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A BOSONIC SYSTEM IN A S DIAGONAL SPACE-TIME

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Abstract
The aim of this paper is to study the SO(3,1) U(1) gauge minimally coupled charged spinless field to a spherically symmetric curved space-time. It is derived the first order analytically approximation solution for the system of Klein-Gordon-Maxwell-Einstein equations. Using these solutions, it evaluated the electric current's components and further the boson system electric charge. The considered space –time manifold generalize an anterior studied one. The chosen metric tensor is of S diagonal type. The anterior results obtained on McCrea space-time, are developed and new aspects are highlighted.
Keywords: Klein-Gordon-Maxwell-Einstein equations, curved space-time .

1. Introduction

The study of boson stars (BS) started with the work of Kaup [1] and Ruffini and Bonazzalo [2], who found asymptotically solutions of the Einstein-Klein-Gordon equations for spherically symmetric equilibrium state. After the mid 1980's, a major interest has been focused on macroscopic stable boson stars, since they have been considered to provide a considerable fraction of the non-baryonic part of dark matter, [3, 4]. These configurations are "macroscopic quantum states" and are only prevented from collapsing gravitationally by the Heisenberg uncertainty principle.

Non-interacting complex scalar fields [1] [2] were originally considered for the constituents composing boson stars. The problem has been successfully worked out in low dimensional gravity [4], whiles in four dimensions, fields interacting via gravity have been investigated mainly by numerical calculations [5]. In present paper we derived an evaluation of the electric charge for a symmetric McCrea space-time. This curved manifold generalizes the anterior studied space-time [4, 5 and 6].

2. Fields equations on curved space-time

Let us consider a spherically symmetric configuration describe by a metric tensor of static conformal type, expressed in Schwarzchild coordinates as

$$ds^2 = e^{\Lambda(r,t)}dr^2 + e^{\mu(r,t)}d\theta^2 + e^{\mu(r,t)}\sin^2(\theta)d\varphi^2 - e^{\nu(r,t)}dt^2 \tag{1}$$

It could be introduced the pseudo-orthonormal tetradic frame $\{e_a\}_{a=1,4}$, with the corresponding dual orthonormal base

$$\omega^1 = e^{\frac{1}{2}\Lambda(r,t)}dr, \quad \omega^2 = e^{\frac{1}{2}\mu(r,t)}d\theta, \quad \omega^3 = e^{\frac{1}{2}\mu(r,t)}\sin(\theta)d\varphi, \quad \omega^4 = e^{\frac{1}{2}\nu(r,t)}dt \tag{2}$$

The connection coefficients derived in this frame are:

$$\Gamma^1{}_{22} = -\Gamma^2{}_{12} = -\frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial r}\right)\frac{1}{e^{\Lambda(r,t)/2}}; \qquad \Gamma^1{}_{33} = -\Gamma^3{}_{13} = -\frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial r}\right)\frac{1}{e^{\Lambda(r,t)/2}}$$

$$\Gamma^1{}_{41} = -\Gamma^4{}_{11} = -\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)\frac{1}{e^{\nu(r,t)/2}}; \qquad \Gamma^1{}_{44} = \Gamma^4{}_{14} = \frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial r}\right)\frac{1}{e^{\Lambda(r,t)/2}}$$

$$\Gamma^2{}_{33} = -\Gamma^3{}_{23} = -\frac{\cos(\theta)}{\sin(\theta)}\frac{1}{e^{\mu(r,t)/2}}; \qquad \Gamma^2{}_{12} = \Gamma^2{}_{21} = \frac{1}{2}\frac{\partial\mu(r,t)}{\partial r}$$

$$\Gamma^2{}_{33} = -\sin(\theta)\cos(\theta); \qquad \Gamma^2{}_{42} = \Gamma^4{}_{22} = \frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\frac{1}{e^{\nu(r,t)/2}}$$

$$\Gamma^3{}_{43} = \Gamma^4{}_{33} = \frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\frac{1}{e^{\nu(r,t)/2}}$$

