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INTERACTIONS BETWEEN A MASSLESS TENSOR FIELD WITH THE MIXED SYMMETRY $(2,2)$ AND A TWO-FORM GAUGE FIELD

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Abstract
The consistent interactions between a single, free, massless tensor gauge field with the mixed symmetry of the Riemann tensor and a massless two-form gauge field are investigated in the framework of the BRST formalism combined with cohomological techniques. Under the assumptions on smoothness, locality, Lorentz covariance, and Poincaré invariance of the deformations, supplemented by the requirement that the interacting Lagrangian is at most second-order derivative, it is proved that there are no consistent cross-interactions between the tensor field with the mixed symmetry of the Riemann tensor and the massless two-form gauge field.

1. Outline

Mixed symmetry type tensor fields [1]--[7] appear in various theories, like superstrings, supergravities, or supersymmetric high spin theories. From the analysis of gauge theories with mixed symmetry type tensor fields several results have been formulated, like the dual formulation of field theories of spin two or higher [8]--[15], the impossibility of consistent cross-interactions in the dual formulation of linearized gravity [16], or a Lagrangian first-order approach [17] – [18] to some classes of free massless mixed symmetry type tensor gauge fields, suggestively resembling to the tetrad formalism of General Relativity. A basic aspect related to this type of gauge models is the analysis of their consistent interactions --- among themselves as well as with higher-spin gauge theories [19]--[28]. The most efficient approach to this matter is the cohomological one, based on the deformation of the solution to the master equation [29]. The aim of the present paper is to investigate the manifestly covariant consistent interactions between a single, free, massless tensor gauge field $t_{\mu\nu|\alpha\beta}$, with the mixed symmetry of the Riemann tensor and a massless two-form gauge field.

Our procedure relies on the deformation of the solution to the master equation by means of the local BRST cohomology. We initially determine the associated free antifield-BRST symmetry s, which splits as the sum between the Koszul-Tate differential and the exterior longitudinal derivative only, $s = \delta + \gamma$. Then, we solve the basic equations of the deformation procedure. Under the supplementary assumptions on smoothness, locality, Lorentz covariance, and Poincar e′ invariance of the deformations as well as on the maximum derivative order of the interacting Lagrangian being equal to two, we prove that there are no consistent cross-interactions between the tensor field with the mixed symmetry of the Riemann tensor and the massless two-form gauge field.

2. Free model

We consider a free theory described by the Lagrangian action

$$
\begin{aligned}
& S_0\left[t_{\mu\nu|\alpha\beta}, B_{\mu\nu}\right] \equiv S_0^{\mathrm{t}}\left[t_{\mu\nu|\alpha\beta}\right] + S_0^{\mathrm{B}}\left[B_{\mu\nu}\right] \\
= \; & \int d^D x \left[\frac{1}{8}\left(\partial^\lambda t^{\mu\nu|\alpha\beta}\right)\left(\partial_\lambda t_{\mu\nu|\alpha\beta}\right) - \left(\partial_\mu t^{\mu\nu|\alpha\beta}\right)\left(\partial_\beta t_{\nu\alpha}\right)\right. \\
& -\frac{1}{2}\left(\partial_\mu t^{\mu\nu|\alpha\beta}\right)\left(\partial^\lambda t_{\lambda\nu|\alpha\beta}\right) - \frac{1}{2}\left(\partial^\lambda t^{\nu\beta}\right)\left(\partial_\lambda t_{\nu\beta}\right) + \left(\partial_\nu t^{\nu\beta}\right)\left(\partial^\lambda t_{\lambda\beta}\right) \\
& \left. -\frac{1}{2}\left(\partial_\nu t^{\nu\beta}\right)\left(\partial_\beta t\right) + \frac{1}{8}\left(\partial^\lambda t\right)\left(\partial_\lambda t\right) - \frac{1}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda}\right],
\end{aligned}
\tag{1}
$$

in a Minkowski-flat space-time of dimension $D \geq 5$, endowed with a metric tensor of \`mostly plus' signature $\sigma_{\mu\nu} = \sigma^{\mu\nu} = (-++++\cdots)$.

