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NO-GO COUPLINGS BETWEEN AN IRREDUCIBLE SPIN-ONE FIELD AND A MASSLESS Q-GRAVITINO

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Abstract
No-go results on the existence of consistent, four-dimensional couplings between an irreducible spin-one field and a Q-gravitino, both massless, are obtained from a deformation approach based on local BRST cohomology.
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1. Introduction

The development of the BRST formalism took a new turn once its cohomological reformulation became available since it made possible, among others, a useful investigation of many interesting aspects related to the perturbative renormalization problem [1]—[4], the anomaly-tracking mechanism [4]—[8], the simultaneous study of local and rigid invariances of a given theory [9] as well as the reformulation of the construction of consistent interactions in gauge theories [10]—[12] in terms of the deformation theory [13]—[15] or, actually, in terms of the deformation of the solution to the master equation. The impossibility of cross-interactions among several Einstein or Weyl gravitons [16]—[17] and of cross-couplings among different Einstein or Weyl gravitons in the presence of matter fields [16], [18]—[21] has recently been shown by means of cohomological arguments. In the same context the uniqueness of $D=4$, $N=1$ supergravity was proved in [22].

The final goal of our research is the investigation of the uniqueness of the simple conformal SUGRA in four spacetime dimensions using the deformation theory.

It is well known that the field spectrum of $D=4$, $N=1$ conformal SUGRA consists in a massless spin-2, a nonmassive spin-$3/2$ and an irreducible spin-one fields. In the free limit the action of simple conformal SUGRA in $D=4$ reduces to the sum between Weyl, massless Q-gravitino, and the standard abelian gauge field actions

$$S_0^{\mathrm{L}}[h_{\mu\nu},\psi_\mu,A_\mu]=S_0^{\mathrm{W}}[h_{\mu\nu}]+S_0^{\mathrm{Q}}[\psi_\mu]+S_0^{\mathrm{1F}}[A_\mu]. \tag{1}$$

In order to determine the consistent interactions that can be added to action (1) we must study, beside the self-interactions, which are known from the literature, also the cross-couplings. The latter problem can be solved in two steps: firstly, we determine the interaction vertices containing only two of the three types of fields, and then the vertices including all the three kinds. In this talk we present one of the ingredients mentioned in the above, namely the problem of constructing consistent interactions among the nonmasive spin-one (described at the Lagrangian level by the standard one-form action) and the massless spin-$3/2$ (described in the free limit by an action with three spacetime derivatives) fields. We investigate these cross-couplings in the framework of the deformation theory [13] based on local BRST cohomology [23].

2. Free model

Our starting point is the Lagrangian action represented by the sum between the abelian one-form and massless "Q"-gravitino [24] actions in four space-time dimensions

$$\begin{aligned} S_0^{\mathrm{L}}[A_\mu,\psi_\mu]&=S_0^{\mathrm{1F}}[A_\mu]+S_0^{\mathrm{Q}}[\psi_\mu]=\\ =&\int \mathrm{d}^4x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-4\mathrm{i}\bar{\phi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\phi_\rho\right), \end{aligned} \tag{2}$$

where we used the notations

$$F_{\mu\nu}=\partial_{[\mu}A_{\nu]},\ \phi_\mu=\frac{\mathrm{i}}{3}\left(\gamma^\nu\partial_{[\nu}\psi_{\mu]}+\frac{1}{2}\gamma_{\mu\nu\rho}\partial^\nu\psi^\rho\right).$$

We employ the flat metric of 'mostly' minus signature and work with the γ-matrices in the Majorana representation [all γ-matrices are purely imaginary, γ_0 is Hermitian and γ_i are anti-Hermitian]. In (2) ψ_μ is a real spin-vector. The theory (2) is invariant under the gauge transformations

$$\delta_\varepsilon A_\mu=\partial_\mu\varepsilon,\ \delta_\varepsilon\psi_\mu=\partial_\mu\xi+\mathrm{i}\gamma_\mu\zeta. \tag{3}$$

The gauge parameter ε is bosonic and ξ, ζ are Majorana spinors. The gauge algebra of the free theory (2) is Abelian.

We observe that there are no non-vanishing local transformations of ε, ξ and ζ that annihilate $\delta_\varepsilon A_\mu$ and $\delta_\varepsilon\psi_\mu$. This remark allows us to conclude that the generating set of gauge transformations (3) is irreducible.

