Analele Universităţii de Vest din Timişoara
Vol. XLVIII, 2006
Seria Fizică

NON-CANONICAL OPERATORS AND GAUGE FIXING PROCEDURE

Radu Constantinescu and Carmen Ionescu

Faculty of Physics, University of Craiova, Romania

Abstract

An approach of the extended BRST Lagrangean formalism via its equivalence with the Hamiltonian one is proposed. This approach will allow a better understanding of the ghost spectrum and of the form of the master equations. We will also point out the key role played by the non-canonical operators which appear in the Lagrangean context for the gauge fixing procedure. To be more concrete, the case of an irreducible first rank theory endowed with a sp (3) BRST symmetry will be considered.

1. Introduction

This paper intends to present a new approach by which one can recover, in a simpler and accurate way, the main results of the extended BRST Lagrangean theory on the basis of the equivalence between the extended Lagrangean [1] and the Hamiltonian [2] formalisms.

In the standard BRST approach [3], [4] the two formalisms impose practically similar constructs. They ask for minimal sets of ghost-type generators which allow writing down a solution of the master equation, but also for a non-minimal sector which have to be used in the gauge fixing procedure. When an extended $sp(n>1)$ BRST symmetry [5] is implemented, the things change and the similarity is broken. The fundamental difference between the two formalisms consists in the existence of the single BRST generator, S, but of n antibrackets $(,)_a,\ a=1,...,n$ in the Lagrangean case, and of n BRST generators $\{\Omega_a, a=1,...,n\}$ and a single structure of bracket (generalized Poisson bracket) in the Hamiltonian case. In the last case the phase space is generated by pairs of canonical variables, ghosts and associated ghost-momenta, and on their basis, the extended Poisson bracket is defined [5]. The BRST operators $\{s_a,\ a=1,...,n\}$ are connected with the BRST charges by equations of the form:

$$s_a * \equiv [*, \Omega_a], a=1,...,n. \tag{1.1}$$

In the Lagrangean approach, the similarity with (1.1) should ask for a relation between the BRST generator and the n different antibrackets of the form:

$$s_a * \overset{?}{=} (*, S)_a , a = 1,...,n. \quad (1.2)$$

It is well known from the sp(2) BRST Lagrangean [1] approach that the relation (1.2) is not valid. The requirement of acyclicity asks for the introduction of same "bar" variables, without canonical pairs, variables which generate the apparition of some "non-canonical" operators V_a in the master equations. Lagrangean equivalent of (1.1) will be not (1.2) but:

$$s_a * \equiv (*, S)_a + V_a * \quad a = 1,...,n. \quad (1.3)$$

The form of the operators V_a has been "guessed" in the sp(2) [1] and sp(3) [6] cases. It depends on the ghost spectrum, but this one becomes as large as the order of the symmetry grows. The non-minimal sector dramatically expand, it too. It is practically impossible to maintain the control of the process. To explain the origin of the non-canonical operators V_a and to find a way of determining their concrete form represents two important aims of this paper. We will also see the importance of these operators in the gauge fixing process and we will propose a manner of controlling the extension of the ghost spectrum, so that to maintain it at the minimal necessary structure. More precisely, we will derive the form of the master equations in the Lagrangean context starting from the Hamiltonian one. We will use three important assets:

(i) a bi-graduation defined by the *ghost number* (*gh*) and the *level number* (*lev*) [7] can be used, and the variables with the same bi-degree in the two formalisms can be identified;

(ii) the action of the BRST operators is the same on the common ghost-field spectrum in the two approaches; *(iii)* in the Lagrangean approach, the Koszul differentials δ_a can be split in two parts: a canonical part, with a non-trivial action on the antifields with canonical conjugates only, and a "non-canonical" part acting on the single "bar" variables:

$$\delta_a = \delta_a^{(can)} + \delta_a^{(nc)} \quad (1.4)$$

The previous decomposition induces a similar one at the level of the whole BRST operator:

$$s_a = s_a^{(can)} + s_a^{(nc)} \quad (1.5)$$

The supposition *(ii)* from behind is based on the remark that the field-spectrum in the Lagrangean and in the Hamiltonian constructions are practically the same. The third supposition does not refer to the Hamiltonian approach which implies canonical pairs of variables only, so that the non-canonical operators loose here any sense. Moreover, because of in the Hamiltonian theory two important problems have been solved: how to introduce in a natural way the non-minimal sector [7] and how to choose the gauge fixing term [8]. These are problems which when we try to built the Lagrangean covariant formalism, we are faced with major difficulties. All these represent important arguments which suggest that, despite its luck of covariance, it is more convenient to work in the Hamiltonian frame and to obtain the Lagrangean formalism on the basis of the equivalence between the two.

