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THE DERIVATION OF THE DIRAC BRACKET FOR GAUGE-FIXED THREE- AND TWO-FORMS WITH STUECKELBERG COUPLING

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Abstract
The Dirac bracket for gauge-fixed three- and two-forms with Stueckelberg coupling is derived initially along an reducible manner and subsequently we folowing an alternative irreducible treatment, we obtain the same results.

1. Introduction

The canonical approach of the systems with reducible second-class constraints represents a difficult problem. This is because not all the second-class constraint functions are independent, hence the matrix of the Poisson brackets among them is not invertible.

In order to construct the Dirac bracket for such systems in a consistent manner we have the following options:

• to isolate a set of independent constraint functions and then build the Dirac bracket in terms of this smaller set;

• to construct the Dirac bracket in terms of a noninvertible matrix without separating the independent constraint functions;

• to substitute the reducible second-class constraints by some equivalent irreducible ones [by appropiately enlarging the original phase-space] and further work with the Dirac bracket based on the irreducible constraints.

2. Second-stage reducible second-class constraints

Our starting point is a system with the phase-space locally parametrized by N canonical pairs $z^a = \left(q^i, p_i\right)$ and subject to the second-stage reducible second-class constraints

$$\chi_{\alpha_0}(z^a) \approx 0, \alpha_0 = \overline{1, M_0},$$

$$Z_{\alpha_1}^{\alpha_0} \chi_{\alpha_0} = 0, \alpha_1 = \overline{1, M_1},$$

$$Z_{\alpha_2}^{\alpha_1} Z_{\alpha_1}^{\alpha_0} = 0, \alpha_1 = \overline{1, M_2}. \qquad (1)(2)(3)$$

These constraints are purely second-class if any maximal, independent set of $M_0 - M_1 + M_2$ constraint functions χ_A, $A = \overline{1, M_0 - M_1 + M_2}$, among the χ_{α_0} is such that the matrix

$$C_{AB} = [\chi_A, \chi_B], \qquad (4)$$

is invertible. In terms of such a set of independent constraints, the Dirac bracket takes the form

$$[F, G]^* = [F, G] - [F, \chi_A] M^{AB} [\chi_B, G], \qquad (5)$$

where $M^{AB} C_{BC} \approx \delta_C^A$.

We can construct the Dirac bracket even without performing such a separation. We denote the matrix of the Poisson brackets among the second-class constraint functions by

$$C_{\alpha_0 \beta_0} = [\chi_{\alpha_0}, \chi_{\beta_0}]. \qquad (6)$$

It is easy to see, on behalf of (2), that the matrix $C_{\alpha_0 \beta_0}$ is not invertible $Z_{\alpha_1}^{\alpha_0} C_{\alpha_0 \beta_0} \approx 0.$ (7)

If $A_{\alpha_0}^{\alpha_1}$ stand for some functions that satisfying

$$\operatorname{rank}\left(Z_{\alpha_1}^{\alpha_0} A_{\alpha_0}^{\beta_1} \right) \equiv \operatorname{rank}\left(D_{\alpha_1}^{\beta_1} \right) = M_1 - M_2, \qquad (8)$$

then we can introduce another matrix $M^{\alpha_0 \beta_0}$ through the relation $M^{\alpha_0 \beta_0} C_{\beta_0 \gamma_0} \approx D_{\gamma_0}^{\alpha_0}$, (9)

with $M^{\alpha_0 \beta_0} = -M^{\beta_0 \alpha_0}$, such that the bracket

$$[F, G]^* = [F, G] - [F, \chi_{\alpha_0}] M^{\alpha_0 \beta_0} [\chi_{\beta_0}, G], \qquad (10)$$

defines the same Dirac bracket like (5) on the surface (1), where

$$D_{\beta_0}^{\alpha_0} = \delta_{\beta_0}^{\alpha_0} - Z_{\beta_1}^{\alpha_0} A_{\beta_0}^{\beta_1}. \qquad (11)$$

