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THE POST – GAUSSIAN WAVELET

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Abstract
One way to decompose arbitrary signals are the procedures of wavelet transforms. The localized contributions to signals are characterized by the scale parameter and the localization parameters. In this brief report we introduced a new family of post – Gaussian wavelets. This new wavelet is an algebraic not a exponential one, so reducing the computation time. The new wavelet for great orders (tending to infinity) describes the classical Morlet wavelet.
Keywords: Signal processing, A post – Gaussian wavelet

1. Introduction

The wavelet theory is a powerful method for processing different signals. In this report a new wavelet was obtained. This post – Gaussian delta wavelet is based on an approximation of a delta window. From the admissibility condition in the case of wavelet regular enough we have:

i) the wavelet Fourier transform in origin equal to zero
ii) the wavelet has zero mean

This transform is a more flexible one because is dependent on three parameters and can cover more combinations of discrete and continuous parameter discretisations (hybrid cases).

2. Method

In processing signals, a part the well known Fourier a method, a more powerful instrument is offered by the wavelet theory [1], [2], [3a], [3b]. The post – Gaussian window is given by:

$$f_{\varepsilon,k}(t) = \frac{\varepsilon}{\varepsilon^2 + t^{2k}\pi^k}, \tag{1a}$$

k-parameter which gives the order of post – Gaussian window.

Also we have:

$$f_\varepsilon(t) = \frac{1}{\varepsilon\sqrt{\pi}}\exp\left[-\left(\frac{t}{\varepsilon}\right)^2\right] \tag{1b}$$

where for ε tending to zero the limit is the Dirac function$\delta(t)$. In Fig.1a the post – Gaussian delta window

$$postgw_{\varepsilon,k,d} = \frac{\varepsilon}{\varepsilon^2 + (t-d)^{2k}\pi^k} \tag{2a}$$

is represented, where d is the window length.

The post Gaussian window and general post Gaussian window use almost the same number of multiplications .

The expression (2a) is represented for different values ofε: $(\varepsilon = 0.1;\ \varepsilon = 0.3;\ k = 2;\ d = 0)$.

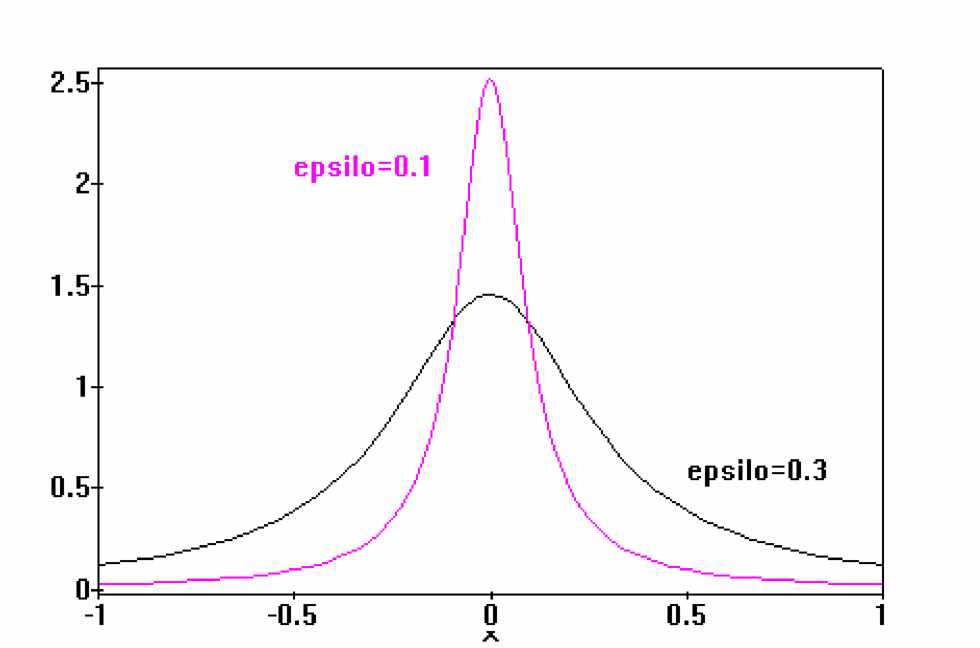

Fig.1a Normalized Post – Gaussian Windows

We construct the mother wavelet which is the derivative of expression of type (2a):

$$h_{\varepsilon,k,0}(t) = \frac{d^2 postgw_{\varepsilon,k,0}(t)}{dt^2} \tag{2b}$$

and

$$\frac{d\ postgw}{dt} = -2\varepsilon \frac{k\,t^{2k}\pi^k}{(\varepsilon^2 + t^{2k}\pi^k)^2\,t} \tag{2c}$$

The first wavelet $\frac{d\ postgw}{dt}$ have almost the double number of multiplications than post Gaussian window.