The massless tensor field $t_{\mu\nu|\alpha\beta}$ has the mixed symmetry of the Riemann tensor and hence transforms according to an irreducible representation of $GL(D, \Re)$, corresponding to a rectangular Young diagram with two columns and two rows. Thus, it has the following properties,

$$
\begin{aligned}
& t_{\mu\nu|\alpha\beta} = -t_{\nu\mu|\alpha\beta}, t_{\mu\nu|\alpha\beta} = -t_{\mu\nu|\beta\alpha}, t_{\mu\nu|\alpha\beta} = t_{\alpha\beta|\mu\nu} \\
& t_{[\mu\nu|\alpha]\beta} \equiv 0 \qquad \text{(BianchiI identity)}
\end{aligned}
\tag{2}
$$

The notation $t_{\nu\beta}$ signifies the simple trace of the original tensor field, $t_{\nu\beta} = \sigma^{\mu\alpha} t_{\mu\nu|\alpha\beta}$, which is symmetric, $t_{\nu\beta} = t_{\beta\nu}$, while t denotes its double trace, $t = \sigma^{\nu\beta} t_{\nu\beta} \equiv t^{\mu\nu}{}_{|\mu\nu}$, which is a scalar. $F_{\mu\nu\lambda}$ is the field strength of the abelian two-form $B_{\mu\nu}$, defined in the standard manner by

$$
F_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}, \quad B_{\mu\nu} = -B_{\nu\mu}. \tag{3}
$$

(The notation $[\mu\cdots\lambda]$ signifies complete antisymmetry with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms.)

A generating set of gauge transformations for our action can be taken of the form

$$\begin{aligned} \delta_\varepsilon t_{\mu\nu|\alpha\beta} &= \partial_\mu \varepsilon_{\alpha\beta|\nu} - \partial_\nu \varepsilon_{\alpha\beta|\mu} + \partial_\alpha \varepsilon_{\mu\nu|\beta} - \partial_\beta \varepsilon_{\mu\nu|\alpha}, \\ \delta_\xi B_{\mu\nu} &= \partial_{[\mu}\xi_{\nu]}, \end{aligned} \qquad (4)(5)$$

where the parameters $\varepsilon_{\alpha\beta|\nu}$ determine a tensor field with the mixed symmetry $(2,1)$,

$$\varepsilon_{\alpha\beta|\nu} = -\varepsilon_{\beta\alpha|\nu}, \quad \varepsilon_{[\alpha\beta|\nu]} \equiv 0. \qquad (6)$$

(The tensor field $\varepsilon_{\mu\nu|\alpha}$ transforms according to an irreducible representation of $GL(D,\Re)$, corresponding to a three-cell Young diagram with two columns and two rows. The last identity is required to ensure the gauge transformations ((4)) check the same Bianchi I identity like the fields, $\delta_\varepsilon t_{[\mu\nu|\alpha]\beta} \equiv 0$.)

The gauge transformations are Abelian and off-shell first stage reducible (it means there are some transformations $\varepsilon = \varepsilon(\theta)$, $\xi = \xi(\theta)$ that make the gauge transformations ((4))--((5)) to vanish identically),

$$\begin{aligned} &\varepsilon_{\mu\nu|\alpha} = 2\partial_\alpha \theta_{\mu\nu} - \partial_{[\mu}\theta_{\nu]\alpha}, \xi_\mu = \partial_\mu \theta \Rightarrow \begin{cases} \delta_{\varepsilon(\theta)} t_{\mu\nu|\alpha\beta} \equiv 0, \\ \delta_{\xi(\theta)} B_{\mu\nu} \equiv 0. \end{cases} \\ &\theta_{\mu\nu} = -\theta_{\nu\mu}. \end{aligned}$$