2. The model

We consider the canonical approach to gauge-fixed three- and two-forms with Stueckelerg coupling, described by the Lagrangian action

$$S_0^L\left[A_{\mu\nu\rho},H_{\mu\nu}\right] \;=\; -\int d^D x\left(\frac{1}{48}F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda}+\right.$$
$$\left.+\frac{1}{12}\left(F_{\mu\nu\rho}-MA_{\mu\nu\rho}\right)\left(F^{\mu\nu\rho}-MA^{\mu\nu\rho}\right)\right), \tag{12}$$

where
$$F_{\mu\nu\rho\lambda}=\partial_{\mu}A_{\nu\rho\lambda]},F_{\mu\nu\rho}=\partial_{\mu}H_{\nu\rho]}, \tag{13}$$

and $D\geq 4$. Everywhere in this presentation the notation $[\mu\ldots\nu]$ signifies complete antisymmetry with respect to the indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. The canonical analysis of this model leads to the first-class constraints

$$\begin{aligned} G_{i_1}^{(1)} &\equiv \Pi_{0i_1}\approx 0, \qquad G_{i_1 i_2}^{(1)}\equiv \pi_{0i_1 i_2}\approx 0,\\ \chi_{i_1}^{(1)} &\equiv 2\partial^k\Pi_{ki_1}\approx 0,\\ \chi_{i_1 i_2}^{(1)} &\equiv -3\partial^k\pi_{ki_1 i_2}+M\Pi_{i_1 i_2}\approx 0,\end{aligned} \tag{14)(15)(16}$$

where the momenta $\Pi_{\mu\nu}$ and $\pi_{\mu\nu\rho}$ are respectively conjugated to $H^{\mu\nu}$ and $A^{\mu\nu\rho}$. In order to fix the gauge, we have to choose a set of canonical gauge conditions. An appropriate set of such gauge conditions is given by

$$\begin{aligned} G^{(2)i_1} &\equiv H^{0i_1}\approx 0, \qquad G^{(2)i_1 i_2}\equiv A^{0i_1 i_2}\approx 0,\\ \chi^{(2)i_1} &\equiv -2\partial_k H^{ki_1}\approx 0,\\ \chi^{(2)i_1 i_2} &\equiv -\partial_k A^{ki_1 i_2}-MH^{i_1 i_2}\approx 0.\end{aligned} \tag{17)(18)(19}$$

The constraints (14)-(19) are second-class and, moreover, second-stage reducible. It is simple to see that (14) and (17) generate a submatrix (of the matrix of the Poisson brackets among the constraint functions) of maximum rank, therefore they form a subset of ireducible second-class constraints, so they are not relevant in view of our appoach. Thus in the sequel we examine only the constraints (15)--(16) and (18)--(19), which we organize as

$$\chi_{\alpha_0}\equiv\begin{pmatrix}\chi_{i_1}^{(1)}\\ \chi_{i_1 i_2}^{(1)}\\ \chi^{(2)j_1}\\ \chi^{(2)j_1 j_2}\end{pmatrix}\approx 0. \tag{20}$$

The second-class constraint functions from (20) are second-stage reducible, with the first-, respectively, second-stage reducibility functions given by

$$Z^{\alpha_0}_{\alpha_1} = \begin{pmatrix} -\partial^{i_1} & 0 & 0 & 0 \\ -M\delta^{i_1}_{k_1} & -\delta^{[i_1}_{k_1}\partial^{i_2]} & 0 & 0 \\ 0 & 0 & \frac{1}{2}\partial_{j_1} & 0 \\ 0 & 0 & -\frac{M}{2}\delta^{l_1}_{j_1} & -\frac{1}{2}\delta^{l_1}_{[j_1}\partial_{j_2]} \end{pmatrix}, \tag{21}$$

$$Z^{\alpha_1}_{\alpha_2} = \begin{pmatrix} -M & \partial^{k_1} & 0 & 0 \\ 0 & 0 & M & \partial_{l_1} \end{pmatrix}. \tag{22}$$

The matrix of the Poisson brackets among the constraints (20) is expressed by

$$C_{\alpha_0\beta_0} = \begin{pmatrix} 0 & 0 & -2\Delta D^{j_1}_{i_1} & -M\delta^{[j_1}_{i_1}\partial^{j_2]} \\ 0 & 0 & M\delta^{j_1}_{[i_1}\partial_{i_2]} & \left(\Delta+M^2\right)\overline{D}^{j_1 j_2}_{i_1 i_2} \\ 2\Delta D^{i_1}_{j_1} & M\delta^{i_1}_{[j_1}\partial_{j_2]} & 0 & 0 \\ -M\delta^{[i_1}_{j_1}\partial^{i_2]} & -\left(\Delta+M^2\right)\overline{D}^{i_1 i_2}_{j_1 j_2} & 0 & 0 \end{pmatrix}, \tag{23}$$