The Einstein tensor G_{ab} has the following non-vanishing components

$$\begin{aligned} G_{11} &= \frac{1}{e^{\mu(r,t)}} + \frac{1}{e^{\Lambda(r,t)}}\left[\left(\frac{\partial\mu(r,t)}{\partial r}\right)\left(\frac{\partial\nu(r,t)}{\partial r}\right) - \frac{1}{4}\left(\frac{\partial\mu(r,t)}{\partial r}\right)^2\right] \\ &+ \frac{1}{e^{\nu(r,t)}}\left[\left(\frac{\partial^2\mu(r,t)}{\partial t^2}\right) - \frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\left(\frac{\partial\nu(r,t)}{\partial t}\right) + \frac{3}{4}\left(\frac{\partial\mu(r,t)}{\partial t}\right)^2\right] \end{aligned} \tag{3}$$

$$G_{22} = \frac{1}{e^{\nu(r,t)}}\left[\frac{1}{4}\left(\frac{\partial\mu(r,t)}{\partial t}\right)^2 + \frac{1}{4}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)\left(\frac{\partial\mu(r,t)}{\partial t}\right) - \right.$$
$$- \frac{1}{4}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)\left(\frac{\partial\nu(r,t)}{\partial t}\right) + \frac{1}{2}\left(\frac{\partial^2\mu(r,t)}{\partial t^2}\right) -$$
$$\left. - \frac{1}{4}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\left(\frac{\partial\nu(r,t)}{\partial t}\right) + \frac{1}{4}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial^2\Lambda(r,t)}{\partial t^2}\right)\right] +$$
$$+ \frac{1}{e^{\Lambda(r,t)}}\left[-\frac{1}{2}\left(\frac{\partial^2\mu(r,t)}{\partial r^2}\right) - \frac{1}{4}\left(\frac{\partial\mu(r,t)}{\partial r}\right)^2 + \frac{1}{4}\left(\frac{\partial\Lambda(r,t)}{\partial r}\right)\left(\frac{\partial\mu(r,t)}{\partial r}\right) - \right.$$
$$- \frac{1}{2}\left(\frac{\partial^2\nu(r,t)}{\partial r^2}\right) + \frac{1}{4}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)\left(\frac{\partial\nu(r,t)}{\partial t}\right) -$$
$$\left. - \frac{1}{4}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\left(\frac{\partial\nu(r,t)}{\partial t}\right) - \frac{1}{4}\left(\frac{\partial\nu(r,t)}{\partial t}\right)^2\right] \quad (4)$$

$$G_{33} = G_{22} \quad (5)$$

$$G_{44} = -\frac{1}{e^{\mu(r,t)}} + \frac{1}{e^{\nu(r,t)}}\left[-\frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\left(\frac{\partial\Lambda(r,t)}{\partial t}\right) - \frac{1}{4}\left(\frac{\partial\mu(r,t)}{\partial t}\right)^2\right] +$$
$$+ \frac{1}{e^{\Lambda(r,t)}}\left[\frac{3}{4}\left(\frac{\partial\mu(r,t)}{\partial r}\right)^2 - \frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial r}\right)\left(\frac{\partial\Lambda(r,t)}{\partial r}\right) + \left(\frac{\partial^2\mu(r,t)}{\partial r^2}\right)\right] \quad (6)$$

$$G_{14} = \frac{\left(\frac{\partial^2\mu(r,t)}{\partial t\partial r}\right)}{e^{\nu(r,t)/2}e^{\Lambda(r,t)/2}} + \frac{1}{2}\frac{\left(\frac{\partial\mu(r,t)}{\partial r}\right)\left(\frac{\partial\mu(r,t)}{\partial t}\right)}{e^{\nu(r,t)/2}e^{\Lambda(r,t)/2}} -$$
$$- \frac{1}{2}\frac{\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)\left(\frac{\partial\mu(r,t)}{\partial r}\right)}{e^{\nu(r,t)/2}e^{\Lambda(r,t)/2}} - \frac{1}{2}\frac{\left(\frac{\partial\nu(r,t)}{\partial r}\right)\left(\frac{\partial\mu(r,t)}{\partial t}\right)}{e^{\nu(r,t)/2}e^{\Lambda(r,t)/2}} \quad (7)$$