3.Construction of consistent interactions

Due to the fact that the solution to the master equation contains all the information on the gauge structure of a given theory, we can reformulate the problem of introducing consistent interactions as a deformation problem of the solution to the master equation corresponding to the ``free'' theory. If an interacting gauge theory can be consistently constructed, then the solution $S$ to the master equation associated with the ``free'' theory can be deformed into a solution $\bar{S}$

$$\begin{aligned} S \rightarrow \bar{S} &= S + \lambda S_1 + \lambda^2 S_2 + \cdots \\ &= S + \lambda \int \mathrm{d}^D x a + \lambda^2 \int \mathrm{d}^D x b + \cdots, \end{aligned} \tag{4}$$

of the master equation for the deformed theory

$$\left(\bar{S}, \bar{S}\right) = 0, \tag{5}$$

such that both the ghost and antifield spectra of the initial theory are preserved. The equation (5) splits, according to the various orders in λ, into

$$(S, S) = 0, \tag{6}$$

$$2(S_1, S) = 0, \tag{7}$$

$$2(S_2, S) + (S_1, S_1) = 0, \tag{8}$$

$$(S_3, S) + (S_1, S_2) = 0, \tag{9}$$

$$\vdots$$

The equation (6) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, S_1, is a co-cycle of the ``free'' BRST differential. However, only cohomologically non-trivial solutions to (7) should be taken into account, as the BRST-exact ones [BRST co-boundaries] correspond to trivial interactions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically nonempty due to its isomorphism to the space of physical observables of the ``free'' theory. It has been shown [on behalf of the triviality of the antibracket map in the cohomology of the BRST differential] that there are no obstructions in finding solutions to the remaining equations [(8)--(9), etc.]. However, the resulting interactions may be nonlocal, and there might even appear obstructions if one insists on their locality. The analysis of these obstructions can be done with the help of cohomological techniques.

For our free model the solution to the master equation reads as

$$S = S_0^{\mathrm{L}}[A_\mu, \psi_\mu] + \int \mathrm{d}^4 x \left[A^{*\mu} \partial_\mu \eta + \psi^{*\mu} \left(\partial_\mu \chi + \mathrm{i} \gamma_\mu \theta \right) \right], \tag{10}$$

where we denoted by η, χ and θ the ghost associated with the gauge parameters ε, ξ and respectively ζ.

4. Main results

By direct computation we obtain that the first-order deformation reads as

$$\begin{aligned} S_1 = \int \mathrm{d}^4 x \Big\{ k \Big[& \eta^* \bar{\chi} \gamma_5 \theta + \frac{1}{4} \left(\chi^* \gamma_5 \chi - \theta^* \gamma_5 \theta \right) \eta \\ & - A^{*\mu} \left(\bar{\chi} \gamma_5 \phi_\mu - \bar{\theta} \gamma_5 \psi_\mu \right) + \frac{1}{4} \psi^{*\mu} \left(\gamma_5 \psi_\mu \eta - \gamma_5 \chi A_\mu \right) \\ & + \bar{\phi}_\mu \gamma_5 \psi_\nu F^{\mu\nu} + \left(\bar{\psi}_\mu \gamma_5 \hat{F}^{\mu\nu} - \bar{\phi}_\mu \gamma_5 \mathbf{F}^{\mu\nu} \right) A_\nu \Big] \\ & + p \bar{\mathbf{F}}^{\mu\nu} \mathbf{F}_{\mu\nu} + q \bar{\mathbf{F}}_{\mu\nu} \gamma^{\mu\nu\rho\lambda} \mathbf{F}_{\rho\lambda} \Big\}, \end{aligned} \tag{11}$$

where k, p and q are arbitrary constants and the objects $\mathbf{F}_{\mu\nu}$ and $\hat{F}_{\mu\nu}$ are given below

$$\mathbf{F}_{\mu\nu} = \partial_{[\mu} \psi_{\nu]} + \mathrm{i} \gamma_{[\mu} \phi_{\nu]}, \quad \hat{F}_{\mu\nu} = \partial_{[\mu} \phi_{\nu]} \tag{12}$$

and pertain to the cohomology of the derivative along the gauge orbits for the free theory. In fact, the cohomology of the derivative along the gauge orbits $H(\gamma)$ is generated by the antifields associated with the original fields/ghosts together with their spacetime derivatives, by the field strength $F_{\mu\nu}$ and (12) together with their spacetime derivatives as well as the undifferentiated ghosts η, χ and θ.