We will try to prove this assertion and to recover some basic results concerning the sp(3) theory. To be more concrete, we will restrict ourselves to the study of the irreducible first class models. The paper is structured in four parts. After this introduction, in the second section we will briefly review some known results on the Lagrangean and, respectively, Hamiltonian BRST extended formalisms. The third section will contain the core of the paper and it will effectively use the equivalence between the two formalism in order to give important details on the main equations and quantities appearing in the sp(3) Lagrangean approach. We will emphasis the key role of the bar" antifields associated to the Lagrange multipliers in the gauge fixing procedure. Some concluding remarks will end the paper.

2. Basic results on the extended BRST theories

Let us consider a gauge theory described by the Lagrangean action $S_0^L[q^i]$, where $\{q^i, i=1,\cdots,r\}$ represents the set of all fields of the theory. The action is invariant in respect with the gauge transformations

$$\delta_\varepsilon q^i = R^i_\alpha[q]\varepsilon^\alpha;\ \alpha=1,\cdots,m. \tag{1.6}$$

In the BRST Lagrangean approach, starting from this action one obtains the solution S^L of the master equation and the path integral Z^L_Y, by an adequate gauge fixing procedure.

In the Hamiltonian case a theory with constraints corresponds to a gauge theory. It is described by an Hamiltonian $H_0(q^i, p_i)$ and by a set of the constraints $\{G_\alpha(q^i, p_i)=0; i=1,...,r; \alpha=1,\cdots,m\}$. We will refer in this paper to the case of an irreducible theory with first class constraints, case in which the extended Hamiltonian $H^{(0)}$ has the form

$$H^{(0)}(q,p,u) = H_0(q,p) + u^\alpha G_\alpha. \tag{1.7}$$

The algebra satisfied by the constraints is given by:

$$[G_\alpha, G_\beta] = f_{\alpha\beta}^{\gamma} G_\gamma ; [H_0, G_\alpha] = V_\alpha^\beta G_\beta \tag{1.8}$$

and we assume that the structure functions $f_{\alpha\beta}^{\gamma}$ and V_α^β are constants. The canonical action will be

$$S_c[q, p, u] = \int dt[\dot{q}^i \, p_i - H^{(0)}(q, p, u)]; i = 1, \cdots, r. \tag{1.9}$$

The action (1.9) is invariant under the gauge transformations

$$\delta_\varepsilon q^i = [q^i, G_\alpha]\varepsilon^\alpha ; \; \delta_\varepsilon p_i = [p_i, G_\alpha]\varepsilon^\alpha \tag{1.10}$$

$$\delta_\varepsilon u^\alpha = \dot{\varepsilon}^\alpha - V_\beta^\alpha \varepsilon^\beta + f_{\beta\gamma}^{\alpha} u^\gamma \varepsilon^\beta . \tag{1.11}$$

The main problems of the Hamiltonian approach consist in determining the form of the BRST symmetry, given as a differential operator or as a canonical charge, as well as of the gauge fixed Hamiltonian which observe this symmetry. When a sp(3) symmetry $\{s_a, a = 1,2,3\}$ is implemented, it can be built in an extended phase space generated by the following set of canonical conjugate variables:

$$P_{\mathbf{A}} \equiv \{\pi_\alpha, \pi_{\alpha a}, P_{\alpha a}, p_i\}; Q^{\mathbf{A}} = \{q^i, Q^{\alpha a}, \lambda^{\alpha a}, \eta^\alpha\}. \tag{1.12}$$