where $D^{j_1}_{i_1} = \delta^{j_1}_{i_1} - \frac{\partial_{i_1}\partial^{j_1}}{\Delta}$, $\overline{D}^{j_1 j_2}_{i_1 i_2} = \frac{1}{2}\left(\delta^{j_1}_{[i_1}\delta^{j_2}_{i_2]} - \frac{\delta^{k_1}_{[i_1}\partial_{i_2]}\delta^{[j_1}_{k_1}\partial^{j_2]}}{\Delta+M^2}\right)$, (24)

and $\Delta = \partial^k\partial_k$.

3."Reducible" Dirac bracket

Now, we construct the Dirac bracket with respect to the constraints (20). In order to construct the matrix $D^{\alpha_0}_{\beta_0}$ defined in (11), we take $A^{\beta_1}_{\beta_0}$ of the form

$$A^{\beta_1}_{\beta_0} = \begin{pmatrix} \frac{1}{\Delta+M^2}\partial_{k_1} & 0 & 0 & 0 \\ \frac{-M}{\Delta+M^2}\delta^{i_1}_{k_1} & \frac{1}{2\left(\Delta+M^2\right)}\delta^{i_1}_{[k_1}\partial_{k_2]} & 0 & 0 \\ 0 & 0 & \frac{-2}{\Delta+M^2}\partial^{l_1} & 0 \\ 0 & 0 & \frac{-2M}{\Delta+M^2}\delta^{l_1}_{j_1} & \frac{1}{\Delta+M^2}\delta^{[l_1}_{j_1}\partial^{l_2]} \end{pmatrix}. \tag{25}$$

Then, by means of (11) we find

$$D^{\alpha_0}_{\beta_0}=\begin{pmatrix} \frac{\Delta}{\Delta+M^2}D^{i_1}_{k_1} & \frac{M}{2\left(\Delta+M^2\right)}\delta^{i_1}_{[k_1}\partial_{k_2]} & 0 & 0 \\ \frac{M}{\Delta+M^2}\delta^{[i_1}_{k_1}\partial^{i_2]} & \bar{D}^{i_1 i_2}_{k_1 k_2} & 0 & 0 \\ 0 & 0 & \frac{\Delta}{\Delta+M^2}D^{l_1}_{j_1} & \frac{M}{2\left(\Delta+M^2\right)}\delta^{[l_1}_{j_1}\partial^{l_2]} \\ 0 & 0 & \frac{M}{\Delta+M^2}\delta^{l_1}_{[j_1}\partial_{j_2]} & \bar{D}^{l_1 l_2}_{j_1 j_2} \end{pmatrix}. \quad (26)$$

Using (23) and (26) it follows that the relation (9) is fulfilled for

$$M^{\alpha_0\beta_0}=\begin{pmatrix} 0 & 0 & \frac{\Delta}{2\left(\Delta+M^2\right)^2}D^{i_1}_{k_1} & \frac{-M}{2\left(\Delta+M^2\right)^2}\delta^{i_1}_{[k_1}\partial_{k_2]} \\ 0 & 0 & \frac{M}{2\left(\Delta+M^2\right)^2}\delta^{[i_1}_{k_1}\partial^{i_2]} & \frac{-1}{\Delta+M^2}\bar{D}^{i_1 i_2}_{k_1 k_2} \\ \frac{-\Delta}{2\left(\Delta+M^2\right)^2}D^{l_1}_{j_1} & \frac{M}{2\left(\Delta+M^2\right)^2}\delta^{[j_1}_{i_1}\partial^{j_2]} & 0 & 0 \\ \frac{-M}{2\left(\Delta+M^2\right)^2}\delta^{l_1}_{[j_1}\partial_{j_2]} & \frac{1}{\Delta+M^2}\bar{D}^{l_1 l_2}_{j_1 j_2} & 0 & 0 \end{pmatrix}.$$

(27)

