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PHOTOELASTIC METHOD TO SOLVE THE TORSION PROBLEM OF TRIANGULAR CROSS SECTION BARS

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Abstract
The stress state inside a shaft with triangular cross section subjected to torsion was determinated by the stress freezing photoelastic method through a wedging process. The results were compared with the ones obtained by the finite element method (FEM).
Keywords: photoelasticity , stress freezing method, finite elements methods.

1. Introduction

Solving the torsion problem for prismatic bars with arbitrary cross sections by means of Saint Venant [2] method consists in finding the stress function $\Phi(z,y)$ which needs to:

1. satisfy the differential type Poisson equation: $$\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\Phi}{\partial y^2}=-2G\theta \qquad (1)$$

   where G is the shear modulus, θ - the torsional strain,
2. be constant on the cross-section boundary.

It is quite difficult to find the stress function, and for this reason one uses approximate solutions. There are however experimental methods allowing to establish the stress function. In this paper the wedging process has been used for models made of optically active materials that are subjected in advance to torsion by means of the stress freezing procedure. It can be shown [1] that the isocromatics (Figure 1) inside the studied model, when viewed into a circular polariscope (Figure 2), are the level lines of the surface described by $x=\Phi(z,y)$, which allow us to find the shear stresses with the equtations: $$\tau_{xz}=\frac{\partial\Phi}{\partial y}; \quad \tau_{yx}=-\frac{\partial\Phi}{\partial z} \qquad (2)$$

2. Experimental method

The geometric dimension of the model is $l = 25.4\ mm$, figure 3, and the experimental model was subjected to a twisting moment $M_t = 233.5\ N\cdot mm$. The isochromatics inside the 45^0 sectional model in dark and light fiedls are represented in figure 4 a) respectively b). The stress function value $\Phi(z,y)$ corresponding to the k order isochromatic is:

$$\Phi(z, y) = \frac{k}{2} \cdot S_0 \quad (3)$$

where $S_0 = 0.27\,N/mm$ is the material photoelastic constant.

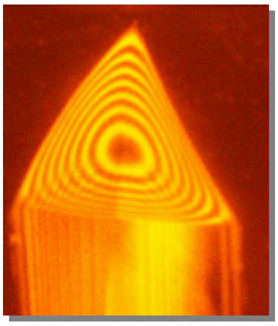


Figure 1

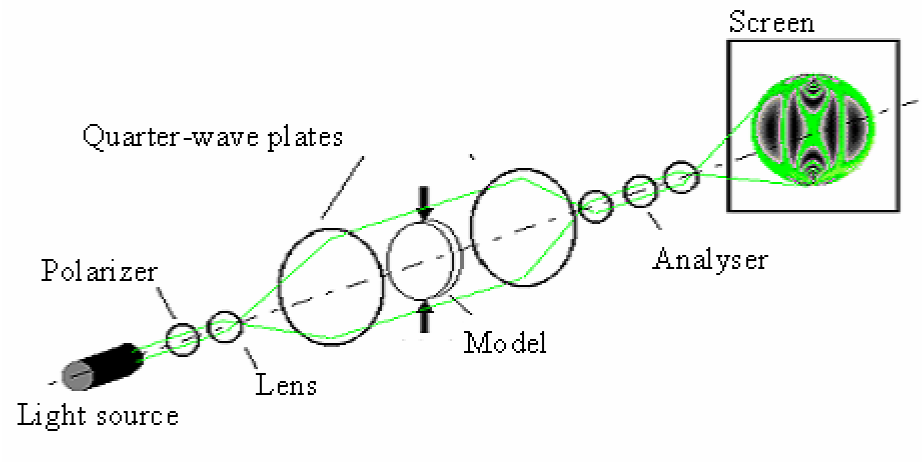


Figure 2

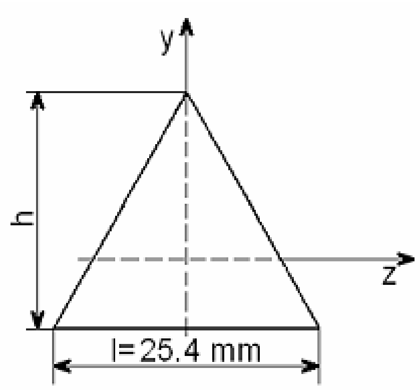


Figure 3

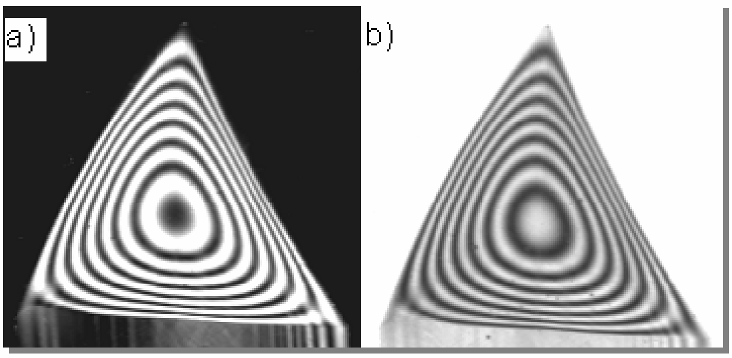
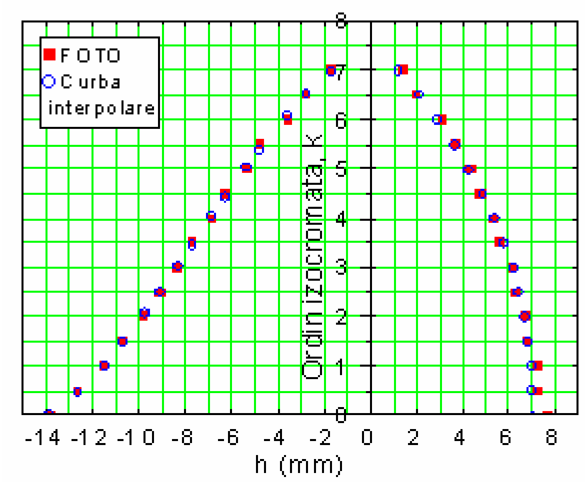