The master equations which allow us to determine s_a or the corresponding charges Ω_a are:

$$s_a s_b + s_b s_a = 0; [\Omega_a, \Omega_b] = 0, a = 1,2,3 \tag{1.13}$$

The concrete forms of the BRST charges are in this case:

$$\begin{aligned} \Omega_a = {} & G_\alpha Q^{\alpha b}\delta_{ba} + \varepsilon_{abc} P_{\alpha c}\lambda^{\alpha b} + \frac{1}{2} f_{\alpha\beta}{}^{\gamma} P_{\gamma c} Q^{\beta c} Q^{\alpha b}\delta_{ba} + \\ & + \pi_{\alpha a}\eta^\alpha + \frac{1}{2} f_{\alpha\beta}{}^{\gamma}\pi_{\gamma c}\lambda^{\beta c} Q^{\alpha b}\delta_{ba} + \frac{1}{12} f_{\sigma\alpha}{}^{\theta} f_{\theta\beta}^{\gamma}\, \varepsilon_{bcd}\pi_{\gamma b} Q^{\alpha c} Q^{\beta d} Q^{\sigma e}\delta_{ea} + \\ & + \frac{1}{2} f_{\alpha\beta}{}^{\gamma}\pi_\gamma \eta^\beta Q^{\alpha b}\delta_{ba} + \frac{1}{12}(f_{\alpha\sigma}{}^{\theta} f_{\theta\beta}{}^{\gamma} + f_{\beta\sigma}{}^{\theta} f_{\theta\alpha}{}^{\gamma})\pi_\gamma \lambda^{\alpha c} Q^{\beta c} Q^{\sigma b}\delta_{ba}, a = 1,2,3. \end{aligned} \tag{1.14}$$

A way of passing towards a Lagrangean approach is given by the fact that in (203) the momenta $\{p^i, i = 1, \cdots, r\}$ can be seen as auxiliary variables and, therefore, can be eliminated on the basis of their equations of motion. One obtain at the end the action

$$S_0[q, u] = \int dt \; L_0(q, \dot{q}, u). \tag{1.15}$$

In this case, the relations (1.10), (1.11) become:

$$\delta q^i = a^i_\alpha(q,\dot{q},u)\varepsilon^\alpha ; \delta u^\alpha = \dot{\varepsilon}^\alpha - V^\alpha_\beta \varepsilon^\beta + f^\alpha_{\beta\gamma} u^\gamma \varepsilon^\beta \tag{1.16}$$

and the Noether's identities will take the form

$$\frac{\delta S_0}{\delta q^i} a^i_\beta + \frac{\delta S_0}{\delta u^\alpha}[-V^\alpha_\beta + f^\alpha_{\beta\gamma} u^\gamma] - \frac{d}{dt}\left(\frac{\delta S_0}{\delta u^\beta}\right) = 0. \tag{1.17}$$

In the sp(3) case, the Lagrangean formalism allows to obtain the solution of the master equation:

$$\frac{1}{2}(S,S)_a + V_a S = 0,\ a = 1,2,3 \tag{1.18}$$

As a very important remark, we have to mention that this extended space is generated by the same set of fields as in the Hamiltonian approach:

$$\{u^\alpha, Q^{\mathbf{A}}\} = \{u^\alpha, q^i, Q^A\} = \{u^\alpha, q^i, Q^{\alpha a}, \lambda^{\alpha a}, \eta^\alpha\}\,. \tag{1.19}$$

A key role is played by the antifields associated with the Lagrange multipliers. This can be identified with the ghost-momenta [8]. The differences start when the total antifield spectrum is compared with the ghost-momenta given in (1.12). The existence of 3 antibrackets will ask for the introduction of three different sets of antifields, $\{Q^*_{Aa}$, a=1,2,3$\}$conjugated with $\{Q^A\}$ from (1.19) in respect with each antibracket, $a=1$, $a=2$ and respectively $a=3$. So, the extended configuration space adequate for implementation of the extended sp(3) BRST will be generated by:

$$\{Q^A, Q^*{}_{Aa}, \bar{Q}_{Aa}, \bar{Q}_A, a = 1,2,3\}. \tag{1.20}$$

The gauge fixing procedure in the extended Lagrangean formalism [1], [6] supposes the introduction of the non-minimal sector, fact which will determine an extra-extension of the generators' spectra.