With $M^{\alpha_0\beta_0}$ at the hand, we can construct the Dirac bracket by means of formula (10). After some computation, we find that the only non-vanishing fundamental Dirac brackets are given by

$$\left[A^{i_1 i_2 i_3}(x),\pi_{j_1 j_2 j_3}(y)\right]^*_{x^0=y^0}=\bar{D}^{i_1 i_2 i_3}_{j_1 j_2 j_3}\delta^{D-1}(\vec{x}-\vec{y}),$$

$$\left[H^{i_1 i_2}(x),\Pi_{j_1 j_2}(y)\right]^*_{x^0=y^0}=\frac{\Delta}{\Delta+M^2}D^{i_1 i_2}_{j_1 j_2}\delta^{D-1}(\vec{x}-\vec{y}),$$

$$\left[A^{i_1 i_2 i_3}(x),\Pi_{j_1 j_2}(y)\right]^*_{x^0=y^0}=\frac{-M}{2\left(\Delta+M^2\right)}\delta^{[i_1}_{j_1}\delta^{i_2}_{j_2}\partial^{i_3]}\delta^{D-1}(\vec{x}-\vec{y}),$$

$$\left[H^{i_1 i_2}(x),\pi_{j_1 j_2 j_3}(y)\right]^*_{x^0=y^0}=\frac{-M}{3!\left(\Delta+M^2\right)}\delta^{i_1}_{[j_1}\delta^{i_2}_{j_2}\partial_{j_3]}\delta^{D-1}(\vec{x}-\vec{y}),$$

(28)(29)(30)(31)

where we use the notations

$$D^{i_1 i_2}_{j_1 j_2} = \frac{1}{2}\left(\delta^{i_1}_{[j_1}\delta^{i_2}_{j_2]} - \frac{\delta^{[i_1}_{k_1}\partial^{i_2]}\delta^{k_1}_{[j_1}\partial_{j_2]}}{\Delta}\right),$$

$$\overline{D}^{i_1 i_2 i_3}_{j_1 j_2 j_3} = \frac{1}{3!}\left(\delta^{i_1}_{[j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3]} - \frac{\delta^{[i_1}_{k_1}\delta^{i_2}_{k_2}\partial^{i_3]}\delta^{k_1}_{[j_1}\delta^{k_2}_{j_2}\partial_{j_3]}}{2\left(\Delta + M^2\right)}\right). \quad (32)(33)$$

In this way, the (reducible) Dirac analysis of this model is complete.

4.Irreducible analysis

In this section we reobtain the non-vanishing fundamental Dirac brackets (28)--(31) in an ireducible manner.

4.1.Original phase-space approach

Initially, we investigate the problem of the construction the Dirac bracket for our model in the original phase-space in terms of an invertible matrix. In this sense, we remark that $D^{\alpha_0}_{\beta_0}$ given in (11) has two nice properties, that can be checked by direct computation. First, it is a projector

$$D^{\alpha_0}_{\beta_0} D^{\beta_0}_{\gamma_0} = D^{\alpha_0}_{\gamma_0}, \quad (34)$$

and second, it satisfies the relations $D^{\alpha_0}_{\beta_0}\chi_{\alpha_0} = \chi_{\beta_0}$. (35)

It can be proved that the following theorem holds for systems with second-stage reducible second-class constraints .

Theorem 1 *There exists an invertible, antisymmetric matrix* $\mu^{\gamma_0\delta_0}$ *such that*

$$M^{\alpha_0\beta_0} = D^{\alpha_0}_{\gamma_0}\mu^{\gamma_0\delta_0}D^{\beta_0}_{\delta_0}. \quad (36)$$

In the case of our model, where the matrices $D^{\alpha_0}_{\beta_0}$ and $M^{\alpha_0\beta_0}$ are expressed by (26) and respectively (27), the matrix $\mu^{\gamma_0\delta_0}$ takes the form

$$\mu^{\gamma_0\delta_0}=\begin{pmatrix} 0 & 0 & \frac{1}{2(\Delta+M^2)}\delta^{i_1}_{k_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{2(\Delta+M^2)}\delta^{i_1}_{k_1}\delta^{i_2}_{k_2]} \\ \frac{-1}{2(\Delta+M^2)}\delta^{l_1}_{j_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2(\Delta+M^2)}\delta^{[l_1}_{j_1}\delta^{l_2]}_{j_2} & 0 & 0 \end{pmatrix}. \tag{37}$$

Replacing (36) in (10) and using (35) we obtain that the Dirac bracket takes the final form

$$[F,G]^*=[F,G]-[F,\chi_{\alpha_0}]\mu^{\alpha_0\beta_0}[\chi_{\beta_0},G]. \tag{38}$$

By computing the fundamental Dirac brackets with the help of (38), we reobtain precisely (28)--(31).

