretical indications that a final theory which would unify the gravity with all the other fundamental interactions should be supersymmetrical. Supersymmetry unifies in a non- trivial manner space-time with internal symmetries by adding new, fermionic symmetries. The main goal of our work is the investigation of the uniqueness of the simple conformal SUGRA in four space-time dimensions using the deformation theory. In $D=4$ the uniqueness of the simple SUGRA was already proved in the framework of the BRST formalism[1].

It is well known that the field spectrum of $D=4$, $N=1$ conformal SUGRA consists in a massless spin-2, a nonmassive spin-$3/2$ and an irreducible spin-one fields. In the free limit the action of simple conformal SUGRA in $D=4$ reduces to the sum between Weyl, massless Q-gravitino, and the standard abelian gauge field actions

$$S_0^{\mathrm{L}}[h_{\mu\nu},\psi_\mu,A_\mu]=S_0^{\mathrm{W}}[h_{\mu\nu}]+S_0^{3/2}[\psi_\mu]+S_0^{\mathrm{1F}}[A_\mu]. \tag{1}$$

In order to determine the consistent interactions that can be added to action (1) we must study, beside the self-interactions, which are known from the literature, also the cross-couplings. The latter problem can be solved in two steps: firstly, we determine the interaction vertices containing only two of the three types of fields, and then the vertices including all the three kinds.

In this talk we present one of the ingredients mentioned in the above, namely the problem of constructing consistent interactions among the Weyl graviton (described at the Lagrangian level by the linearized Weyl gravity action) and the massless spin-$3/2$ (described in the free limit by an action with three space-time derivatives) fields. We investigate these cross-couplings in the framework of the deformation theory [2] based on local BRST cohomology [3] and show that, if one requires (reasonables) assumptions like locality, Lorentz-Poincare invariance, and preservation of the number of derivatives on each field, all the cross-interactions between the Weyl graviton and Q-gravitino are forbidden. One really needs to introduce more fields in the theory in order to get consistent interactions between the Weyl graviton and its "supersymmetric partner", namely Q-gravitino.

2.Free model

We start with a "free" lagrangian action written as the sum between the linearized Weyl gravity and massless Q-gravitino actions in four space-time dimensions

$$\begin{aligned} S_0^{\mathrm{L}}\left[h_{\mu\nu},\psi_\mu\right] &= \int d^4x\left(\frac{1}{2}W_{\mu\nu\alpha\beta}W^{\mu\nu\alpha\beta}-4i\phi_\mu\gamma^{\mu\nu\rho}\partial_\nu\phi_\rho\right)\equiv \\ &\equiv \int d^4x\left(L_0^{(\mathrm{W})}+L_0^{(3/2)}\right). \end{aligned} \tag{2}$$

where we used the notations

$$\begin{aligned} W_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta}-\frac{1}{2}\left(\sigma_{\mu\alpha}R_{\beta]\nu}-\sigma_{\nu\alpha}R_{\beta]\mu}\right)+\frac{1}{6}R\sigma_{\mu\alpha}\sigma_{\beta]\nu}, \\ R_{\mu\nu\alpha\beta} &= \frac{1}{2}\left(\partial_\mu\partial_\beta h_{\nu\alpha}+\partial_\nu\partial_\alpha h_{\mu\beta}-\partial_\nu\partial_\beta h_{\mu\alpha}-\partial_\mu\partial_\alpha h_{\nu\beta}\right), \\ R_{\mu\nu} &= \sigma^{\alpha\beta}R_{\mu\alpha\nu\beta}, \qquad R=\sigma^{\mu\nu}R_{\mu\nu}, \\ W_{\mu\nu\alpha\beta} &= R_{\mu\nu\alpha\beta}-\left(\sigma_{\mu\alpha}\mathrm{K}_{\beta]\nu}-\sigma_{\nu\alpha}\mathrm{K}_{\beta]\mu}\right), \\ \mathrm{K}_{\mu\nu} &= \frac{1}{2}\left(R_{\mu\nu}-\frac{1}{6}\sigma_{\mu\nu}R\right), \qquad \mathrm{K}=\frac{1}{6}R, \\ \mathrm{C}_{\alpha\beta\mu} &= \partial_\mu\mathrm{K}_{\beta]\alpha}, \qquad \partial W^{\alpha}_{\alpha\beta\mu\nu}=-\mathrm{C}_{\beta\mu\nu}, \\ \phi_\mu &= \frac{i}{3}\gamma^\nu\partial_\nu\psi_{\mu]}+\frac{i}{3!}\gamma_{\mu\nu\rho}\partial^\nu\psi^\rho. \end{aligned} \tag{3)(4)(5)(6)(7)(8)(9}$$