The object invariant under the gauge transformations of the field tensor $t_{\mu\nu|\alpha\beta}$ and containing the minimum number of field derivatives is called " the curvature tensor" and is given by

$$K_{\lambda\mu\nu|\alpha\beta\gamma} = \partial_{[\lambda} t_{\mu\nu]|[\beta\gamma,\alpha]}, \qquad ,\mu \equiv \frac{\partial}{\partial x^\mu}.$$

It satisfies the identities

$$\begin{aligned} K_{[\lambda\mu\nu|\alpha]\beta\gamma} &\equiv 0 \,(\text{BianchiI}), \\ \partial_{[\rho} K_{\lambda\mu\nu]|\alpha\beta\gamma} &\equiv 0 \,(\text{BianchiII}). \end{aligned} \qquad (7)$$

3. Cohomological computation of the interactions

It is known that at the level of the BRST formalism the entire gauge structure of a theory is completely captured by the BRST differential, s, which is nilpotent, $s^2 = 0$. For the free theory ((1)) s splits into

$$s = \delta + \gamma, \tag{8}$$

where δ represents the Koszul-Tate differential ($\delta^2 = 0$), graded by the antighost number agh ($\mathrm{agh}(\delta) = -1$), and γ stands for the exterior derivative along the gauge orbits, whose degree is named pure ghost number pgh ($\mathrm{pgh}(\gamma) = 1$). These two degrees do not interfere ($\mathrm{agh}(\gamma) = 0$, $\mathrm{pgh}(\delta) = 0$). The overall degree that grades the BRST differential is known as the ghost number (gh) and is defined like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}(s) = \mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = 1$. The generators of the BRST algebra are the fields/ghosts (the fermionic ghosts $\eta_{\alpha\beta|\mu}$ and C_μ associated with the gauge parameters $\varepsilon_{\alpha\beta|\mu}$ and ξ_μ together with the bosonic ghosts for ghosts $C_{\mu\nu}$ and C due to the first-stage reducibility parameters $\theta_{\mu\nu}$ and θ) as well as the antifields ($t^{*\mu\nu|\alpha\beta}$ and $B^{*\mu\nu}$ associated with the original tensor fields, respectively $\eta^{*\mu\nu|\alpha}$, $C^{*\mu}$, $C^{*\mu\nu}$, and C^* corresponding to the ghosts). The statistics of the antifields is opposite to that of the associated fields/ghosts, being understood that the antifields have the same mixed symmetry properties like the corresponding fields/ghosts.

According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued like

$$\begin{aligned}
\mathrm{pgh}(t_{\mu\nu|\alpha\beta}) &= 0 = \mathrm{pgh}(B_{\mu\nu}), \quad \mathrm{pgh}(\eta_{\mu\nu|\alpha}) = 1 = \mathrm{pgh}(C_\mu), \\
\mathrm{pgh}(C_{\mu\nu}) &= 2 = \mathrm{pgh}(C), \quad \mathrm{pgh}(t^{*\mu\nu|\alpha\beta}) = 0 = \mathrm{pgh}(B^{*\mu\nu}), \\
\mathrm{pgh}(\eta^{*\mu\nu|\alpha}) &= \mathrm{pgh}(C^{*\mu}) = \mathrm{pgh}(C^{*\mu\nu}) = \mathrm{pgh}(C^*) = 0, \\
\mathrm{agh}(t_{\mu\nu|\alpha\beta}) &= \mathrm{agh}(B_{\mu\nu}) = \mathrm{agh}(\eta_{\mu\nu|\alpha}) = \mathrm{agh}(\eta_\mu) = 0, \\
\mathrm{agh}(C_{\mu\nu}) &= 0 = \mathrm{agh}(C), \quad \mathrm{agh}(t^{*\mu\nu|\alpha\beta}) = 1 = \mathrm{agh}(B^{*\mu\nu}), \\
\mathrm{agh}(\eta^{*\mu\nu|\alpha}) &= 2 = \mathrm{agh}(C^{*\mu}), \quad \mathrm{agh}(C^{*\mu\nu}) = 3 = \mathrm{agh}(C^*),
\end{aligned} \tag{9)(10)(11)(12}$$