4.2.Extended phase-space approach

In the sequel we construct some equivalent irreducible second-class constraints associated with (1) such that the Dirac bracket constructed with respect to the irreducible set coincides with the Dirac bracket corresponding to the reducible second-class model. Firstly, we introduce some new variables $(y_{\alpha_1})_{\alpha_1=\overline{1,M_1}}$ with the Poisson brackets

$$[y_{\alpha_1},y_{\beta_1}]=\omega_{\alpha_1\beta_1}, \tag{39}$$

and consider the system subject to the reducible second-class constraints

$$\chi_{\alpha_0}\approx 0,\qquad y_{\alpha_1}\approx 0. \tag{40}$$

The Dirac bracket corresponding to the above second-class constraints on the phase-space locally parametrized by (z^a,y_{α_1}) reads as

$$[F,G]^*\big|_{z,y}=[F,G]-[F,\chi_{\alpha_0}]\mu^{\alpha_0\beta_0}[\chi_{\beta_0},G]-[F,y_{\alpha_1}]\omega^{\alpha_1\beta_1}[y_{\beta_1},G], \tag{41}$$

where the Poisson brackets from the right-hand side of (41) contain derivatives with respect to all z^a and y_{α_1}. After some computation we infer that $[F,G]^*\big|_{z,y}\approx[F,G]^*$, where $[F,G]^*$ is given by (38). Under these considerations, the following theorem can be proved to hold.

Theorem 2 *There exists a set of irreducible second-class constraints*

$$\tilde{\chi}_{\alpha_0} \equiv \chi_{\alpha_0} + a_{\alpha_0}^{\alpha_1} y_{\alpha_1} \approx 0,$$
$$\tilde{\chi}_{\alpha_2} \equiv Z_{\alpha_2}^{\alpha_1} y_{\alpha_1} \approx 0, \tag{42)(43}$$

such that:

• (42)--(43) is equivalent with (40) [this means that both sets describe the same surface on the larger phase-space];

• the Dirac bracket with respect to the irreducible second-class constraints is given by

$$\begin{aligned}\left[F,G\right]^*\Big|_{irred} &= \left[F,G\right] - [F,\tilde{\chi}_{\alpha_0}]\mu^{\alpha_0\beta_0}[\tilde{\chi}_{\beta_0},G] - [F,\tilde{\chi}_{\alpha_0}]Z_{\alpha_1}^{\alpha_0}\omega^{\alpha_1\gamma_1}A_{\gamma_1}^{\gamma_2}\bar{D}_{\gamma_2}^{\beta_2}[\tilde{\chi}_{\beta_2},G] \\ &- [F,\tilde{\chi}_{\alpha_2}]\bar{D}_{\beta_2}^{\alpha_2}A_{\gamma_1}^{\beta_2}\omega^{\gamma_1\beta_1}Z_{\beta_1}^{\beta_0}[\tilde{\chi}_{\beta_0},G] - [F,\tilde{\chi}_{\alpha_2}]\bar{D}_{\gamma_2}^{\alpha_2}A_{\gamma_1}^{\gamma_2}\omega^{\gamma_1\lambda_1}A_{\lambda_1}^{\lambda_2}\bar{D}_{\lambda_2}^{\beta_2}[\tilde{\chi}_{\beta_2},G].\end{aligned} \tag{44}$$

and coincides with (41) $\left[F,G\right]^*\Big|_{irred} = \left[F,G\right]^*\Big|_{z,y}$ (45)

on the surface (40).

In the formula (44) $a_{\alpha_0}^{\alpha_1}$ is a matrix that fulfills $rank\left(a_{\alpha_0}^{\alpha_1} Z_{\beta_1}^{\alpha_0}\right) = M_1 - M_2$ and $\bar{D}_{\beta_2}^{\alpha_2}$ is the inverse of $D_{\alpha_2}^{\beta_2} = Z_{\alpha_2}^{\alpha_1} A_{\alpha_1}^{\beta_2}$ and $A_{\alpha_1}^{\beta_2}$ are some function taken such that $rank\left(D_{\alpha_2}^{\beta_2}\right) = M_2$. The existence of such functions is guaranteed by the fact that the second-class constraints are by assumption second-stage reducible.