Remark 1: Considering the action (1.15), the solution of (1.18) will start as:

$$S^E[q,u,Q,\ q^*,u^*,Q^*,\bar{q},\bar{u},\bar{Q}] = S_0[q,u] +$$

$$+\int dt\ (q^*_{ia} a^i_\alpha Q^{\alpha a} + u^*_{\alpha a}(\dot{Q}^{\alpha a} - V^\alpha_\beta Q^{\beta a} + f^\alpha_{\beta\gamma} u^\gamma Q^{\beta a}) + \frac{1}{2} f^\alpha_{\beta\gamma}(Q^*_{\alpha ab} Q^{\gamma a} Q^{\beta b} +$$

$$+ \lambda^*_{\alpha ab} Q^{\gamma a} \lambda^{\beta b} + \eta^*_{\alpha ab} Q^{\gamma a} \eta^{\beta b}) + \ldots).$$

Remark 2: A very useful tool in the construction of the Lagrangean formalism will be the structuring of the configuration space using the same rules as we proposed for the Hamiltonian approach. It will be based on a double graduation following the ghost number, gh, and the level number, lev . This graduation will allow the identification of the generators and will smooth the transfer process from the Hamilton to Lagrange.

3. From Hamilton to Lagrange

The study of the equivalence will impose the identification of the action of the BRST operators, Hamiltonian $\{s_a^H, a = 1,2,3\}$ and Lagrangean $\{s_a^L, a = 1,2,3\}$, on the common fields $Q^{\mathbf{A}} = \{q^i, Q^A\}$:

$$s_a^H Q^{\mathbf{A}} = s_a^L Q^{\mathbf{A}} \tag{1.22}$$

where Q^A represents the set of the ghosts, set which in the $sp(3)$ case is given by (1.19).

As a general rule, the proof of the equivalence between the two formalisms imposes the following steps:

1) to obtain the antifield spectrum, the proper solution of the BRST generator S and the concrete form of the non-canonical operators V_a specific for the Lagrangean formalism, knowing the expression of the BRST charges Ω_a and using the relation (1.22).
2) to recover the main quantities of the Hamiltonian formalism on the basis of a Lagrangean annalysis of the action obtained after eliminating the original momenta $\{p^i, i = 1,\cdots,r\}$, and considering the Lagrange multipliers as usual fields in the canonical action.

In this paper we are interested to follow the first step only and we will do it considering the case of an irreducible first rank gauge theory described by the canonical action (1.9).

It is remarkable that, following the ansatz *(i)* from the introduction concerning the bigraduation (gh, lev), we can transfer the ghost-momenta from Hamiltonian approach as antifields associated with Lagrange multipliers in Lagrangean one:

$$\overset{(-1,1-a)}{P_{\alpha a}} \equiv \overset{(-1,1-a)}{u^*_{\alpha a}} \;,\; \overset{(-2,a-4)}{\pi_{\alpha a}} \equiv \overset{(-2,a-4)}{\bar{u}_{\alpha a}} \;,\; \overset{(-3,-3)}{\pi_{\alpha}} \equiv \overset{(-3,-3)}{\bar{u}_{\alpha}} \tag{1.23}$$

This identifications suggests the possibility of extending (1.22) with

$$s_b^H P_{\alpha a} = s_b^L u^*_{\alpha a},\;\; s_b^H \pi_{\alpha a} = s_b^L \bar{u}_{\alpha a},\; s_b^H \pi_\alpha = s_b^L \bar{u}_\alpha . \tag{1.24}$$

We observe that: *(i)* each ghost-momentum $\{\pi_\alpha\}$ which is not affected by the $sp(3)$ index is in correspondence with the new bar antifield $\{\bar{u}_\alpha\}$; *(ii)* the antifields $\{\bar{u}_{\alpha a}, a = 1,2,3\}$ are in correspondence with the ghost-momenta $\{\pi_{\alpha a}, a=1,2,3\}$. These correspondences are established following the Grassmann parity and the bi-graduation (gh, lev) .

For recovering the Lagrangean formalism from the Hamiltonian one, we start from the expressions of the BRST charges (1.14) and we will assume that the relations (1.22) and (1.23) are observed. We are interested in recovering: *(a)* the antifields spectra, generators of the Koszul-Tate tricomplex; *(b)* the $sp(3)$ BRST generator (1.14) and *(c)* the form of the master equations used in the Lagrangean $sp(3)$ BRST approach. The term from the left hand side of (1.22) has the form

$$s_a^H Q^{\mathtt{A}} \equiv [Q^{\mathtt{A}}, \Omega_a] \tag{1.25}$$

Conclusion 1: *Our approach of the equivalence between the* $sp(3)$ BRST *Hamiltonian and Lagrangean formalisms allowed to recover and to justify the structure of the configuration space in the* $sp(3)$ *antibracket-antifield formalism.*