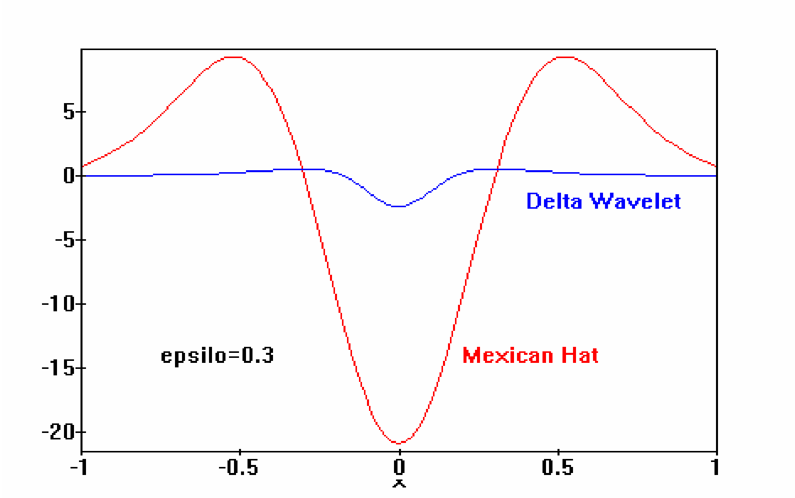

Fig.1b Delta and Mexican Hat Wavelets

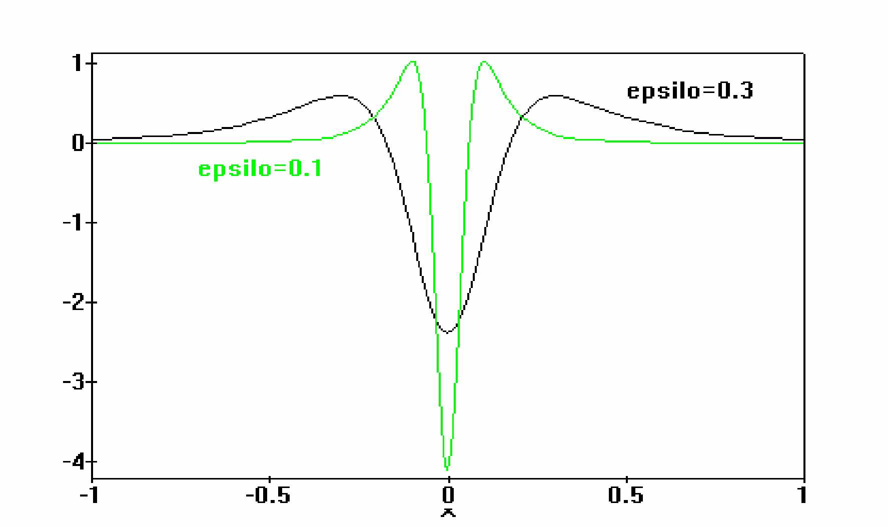

Fig.2a Normalized Post – Gaussian Wavelets $(k = 2)$

From the admissibility condition [4] in case of wavelet regular enough we have:

i) the Fourier transform $\hat{h}_{\varepsilon,k,0}(0) = 0$

ii) the mother wavelet: $$\int_{-\infty}^{\infty} h_{\varepsilon,k,0}(t)dt = 0 \qquad (3)$$

In Fig.1b the wavelets Mexican hat and delta $(\varepsilon = 0.3;\ d = 0)$ were shown.

In Fig.2a the mother wavelets $(\varepsilon = 0.1;\ \varepsilon = 0.3;\ k = 2;\ d = 0)$ were also shown.

The set of daughter wavelets are generated from the mother wavelet by shift operations:

$$h_{\varepsilon,k,d}(t) = \frac{1}{\sqrt{F}} h_{\varepsilon,k,0}(t) \qquad (4a)$$

where d is the shift. The normalization factor is given by:

$$F = \int_{-\infty}^{\infty} \left| h_{\varepsilon,k,0}(t) \right|^2 dt \qquad (4b)$$

The (1-D) wavelet transform of $f(t)$ is defined as:

$$W(\varepsilon,k,d) = \int_{-\infty}^{\infty} f(t) h^*{}_{\varepsilon,k,d}(t)dt \qquad (5)$$

This is a correlation operation between the signal $f(t)$ and the shifted (d) and scaled (ε) mother wavelet $h_{\varepsilon,k,d}(t)$. The $h_{\varepsilon,k,d}(t)$ wavelet depends of three parameters (ε,k,d). Also for the same number of parameters we obtained less operation in post – Gaussian wavelet evaluation than for Mexican hat one (the presence of time consuming exponential evaluation). For $k = 1$ we have the particular case of delta wavelet [5], for k tending to infinity we describe the Mexican Hat wavelet and for k intermediary we have the hole family.

Acknowledgements

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