In the above, $W_{\mu\nu\alpha\beta}$, $R_{\mu\nu\alpha\beta}$ and $\mathrm{C}_{\alpha\beta\mu}$ reprezent the (linearized) Weyl, Riemann and respectively Cotton tensors, while ϕ_μ and ψ_μ are real (fermionic) Majorana spinors. We employ the flat metric of 'mostly' minus signature and work with the γ-matrices in the Majorana representation. The theory (2) is invariant under the gauge transformations

$$\begin{aligned} \delta_\varepsilon h_{\mu\nu} &= \partial_{(\mu}\varepsilon_{\nu)}+2\sigma_{\mu\nu}\varepsilon, \\ \delta_\varepsilon\psi^\mu &= \partial^\mu\varepsilon+i\gamma^\mu\zeta. \end{aligned} \tag{10)(11}$$

The gauge parameters ε_μ and ε are bosonic while ξ, ζ are Majorana spinors. The gauge algebra of the free theory (2) is Abelian.

We observe that there are no non-vanishing local transformations of ε_μ, ε, ξ and ζ that annihilate $\delta_\varepsilon h_{\mu\nu}$ and $\delta_\varepsilon \psi_\mu$. This remark allows us to conclude that the generating set of gauge transformations (10), (11) is irreducible.

3.Construction of consistent interactions

Due to the fact that the solution to the master equation contains all the information on the gauge structure of a given theory, we can reformulate the problem of introducing consistent interactions as a deformation problem of the solution to the master equation corresponding to the " free" theory. If an interacting gauge theory can be consistently constructed, then the solution S to the master equation associated with the " free" theory can be deformed into a solution $\bar{S}$

$$\begin{aligned} S \rightarrow \bar{S} &= S + \lambda S_1 + \lambda^2 S_2 + \cdots \\ &= S + \lambda \int \mathrm{d}^D x a + \lambda^2 \int \mathrm{d}^D x b + \cdots, \end{aligned} \tag{12}$$

of the master equation for the deformed theory

$$\left(\bar{S}, \bar{S}\right) = 0, \tag{13}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The equation (13) splits, according to the various orders in λ, into

$$\begin{aligned} (S,S) &= 0, \\ 2(S_1,S) &= 0, \\ 2(S_2,S)+(S_1,S_1) &= 0, \\ (S_3,S)+(S_1,S_2) &= 0, \\ &\vdots \end{aligned} \qquad (14)(15)(16)(17)$$

The equation (14) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, S_1, is a co-cycle of the " free" BRST differential. However, only cohomologically non-trivial solutions to (15) should be taken into account, as the BRST-exact ones (BRST co-boundaries) correspond to trivial interactions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the " free" theory. It has been shown (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations (16--17), etc.). However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques.