(13)(14)

and the actions of δ and γ on them are given by

$$
\begin{aligned}
\gamma t_{\mu\nu|\alpha\beta} &= \partial_\mu \eta_{\alpha\beta|\nu} - \partial_\nu \eta_{\alpha\beta|\mu} + \partial_\alpha \eta_{\mu\nu|\beta} - \partial_\beta \eta_{\mu\nu|\alpha}, \\
\gamma \eta_{\mu\nu|\alpha} &= 2\partial_\alpha C_{\mu\nu} - \partial_{[\mu} C_{\nu]\alpha}, \quad \gamma C_{\mu\nu} = 0, \\
\gamma t^{*\mu\nu|\alpha\beta} &= \gamma \eta^{*\mu\nu|\alpha} = \gamma C^{*\mu\nu} = 0, \\
\delta t_{\mu\nu|\alpha\beta} &= \delta \eta_{\mu\nu|\alpha} = \delta C_{\mu\nu} = 0, \\
\delta t^{*\mu\nu|\alpha\beta} &= \frac{1}{4} T^{\mu\nu|\alpha\beta}, \quad \delta \eta^{*\alpha\beta|\nu} = -4\partial_\mu t^{*\mu\nu|\alpha\beta}, \quad \delta C^{*\mu\nu} = 3\partial_\alpha \eta^{*\mu\nu|\alpha}, \text{(15)(16)(17)} \\
\gamma B_{\mu\nu} &= \partial_{[\mu} C_{\nu]}, \quad \gamma C_\mu = \partial_\mu C, \quad \gamma C = 0, \\
\gamma B^{*\mu\nu} &= \gamma C^{*\mu} = \gamma C^* = 0, \quad \delta B_{\mu\nu} = \delta \eta_\mu = \delta C = 0, \\
\delta B^{*\mu\nu} &= -\frac{1}{2}\partial_\lambda F^{\mu\nu\lambda}, \quad \delta C^{*\mu} = 2\partial_\nu B^{*\mu\nu}, \quad \delta C^* = \partial_\mu C^{*\mu},
\end{aligned}
$$

(18)(19)(20)(21)(22)

with $1/4T^{\mu\nu|\alpha\beta} = -\delta S_0/\delta t_{\mu\nu|\alpha\beta}$. Both δ and γ are taken to act like right derivations.

It is well-known that the BRST symmetry has a canonical action in a structure named antibracket

$$sF = (F, S). \tag{23}$$

The bosonic functional of ghost number zero denoted by S is the canonical generator of the BRST symmetry and must satisfy the master equation (equivalent with the second order nilpotency of s),

$$(S, S) = 0,$$

which in the case of our free theory reads as

$$
\begin{aligned}
&S = S_0\left[t_{\mu\nu|\alpha\beta}, B_{\mu\nu}\right] + \\
&\int d^D x \left[t^{*\mu\nu|\alpha\beta}\left(\partial_\mu \eta_{\alpha\beta|\nu} - \partial_\nu \eta_{\alpha\beta|\mu} + \partial_\alpha \eta_{\mu\nu|\beta} - \partial_\beta \eta_{\mu\nu|\alpha}\right) + \right. \\
&\left.\eta^{*\mu\nu|\alpha}\left(2\partial_\alpha C_{\mu\nu} - \partial_{[\mu} C_{\nu]\alpha}\right) + B^{*\mu\nu}\partial_{[\mu} C_{\nu]} + C^{*\mu}\partial_\mu C\right]
\end{aligned} \tag{24}
$$

Our aim is to deform (see [29]) the solution S to the master equation of the " free" theory into a solution $\bar{S}$

$$S \to \bar{S} = S + gS_1 + g^2 S_2 + \cdots,$$

of the master equation for an interacting theory,

$$(\bar{S}, \bar{S}) = 0. \tag{25}$$

(If this procedure succeeds, the gauge structure of the interacting theory can be read from the deformed solution $\bar{S}$.) By projecting the equation ((25)) on the various orders of the coupling constant g, this splits into an equivalent chain of equations,

$$\begin{aligned} (S,S) &= 0, \\ 2(S_1,S) &= 0, \\ 2(S_2,S)+(S_1,S_1) &= 0, \\ (S_3,S)+(S_1,S_2) &= 0, \\ &\vdots \end{aligned} \tag{26)(27)(28)(29}$$