The natural question which appear in this moment is connected with *the action of the BRST operators* $\{ s_a,\ a=1,2,3 \}$ *on the bar antifields which do not have canonical conjugate pairs?* It is clear that for these variables the relation (1.25) is not true. So, we will decompose the BRST operators after the *resolution degree (res).* It is defined as being *0* for ghosts and "*-gh*" for ghost-momenta. The decomposition will have the form:

$$s_a = \sum_{k=-1}^{\infty} \overset{(k)}{s}_a = \overset{(-1)}{\delta}_a + \overset{(0)}{d}_a + \overset{(1)}{s}_a + \overset{(2)}{s}_a + \cdots,\ res(\overset{(k)}{s}_a) = k \tag{1.26}$$

We will have in mind from now on the graduation *(res,lev)* of the ghost-momenta

$$\pi_\alpha = \overset{(2,a-4)}{\pi_a} \tag{1.27}$$

The relation

$$\begin{aligned} s_a \pi_{\alpha b} &= (\overset{(-1,a-1)}{\delta_a} + \overset{(0,a-1)}{d_a} + \overset{(1,a-1)}{s_a} + \cdots) \overset{(2,a-4)}{\pi_{ab}} = \\ &= \varepsilon_{abc} \overset{(1,1-c)}{P_{\alpha c}} + \frac{1}{2} f^{\gamma}_{\alpha\beta} \overset{(2,b-4)}{\pi_{\alpha b}} \overset{(0,a-1)}{Q^{\beta c}} \delta_{ca} + \\ &+ \frac{1}{12} (f^{\theta}{}_{\alpha\sigma} f^{\gamma}{}_{\theta\beta} + f^{\theta}{}_{\beta\sigma} f^{\gamma}{}_{\theta\alpha}) \overset{(2,-3)}{\pi_\gamma} \overset{(0,b-1)}{Q^{\beta b}} \overset{(0,c-1)}{Q^{\sigma c}} \delta_{ca} \end{aligned} \tag{1.28}$$

leads to the conclusion that:

$$\overset{(-1,a-1)}{\delta_a} \overset{(2,b-4)}{\pi_{ab}} = \varepsilon_{abc} \overset{(1,1-c)}{P_{\alpha c}},\ c = 6 - a - b \tag{1.29}$$

$$\overset{(0,a-1)}{d_a}\ \overset{(2,b-4)}{\pi_{ab}} = \frac{1}{2} f^{\gamma}_{\alpha\beta}\ \overset{(2,b-4)}{\pi_{\alpha b}}\ \overset{(0,c-1)}{Q^{\beta c}}\ \delta_{ca} + \\ + \frac{1}{12}(f^{\theta}{}_{\alpha\sigma} f^{\gamma}{}_{\theta\beta} + f^{\theta}{}_{\beta\sigma} f^{\gamma}{}_{\theta\alpha})\ \overset{(2,-3)}{\pi_{\gamma}}\ \overset{(0,b-1)}{Q^{\beta b}}\ \overset{(0,c-1)}{Q^{\sigma c}}\ \delta_{ca}. \tag{1.30}$$

On the basis of this relations and using (1.24) and the identifications (1.23) we obtain

$$s^{L}_{a}\bar{u}_{\alpha b} = (\overset{(-1,a-1)}{\delta_a} + \overset{(0,a-1)}{d_a} + \overset{(1,a-1)}{s_a} + \cdots)\ \overset{(2,b-4)}{\bar{u}_{\alpha b}} \equiv s^{H}_{a}\pi_{\alpha b} = \\ = \varepsilon_{abc}\ \overset{(1,1-c)}{u^{*}_{\alpha c}} + \frac{1}{2} f^{\gamma}_{\alpha\beta}\ \overset{(2,b-4)}{\bar{u}_{\alpha b}}\ \overset{(0,c-1)}{Q^{\beta c}}\ \delta_{ca} + \\ + \frac{1}{12}(f^{\theta}{}_{\alpha\sigma} f^{\gamma}{}_{\theta\beta} + f^{\theta}{}_{\beta\sigma} f^{\gamma}{}_{\theta\alpha})\ \overset{(2,-3)}{\bar{u}_{\gamma}}\ \overset{(0,b-1)}{Q^{\beta b}}\ \overset{(0,c-1)}{Q^{\sigma c}}\ \delta_{ca} \tag{1.31}$$