In order to construct the irreducible second-class constraints for our model we take the matrix $a_{\alpha_0}^{\alpha_1}$ of the form

$$a_{\beta_0}^{\beta_1} = \begin{pmatrix} \partial_{k_1} & 0 & 0 & 0 \\ -M\delta_{k_1}^{i_1} & \frac{1}{2}\delta_{k_1}^{i_1}\partial_{k_2]} & 0 & 0 \\ 0 & 0 & -2\partial^{l_1} & 0 \\ 0 & 0 & -2M\delta_{j_1}^{l_1} & \delta_{j_1}^{[l_1}\partial^{l_2]} \end{pmatrix}, \tag{46}$$

we introduce the new variables y_{α_1} as $y_{\alpha_1} = \begin{pmatrix} p \\ P_{i_1} \\ \varphi \\ B^{j_1} \end{pmatrix},$ (47)

and take
$$\omega_{\alpha_1\beta_1} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -\delta_{i_1}^{j_1} \\ 1 & 0 & 0 & 0 \\ 0 & \delta_{j_1}^{i_1} & 0 & 0 \end{pmatrix}. \tag{48}$$

Then, the equivalent ireducible second-class constraints are expressed by

$$\begin{aligned} &\overset{(1)}{\tilde{\chi}}_{i_1} \equiv 2\partial^k \Pi_{ki_1} - \partial_{i_1} p - MP_{i_1} \approx 0, \\ &\overset{(1)}{\tilde{\chi}}_{i_1 i_2} \equiv -3\partial^k \pi_{ki_1 i_2} + M\Pi_{i_1 i_2} + \frac{1}{2}\partial_{i_1} P_{i_2]} \approx 0, \\ &\overset{(2)j_1}{\tilde{\chi}} \equiv -2\partial_l H^{lj_1} + 2\partial^{j_1}\varphi - 2MB^{j_1}, \\ &\overset{(2)j_1 j_2}{\tilde{\chi}} \equiv -\partial_l A^{lj_1 j_2} - MH^{j_1 j_2} + \partial^{j_1} B^{j_2]}, \end{aligned} \tag{49)(50)(51)(52}$$

$$\begin{aligned} \overset{(1)}{\tilde{\chi}} &\equiv -Mp + \partial^k P_k, \\ \overset{(2)}{\tilde{\chi}} &\equiv M\varphi + \partial_l B^l. \end{aligned} \tag{53)(54}$$

Now, we construct the Dirac bracket with respect to the irreducible second-class constraints (49)--(54). In order to construct the matrix $\bar{D}_{\beta_2}^{\alpha_2}$, we take $A_{\beta_1}^{\beta_2}$

$$A_{\beta_1}^{\beta_2} = \begin{pmatrix} -M & -\partial_{k_1} & 0 & 0 \\ 0 & 0 & M & -\partial^{l_1} \end{pmatrix}, \tag{55}$$

and obtain for $D_{\alpha_2}^{\beta_2}$ the following form $D_{\alpha_2}^{\beta_2} = \begin{pmatrix} \Delta + M^2 & 0 \\ 0 & \Delta + M^2 \end{pmatrix}$. (56)

Thus, taking into account that $\bar{D}_{\beta_2}^{\alpha_2}$ is the invers of $D_{\alpha_2}^{\beta_2}$, it results that

$$\bar{D}_{\beta_2}^{\alpha_2} = \begin{pmatrix} \frac{1}{\Delta + M^2} & 0 \\ 0 & \frac{1}{\Delta + M^2} \end{pmatrix}. \tag{57}$$

If we compute the Dirac brackets among the original field/momenta on behalf of (44), we reobtain the same fundamental non-vanishing Dirac brackets like in the reducible situation, namely, (28)--(31).

5.Conclusion

In this paper we have presented three equivalent approaches to the problem of constructing the Dirac bracket for gauge-fixed three- and two-forms with Stueckelberg coupling:

• we constructed the Dirac bracket in terms of a noninvertible matrix $M^{\alpha_0\beta_0}$;

• we derived the Dirac bracket based on an invertible matrix $\mu^{\alpha_0\beta_0}$;

• we substituted the original second-class constraints by some equivalent ireducible ones on a larger phase-space and observed that the Dirac bracket is in this case equivalent with the Dirac brackets emerging from the previously mentioned approaches.

In conclusion, for gauge-fixed three- and two-forms with Stueckelberg coupling, the fundamental Dirac brackets with respect to the original variables derived within the three approaches coincide.

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