The second order deformation is governed by the equation (8). If we denote by Λ and b the nonintegrated densities of the functionals (S_1, S_1) and respectively S_2, the local form of (8) becomes

$$\Lambda = -2sb + \partial_\mu n^\mu, \tag{13}$$

with

$$\mathrm{gh}(\Delta) = 1, \quad \mathrm{gh}(b) = 0, \quad \mathrm{gh}(n^\mu) = 1, \tag{14}$$

for some local currents n^μ. Developping Λ and b with respect to the antighost number and projecting the equation (13) on various antighost numbers with obtain the equivalent tower of equations

$$\Lambda_2 = -2\gamma b_2 + \partial_\mu n_2^\mu, \tag{15}$$

$$\Lambda_I = -2(\delta b_{I+1} + \gamma b_I) + \partial_\mu n_I^\mu, \quad I = \overline{0,1}. \tag{16}$$

By direct computation we obtain that

$$\Lambda_2 = \frac{k^2}{2}\left(\chi^* \gamma_5 \chi - \theta^* \gamma_5 \theta\right)\bar{\chi}\gamma_5\theta, \tag{17}$$

$$\begin{aligned}\Lambda_1 = {} & \gamma\left[\frac{\mathrm{i}k^2}{6}A^{*\mu}\left(\bar{\chi}\gamma^\nu \psi_{[\mu}A_{\nu]} - \frac{1}{2}\bar{\chi}\gamma_{\mu\nu\rho}\psi^\nu A^\rho\right)\right] \\ & + \frac{k^2}{2}\psi^{*\mu}\gamma_5\psi_\mu\bar{\chi}\gamma_5\theta + \frac{\mathrm{i}k^2}{6}A^{*\mu}\bar{\chi}\gamma^\nu\chi F_{\mu\nu} \\ & + \frac{k^2}{2}\psi^{*\mu}\gamma_5\chi\left(\bar{\chi}\gamma_5\phi_\mu - \bar{\theta}\gamma_5\psi_\mu\right),\end{aligned} \tag{18}$$

$$\begin{aligned}\Lambda_0 = {} & k\left(p\bar{\mathbf{F}}_{\mu\nu} + q\bar{\mathbf{F}}^{\rho\lambda}\gamma_{\mu\nu\rho\lambda}\right)\gamma_5\left[\partial^\mu\left(\psi^\nu\eta - \chi A^\nu\right)\right. \\ & \left. + \frac{\mathrm{i}}{3}\gamma^\mu\gamma_\rho\partial^{[\rho}\left(\psi^{\nu]}\eta - \chi A^{\nu]}\right) + \frac{\mathrm{i}}{6}\gamma^\mu\gamma^{\nu\sigma\varepsilon}\partial_\sigma\left(\psi_\varepsilon\eta - \chi A_\varepsilon\right)\right] \\ & + 2k^2\left(\bar{\psi}_\mu\gamma_5\hat{F}^{\mu\nu} - \bar{\phi}_\mu\gamma_5\mathbf{F}^{\mu\nu}\right)\left(\bar{\theta}\gamma_5\psi_\nu - \bar{\chi}\gamma_5\phi_\nu\right) + \frac{k^2}{2}\bar{\psi}_\mu\hat{F}^{\mu\nu}\eta A_\nu \\ & + 2k^2\bar{\phi}^\mu\gamma_5\psi^\nu\left(\bar{\theta}\gamma_5\mathbf{F}_{\mu\nu} - \bar{\chi}\gamma_5\hat{F}_{\mu\nu}\right) + \frac{k^2}{2}F^{\mu\nu}\left(\bar{\phi}_\mu\psi_\nu\eta - \bar{\phi}_\mu\chi A_\nu\right) \\ & + \gamma\left[k^2\bar{\phi}^{[\mu}\gamma_5\psi^{\nu]}\bar{\phi}_\mu\gamma_5\psi_\nu\right] + \frac{\mathrm{i}k^2}{6}A_{[\mu}\bar{\phi}_{\nu]}\partial^\mu\left(\psi^\nu\eta - \chi A^\nu\right) \\ & + \frac{\mathrm{i}k^2}{6}F^{\mu\lambda}\left[\bar{\psi}_\lambda\gamma^\nu\partial_{[\mu}\left(\psi_{\nu]}\eta - \chi A_{\nu]}\right) - \frac{1}{2}\bar{\psi}_\lambda\gamma_{\mu\nu\rho}\partial^\nu\left(\psi^\rho\eta - \chi A^\rho\right)\right] \\ & + \frac{\mathrm{i}k^2}{6}A_{[\mu}\bar{\psi}_{\nu]}\left[\gamma_\rho\partial^\rho\partial^\mu\left(\psi^\nu\eta - \chi A^\nu\right) + \frac{1}{2}\gamma^{\nu\rho\lambda}\partial^\mu\partial_\rho\left(\psi_\lambda\eta - \chi A_\lambda\right)\right] \\ & - \frac{\mathrm{i}k^2}{6}A_\mu\bar{\mathbf{F}}^{\mu\nu}\left[\gamma^\rho\partial_{[\rho}\left(\psi_{\nu]}\eta - \chi A_{\nu]}\right) + \frac{1}{2}\gamma_{\nu\rho\lambda}\partial^\rho\left(\psi^\lambda\eta - \chi A^\lambda\right)\right] \\ & - \frac{k^2}{6}A_{[\mu}\bar{\phi}_{\nu]}\gamma^\mu\left[\gamma_\rho\partial^{[\rho}\left(\psi^{\nu]}\eta - \chi A^{\nu]}\right) + \frac{1}{2}\gamma^{\nu\rho\lambda}\partial_\rho\left(\psi_\lambda\eta - \chi A_\lambda\right)\right]\end{aligned} \tag{19}$$