The last relation leads to

$$\overset{(-1,a-1)}{\delta^{L}_{a}}\ \overset{(2,b-4)}{\bar{u}_{\alpha b}} = \varepsilon_{abc}\ \overset{(1,1-c)}{u^{*}_{\alpha c}} \tag{1.32}$$

$$\overset{(0,a-1)}{d^{L}_{a}}\ \overset{(2,b-4)}{\bar{u}_{\alpha b}} = \frac{1}{2} f^{\gamma}_{\alpha\beta}\ \overset{(2,b-4)}{\bar{u}_{\alpha b}}\ \overset{(0,c-1)}{Q^{\beta c}}\delta_{ca} + \\ + \frac{1}{12}(f^{\theta}{}_{\alpha\sigma} f^{\gamma}{}_{\theta\beta} + f^{\theta}{}_{\beta\sigma} f^{\gamma}{}_{\theta\alpha})\ \overset{(2,-3)}{\bar{u}_{\gamma}}\ \overset{(0,b-1)}{Q^{\beta b}}\ \overset{(0,c-1)}{Q^{\sigma c}}\ \delta_{ca}. \tag{1.33}$$

Similarly we obtain

$$\overset{(-1,a-1)}{\delta^{L}_{a}}\ \overset{(2,-3)}{\bar{u}_{\alpha}} = \delta_{ab}\ \overset{(2,b-4)}{\bar{u}_{\alpha b}} \tag{1.34}$$

$$\overset{(0,a-1)}{d^{L}_{a}}\ \overset{(3,-3)}{\bar{u}_{\alpha}} = \frac{1}{2} f^{\gamma}_{\alpha\beta}\ \overset{(3,-3)}{\bar{u}_{\alpha}}\ \overset{(0,c-1)}{Q^{\beta c}}\delta_{ca}. \tag{1.35}$$

In conclusion, the Koszul operators can be decomposed in a canonical part and a non-canonical part: $\delta_a * = (\delta^{can}_{a} + \delta^{nc}_{a}) * \equiv (* \quad , S)_a\big|_{ghosturi=0} + V_a *$ (1.36)

The relation (1.36) implies the decomposition of the BRST operators as:

$$s_a * = (s^{can}_{a} + s^{nc}_{a}) * \equiv (* \quad , S)_a + V_a * \tag{1.37}$$

The canonical part of the BRST operator has non-trivial action on the pairs of conjugate variables $\{Q^{A}, Q^{*}_{Aa}\}$ and it is expressed by the antibrackets $(\ ,\ )_a$, $a = 1,2,3$. By contrary, the non-canonical operators V_a have non-trivial action on the "bar" antifields $\{\bar{Q}_{Aa}, a = 1,2,3\}$ and $\{\bar{Q}_{A}\}$ only: $(*, \bar{Q}_{A})_a \equiv 0; V_a Q^{*}_{Ab} \equiv 0; V_a \bar{Q}_{Ab} \neq 0, V_a \bar{Q}_{A} \neq 0$ (1.38)

The BRST variation of the $\{Q^{A}\}$ fields can be expressed now by

$$s_a^L Q^A = (s_a^{can} + s_a^{nc})Q^A \equiv (Q^A, S)_a + V_a Q^A \quad (1.40)$$

where $$s_a^{can} * = (*, S)_a, \; s_a^{nc} * \equiv V_a * . \quad (1.41)$$

So, our approach of the equivalence between the two formalisms allows us to obtain the usual form of the action of BRST operators on the fields which appear in the $sp(3)$ antibracket-antifield formalism. We pass now to the next aim of the paper: to obtain the concrete form of the master equations. We know that in the Hamiltonian formalism we have:

$$s_a^H \pi_{\alpha b} = [\pi_{\alpha b}, \Omega_a] = \varepsilon_{abc} P_{\alpha c} + \cdots \quad (1.42)$$

$$s_a^H \pi_\alpha = [\pi_\alpha, \Omega_a] = \delta_{ab} \pi_{\alpha b} + \cdots \quad (1.43)$$

Similarly, in the Lagrangean formalism we obtain

$$\begin{aligned} s_a^L \bar{u}_{\alpha b} &= (\bar{u}_{\alpha b}, S)_a + V_a \bar{u}_{\alpha b} \overset{(1.38)}{=} V_a \bar{u}_{\alpha b} \\ s_a^L \bar{u}_\alpha &= (\bar{u}_\alpha, S)_a + V_a \bar{u}_\alpha \overset{(1.38)}{=} V_a \bar{u}_\alpha . \end{aligned} \quad (1.44)$$