The first equation from this chain is already solved. Regarding the second equation, we try to solve it in its local form,

$$\left.\begin{array}{l} S_1 = \int d^D x a \\ (S_1,S)=0 \end{array}\right\} \Rightarrow sa = \partial_\mu j^\mu . \tag{30}$$

For this purpose, first we decompose the first-order deformation according to the antighost number

$$a = a_0 + a_1 + ... + a_I,\ \mathrm{agh} a_k = k, \tag{31}$$

and second we take into account the form ((8)) of the BRST differential of the " free" theory. The Cauchy order of the theory ((1)) is two, so we can safely take $I \le 3$ in ((31)) (see [35,36]). Again, we get a chain of equations by the projection of the equation ((30)) on the values of the antighost number,

$$\begin{aligned} &\begin{cases} \text{if } I>0\ \gamma a_I = 0, \\ \text{if } I=0\ \gamma a_0 = \partial_\mu \overset{(0)\mu}{j}\ \text{(homogenousequation)} \end{cases} \\ &\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)\mu}{j}, \\ &\delta a_k + \gamma a_{k-1} = \partial_\mu \overset{(k-1)\mu}{j},\ 1 \le k < I. \end{aligned} \tag{32)(33)(34}$$

To solve the last chain, we need the local cohomology of the Koszul-Tate differential, $H(\delta | d)$, and the cohomology of the exterior longitudinal derivative, $H(\gamma)$. For our " free" theory we have the following results [30]:

$$H_k(\delta | d) = 0, k > 3,\ H^{2l+1}(\gamma) = 0,$$

which help us to conclude in the equation ((31)) $I = 2, \mathrm{or} I = 0$.

After short calculations we eliminate the possibility $I = 2$. In order to analyse the case $I = 0$ we introduce the counting operator $N^{(t)}$ for the tensor field $t_{\mu\nu|\alpha\beta}$ and its derivatives and the operator $N^{(B)}$ for the abelian two-form and its derivatives. We search for the solutions to the second equation in ((32)) that are simultaneously eigen solutions of both counting operators,

$$a_0 \equiv a_{mn},\, N^{(t)}a_{mn} = ma_{mn},\, N^{(A)}a_{mn} = na_{mn}.$$

The exterior longitudinal derivative (at pure ghost number zero) splits into

$$\gamma = \gamma^{(t)} + \gamma^{(B)},$$

where $\gamma^{(t)}$ acts only on t and $\gamma^{(B)}$ only on B, hence the homogenous equation $\gamma a_0 = \partial_\mu \overset{(0)\mu}{j}$ splits into

$$\gamma^{(t)}a_{mn} = \partial_\mu u^\mu,\, \gamma^{(B)}a_{mn} = \partial_\mu v^\mu.$$

It can be shown that the only possible solution to this system is trivial $a_0 = 0$, therefore the first-order deformation (as well as all the higher order deformations) can be taken to be trivial,

$$S_1 = 0, S_2 = 0, \cdots$$

4. Conclusions

The general conclusion of this paper is that the powerful reformulation of the problem of constructing interactions in gauge theories in terms of the local BRST cohomology reveals that the massless tensor field with the mixed symmetry of the Riemann tensor cannot be coupled in a consistent, nontrivial manner to a massless two-form gauge field. Our analysis was constantly based on the assumptions that the resulting deformations are smooth, local, Lorentz-covariant, and Poincaré-invariant and on the natural requirement that the maximum derivative order of the interacting Lagrangian is equal to two. Our approach opens the perspective to investigate the interactions between the tensor field $t_{\mu\nu|\alpha\beta}$ and one p-form ($p > 2$) or, more general, between a tensor field with the mixed symmetry (k,k) and a p-form. These problems are under consideration.

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