Figure 4



1. *Figure 5*

The shear stresses τ_{zx} along the y axis have been determined. In order to do so, the stress function values have been drawn along the y axes only for experimental values, as indicated in figures 5, where the origin of the reference system is set in the centroid of the section. To calculate the stresses with the relations (2), the values were approximated with interpolation curves as:

$$k(y) = b \cdot \left(1 - \frac{y^{m1}}{a^{m1}}\right)^{\frac{m2}{2}} \quad (4)$$

where the values of the parameters are $a = 7.61$, $b = 7$, $m1 = 2.2$, $m2 = 2$ for one third of the height of the triangle and equation (5) for the rest of the height of the triangle.

$$k(y) = 7.509 + 0.299 \cdot x - 0.052 \cdot x^2 - 1.19 \cdot 10^{-3} \cdot x^{-3} + 6.688 \cdot 10^{-5} \cdot x^4 \quad (5)$$

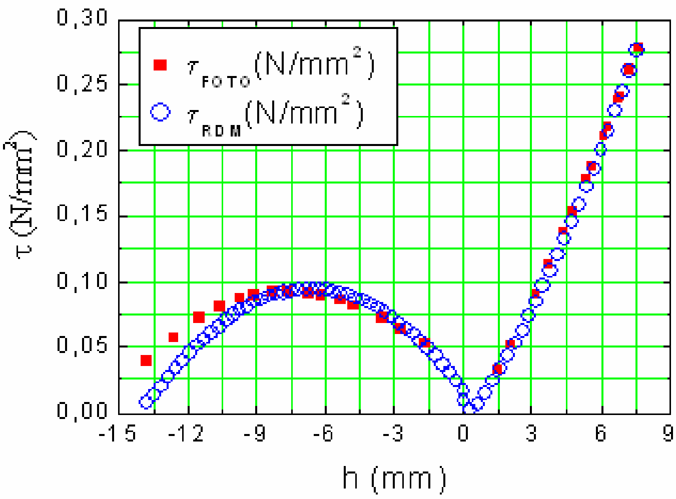
The shear stresses τ_{zx} that have been calculated by the photoelastic method with the help of:

$$\tau_{zx} = \frac{S_0}{2} \cdot \frac{\Delta k}{\Delta y} \quad (6)$$

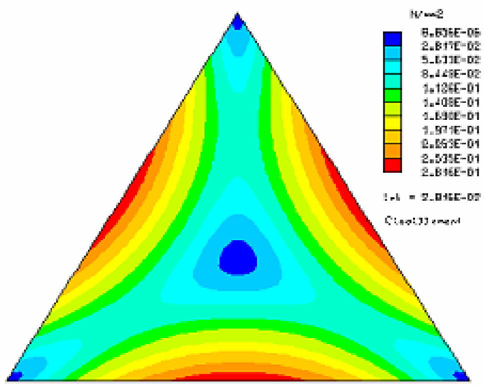
are represented in figure 6. To check the validity of these results, the stresses have also been evaluated by means of the RDM finite elements software and are plotted comparatively on the same figures. The shear stresse inside a triangular cross section shaft calculated by FEM means (with RDM software) are presented in a full field representation in figure 7.

3. Results and Discussions

It is important to know what are the shear stresses inside a torsioned bar. And if the analitic solution is not set, the is important to find a way to determineit. Using photoelasticimetry is one of the best solution, becouse:



2. Figure 6



3. Figure 7

- You can use it on geometrical identic pieces with the real ones
- You can correlate the photoelasticimetry shear stresses with the real ones
- Is an noninvasive methode for the real model.

As you can see in the figure 6 one obtain good correlation between experimental and FEM simulation and the deviations presented in the figure 8.

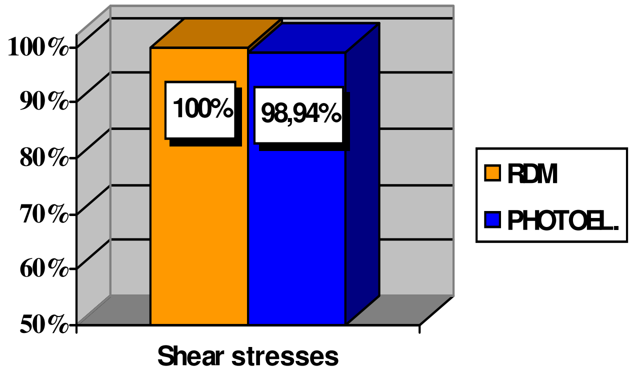


Figure 8.

4. Conclusions

The method which has been used to determine the stress state proved to be both efficient and accurate, the relative errors between experimental and finite element predictions being less than *1.06 %* .

Acknowledgements

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