For our free model the solution to the master equation reads as

$$
\begin{aligned}
\bar{S} \;=\;& S_0^{\mathrm{L}}\left[h_{\mu\nu},\psi_{\mu}\right]+ \\
&+\int d^4x\left[h^{*\mu\nu}\left(\partial_{(\mu}\eta_{\nu)}+2\sigma_{\mu\nu}\xi\right)+\psi^{*\mu}\left(\partial_{\mu}\chi+i\gamma_{\mu}\theta\right)\right]
\end{aligned}
\tag{18}
$$

In (18) $h^{*\mu\nu}$ and $\psi^{*\mu}$ are the antifields corresponding to Weyl graviton field $h_{\mu\nu}$ an Q-gravitino field ψ^{μ} while η_{μ}, ξ, χ and θ stand for the ghosts associated with the gauge parameters ε_{μ}, ε, ε and ζ. Consequently, $\psi^{*\mu}$, χ and θ are also Majorana spinors. We work in the sequel under the assumptions of:

- space-time locality,
- smoothness of the deformations in the coupling constant,
- (background) Lorentz invariance,
- Poincare′ invariance (i.e. we do not allow explicit dependence on the space-time coordinates),
- the maximum number of derivatives in the interacting Lagrangian is four,
- the derivative order of the equations of motion on each field is the same for the free and respectively for the interacting theory.

If some of these requirements are relaxed, various interactions may appear.

4.Main results

By direct computation we obtain the solution to the equation (15), the first-order deformation of the solution to the master equation for the free theory

$$
\begin{aligned}
S_1 \;=\;& \int \mathrm{d}^4x\{k[4\xi^{*}\bar{\theta}\chi+2i\eta^{*\mu}\bar{\chi}\gamma_{\mu}\chi+ \\
&+\theta^{*}\left(\gamma^{\mu\nu}\theta\partial_{\mu}\eta_{\nu]}-4\theta\xi+4i\gamma^{\mu}\chi\partial_{\mu}\xi\right) \\
&+\chi^{*}\left(-8i\gamma^{\mu}\theta\eta_{\mu}+\gamma^{\mu\nu}\chi\partial_{\mu}\eta_{\nu]}+\chi\xi\right)+ \\
&+\psi^{*\rho}\left(-8i\gamma^{\nu}\phi_{\rho}\eta_{\nu}+4i\gamma^{\nu}\theta h_{\rho\nu}+\gamma^{\mu\nu}\psi_{\rho}\partial_{\mu}\eta_{\nu]}-\right. \\
&\left.-2\gamma^{\mu\nu}\chi\partial_{\mu}h_{\nu\rho}+4\psi_{\rho}\xi\right)-8ih^{*\mu\nu}\bar{\chi}\gamma_{\mu}\psi_{\nu}+ \\
&+16iW^{\mu\alpha\nu\beta}\partial_{\alpha}\bar{\psi}_{\nu}\gamma_{\mu}\psi_{\beta}+16W^{\mu\alpha\nu\beta}\bar{\phi}_{\nu}\gamma_{\mu\beta}\psi_{\alpha}- \\
&-16i\bar{F}^{\mu\rho}\gamma^{\nu}\phi_{\rho}h_{\mu\nu}+8\bar{F}^{\rho\lambda}\gamma^{\mu\nu}\psi_{\lambda}\partial_{\mu}h_{\nu\rho}- \\
&-8\bar{\mathbf{F}}_{\lambda\rho}\gamma_{\mu\nu}\phi^{\lambda}\partial^{\mu}h^{\nu\rho}-16\bar{\psi}^{\lambda}\gamma_{\mu}\mathbf{F}_{\lambda\nu}K^{\mu\nu}]\},
\end{aligned}
\tag{19}
$$

where k is a complex constant and the objects $\eta^{*\mu}$, ξ^{*}, χ^{*} and θ^{*} stand for the antifields associated respectively to the ghosts η_{μ}, ξ, χ and θ. Thus, χ^{*} and θ^{*} are

Majorana spinors. The terms $\mathbf{F}_{\mu\nu}$ and $F_{\mu\nu}$ are expressed by

$$\mathbf{F}_{\mu\nu}=\partial_{[\mu}\psi_{\nu]}+\mathrm{i}\gamma_{[\mu}\phi_{\nu]},\quad F_{\mu\nu}=\partial_{[\mu}\phi_{\nu]}. \tag{20}$$