By virtue of the above discussion concerning the cohomology of the derivative along the gauge orbit, we observe that Λ_2 given in (17) is a nontrivial object from $H(\gamma)$. On the othe hand, the equation (15) requires for Λ_2 to be γ-exact modulo d so it has to be zero. This implies that

$$k = 0. \tag{20}$$

Replacing (20) in (11) it simply reduces to

$$S_1 = \int \mathrm{d}^4 x\left(p\bar{\mathbf{F}}^{\mu\nu}\mathbf{F}_{\mu\nu} + q\bar{\mathbf{F}}_{\mu\nu}\gamma^{\mu\nu\rho\lambda}\mathbf{F}_{\rho\lambda}\right).$$

Performing the same operation in (17)--(19) we conclude that Λ is zero so

$$S_2 = 0.$$

The previous results can be summarized in the following theorem.

Theorem 1 *Under the assumptions of:*

i) space-time locality,

ii) smoothness of the deformations in the coupling constant,

iii) (background) Lorentz invariance,

iv) Poincaré invariance (i.e. we do not allow explicit dependence on the space-time coordinates) and

v) the maximum number of derivatives in the interacting Lagrangian is three,

the only consistent deformation of (2) involving a massless spin-3/2 field and an abelian one-form gauge field reads as

$$\begin{aligned} S^{\mathrm{L}}[A_\mu, \psi_\mu] = \int \mathrm{d}^4x \Big[& -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - 4\mathrm{i}\phi_\mu \gamma^{\mu\nu\rho} \partial_\nu \phi_\rho \\ & + \lambda p \overline{\mathbf{F}}^{\mu\nu} \mathbf{F}_{\mu\nu} + \lambda q \overline{\mathbf{F}}_{\mu\nu} \gamma^{\mu\nu\rho\lambda} \mathbf{F}_{\rho\lambda} \Big], \end{aligned} \tag{21}$$

and it is invariant under the original gauge transformations.

5.Conclusions

In this paper we have discussed the cohomological approach to the problem of constructing consistent interactions between the abelian one-form gauge field A_μ and the massless "Q"-gravitino ψ_μ. Under the assumptions of smoothness of the deformations in the coupling constant, (background) Lorentz invariance, Poincaré invariance (i.e. we do not allow explicit dependence on the space-time coordinates) and the maximum number of derivatives in the interacting Lagrangian is three, we have exhausted all the consistent, non-trivial couplings.

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