From relations (1.32), (1.34) and (1.44) we could identify

$$V_a \bar{u}_{\alpha b} = \varepsilon_{abc} u^*_{\alpha c}, \; V_a \bar{u}_\alpha = \delta_{ab} \bar{u}_{\alpha b} . \quad (1.45)$$

A possible solution for the non-canonical operators V_a , $a = 1,2,3$ which satisfy (1.45) is

$$V_a * = \varepsilon_{abc} u^*_{\alpha c} \frac{\delta}{\delta \bar{u}_{\alpha b}} * + \delta_{ab} \bar{u}_{\alpha b} \frac{\delta}{\delta \bar{u}_\alpha} * , a = 1,2,3. \quad (1.46)$$

We will extend this result for all the antifields and we will consider V_a of the form

$$V_a * = (-)^{\varepsilon(Q^A)} \varepsilon_{abc} Q^*_{Ac} \frac{\delta^R}{\delta \bar{Q}_{Ab}} * + (-)^{\varepsilon(Q^A)} \delta_{ab} \bar{Q}_{Ab} \frac{\delta}{\delta \bar{Q}_A} *, \; a = 1,2,3 \quad (1.47)$$

with the properties

$$V_a V_b + V_b V_a = 0, \; V_a \delta_b + \delta_b V_a = 0, \; a,b = 1,2,3. \quad (1.48)$$

From nilpotency condition for s_a^L expressed by (1.37) we obtain the form of the master equations

$$\frac{1}{2}(S,S)_a + V_a S = 0, \; a = 1,2,3. \quad (1.49)$$

Conclusion 2: *The action of the BRST operators on the generators of the extended space in the two formalisms, Hamiltonian and Lagrangean, has the form*

$$\begin{aligned} [q^i, \Omega_a] &= (q^i, S)_a ; \; [Q^{\alpha b}, \Omega_a] = (Q^{\alpha b}, S)_a \\ [\lambda^{\alpha b}, \Omega_a] &= (\lambda^{\alpha b}, S)_a ; [\eta^\alpha, \Omega_a] = (\eta^\alpha, S)_a \end{aligned} \quad (1.50)$$

$$[P_{\alpha b},\Omega_a]=(u^*_{\alpha b},S)_a+\cdots \qquad (1.51)$$

$$[\pi_{\alpha b},\Omega_a]=V_a\bar{u}_{\alpha b}+\cdots;\ [\pi_\alpha,\Omega_a]=V_a\bar{u}_\alpha+\cdots. \qquad (1.52)$$

Conclusion 3: *The non-canonical operators which appear* ***in a natural way*** *in the theory can be used in the gauge fixing procedure. How to implement this original gauge fixing procedure will represent the aim of a forthcoming paper.*

4. Concluding remarks

The study of the equivalence between the Lagrangean and Hamiltonian formalisms in the BRST theory seems to be not a formal problem, but an important manner of understanding the theory itself. What is the significances of the non pairs variables, what is the complete action of the BRST operators, what non-canonical part appears in the master equations, what is the importance of the non-canonical operators from the master equations are some of the problems to whom our comparative study of the two formalisms offered clear answers.

References

[1] I.A. Batalin, P. M. Lavrov, I.V. Tyutin, J. Math. Phys.31, (1990) 1487 ;

[2] I.A. Batalin, P. M. Lavrov, I.V. Tyutin, J. Math. Phys.31, (1990) 6

[3] C. Becchi, A. Rouet and R. Stora, Phys. Lett. B52, 344 (1974); C. Mat. Phys. 42,127 (1975);

[4] I. V. Tyutin, Lebedev Preprint 39 (1975), unpublished;

[5] R. Constantinescu, C. Ionescu, Int. J. of Mod. Phys. A Vol. 21, No. 7 (2006) 1567

[6] C. Bizdadea, S. O. Saliu, Phys. AUC 12 part III (2002) 150

[7] R. Constantinescu, J. Math. Phys. 38 (1997) 2786

[8] R. Constantinescu, C. Ionescu, Int. J. of Mod. Phys. A Vol. 21, No. 32 (2006) 6629