The second order deformation is governed by the equation (16). If we denote by Λ and b the nonintegrated densities of the functionals $\frac{1}{2}(S_1,S_1)$ and respectively S_2, the local form of (16) becomes

$$\Lambda=-sb+\partial_\mu n^\mu, \tag{21}$$

with

$$\mathrm{gh}(\Delta)=1,\quad \mathrm{gh}(b)=0,\quad \mathrm{gh}(n^\mu)=1, \tag{22}$$

for some local currents n^μ. Developing Λ and b with respect to the antighost number and projecting the equation (21) on various antighost numbers we obtain the equivalent tower of equations

$$\begin{aligned}\Lambda_2 &= -\gamma b_2+\partial_\mu n_2^\mu,\\ \Lambda_I &= -(\delta b_{I+1}+\gamma b_I)+\partial_\mu n_I^\mu,\quad I=\overline{0,1}.\end{aligned} \tag{23)(24}$$

By direct computation we obtain that

$$\Lambda_2=\gamma(d)+e, \tag{25}$$

where the terms d and e are expressed by

$$\begin{aligned}d &= k^2\big[16i\theta^*\gamma^\mu\chi\bar{\theta}\psi_\mu-16i\theta^*\gamma^\mu\chi\bar{\chi}\phi_\mu+\\ &\quad +8i\theta^*\gamma^{\mu\nu}\theta\bar{\chi}\gamma_\nu\psi_\mu+8i\chi^*\gamma^{\mu\nu}\chi\bar{\chi}\gamma_\nu\psi_\mu\big],\end{aligned} \tag{26}$$

$$\begin{aligned}e &= k^2\Big[\left(-8\theta^*\gamma^\mu\theta-8\chi^*\gamma^\mu\chi\right)\bar{\chi}\gamma_\mu\theta+\\ &\quad +\left(-8\theta^*\gamma^{\mu\nu}\theta-8\chi^*\gamma^{\mu\nu}\chi\right)\bar{\chi}\gamma_{\mu\nu}\theta+\\ &\quad \left(\frac{4}{3}\theta^*\gamma^{\mu\nu\rho}\theta-\frac{4}{3}\chi^*\gamma^{\mu\nu\rho}\chi\right)\bar{\chi}\gamma_{\mu\nu\rho}\theta+\\ &\quad \left(\frac{2}{3}\theta^*\gamma^{\mu\nu\rho\lambda}\theta-\frac{2}{3}\chi^*\gamma^{\mu\nu\rho\lambda}\chi\right)\bar{\chi}\gamma_{\mu\nu\rho\lambda}\theta\Big].\end{aligned} \tag{27}$$

Due to the fact that the antifields θ^*, χ^* and the ghosts θ, χ, pertain to the cohomology of the derivative along the gauge orbits, we observe that e given in (27) is a nontrivial object from $H(\gamma)$. On the other hand, the equation (23) requires e to be a γ-trivial object. Thus, it has to be zero. This implies that

$$k=0 \tag{28}$$

and moreover,

$$S_2=S_1=0. \tag{29}$$

Inserting the solutions (29) in the next equation (17), one can easily notice that this one reduces to the previous equation (15). Thus, using the same procedure as the one employed in the above, we get $S_3=0$. Therefore, all the deformation terms that can be added to the solution to the master equation for the free theory S, can be made to vanish

$$S_1 = S_2 = \cdots = 0. \tag{30}$$

5. Conclusions

To conclude with, if one imposes usual requirements as space-time locality, Lorentz-Poincare invariance, smoothness of the deformations in the coupling constant and the preservation of the number of derivatives on each field and for the interacting vertices, with respect to the free theory, there are no consistent interactions between the Weyl graviton and the Q-gravitino. Still, consistent interactions can be achieved, by adding new fields to the original spectrum. In fact, it is sufficient to add only one field, namely the abelian vector field.

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