Further is considered a charged massive boson, coupled to the electromagnetic field. The $SO(3,1)\times U(1)$ gauge invariant Lagrangean density has the form

$$L = \eta^{ab}\overline{\Phi}_{;a}\Phi_{;b} + m_0^2\overline{\Phi}\Phi + \frac{1}{4}F^{ab}F_{ab} \quad (8)$$

where we used the definitions $\Phi_{;a} = \Phi_{|a} - ieA_a\Phi$ and $\overline{\Phi}_{;a} = \overline{\Phi}_{|a} + ieA_a\overline{\Phi}$.

The Maxwell tensor $F_{ab} = A_{b:a} - A_{a:b}$ is expressed in the terms of the Levi-Civita covariant derivative of the four-potential $\{A_a\}_{a=\overline{1,4}}$, i.e.

$$A_{a:b} = A_{a|b} - A_c\Gamma^c{}_{ab} \quad (9)$$

Working in the minimally symmetric *ansatz* $A_1 = A_1(r,t)$, $A_4 = A_4(r,t)$, $\Phi = \Phi(r,t)$, the single non-vanishing Maxwell tensor (8) component is

$$F_{14} = -F_{41} = -\frac{\left[e^{\Lambda(r,t)/2}\left(2A_{1,t} + \frac{\partial\Lambda(r,t)}{\partial t}A_1\right) - e^{\nu(r,t)/2}\left(\frac{\partial\nu(r,t)}{\partial r}A_4 - 2A_{4,r}\right)\right]}{e^{\nu(r,t)/2}e^{\Lambda(r,t)/2}} \tag{10}$$

Further it was derived the Klein - Gordon equation, for the scalar spinless field Φ. The considered evolution equation can be read as [7]:

$$\Box\Phi - m_0{}^2\Phi = 2ieA^c\Phi_{|c} + e^2A^cA_c\Phi \tag{11}$$

and it's hermitic conjugated.

The explicit form for the Klein - Gordon equation (11) can be written as

$$\begin{aligned}
& m_0^2\Phi(r,t) - \frac{1}{e^{\nu(r,t)}}\left(\frac{\partial^2\Phi(r,t)}{\partial t^2}\right) + \frac{1}{e^{\Lambda(r,t)}}\left(\frac{\partial^2\Phi(r,t)}{\partial r^2}\right) + \\
& + \frac{1}{e^{\nu(r,t)}}\left(\frac{\partial\Phi(r,t)}{\partial t}\right)\left[-\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right) - \left(\frac{\partial\mu(r,t)}{\partial t}\right) + \frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial t}\right)\right] + \\
& + \frac{1}{e^{\Lambda(r,t)}}\left(\frac{\partial\Phi(r,t)}{\partial r}\right)\left[-\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial r}\right) + \left(\frac{\partial\mu(r,t)}{\partial r}\right) + \frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial r}\right)\right] = \\
& = 2ie\left[\frac{1}{e^{\Lambda(r,t)/2}}A_1\frac{\partial\Phi(r,t)}{\partial r} - \frac{1}{e^{\nu(r,t)/2}}A_4\frac{\partial\Phi(r,t)}{\partial t}\right] + e^2\Phi(r,t)\left[A_1^2 - A_4^2\right]
\end{aligned} \tag{12}$$

and respective it's hermitic conjugated.

The Maxwell system equations

$$F^{ab}{}_{:c} = -ie\eta^{ab}\left[\overline{\Phi}\left(\Phi_{|b} - ieA_b\Phi\right) - \left(\overline{\Phi}_{|b} - ieA_b\overline{\Phi}\right)\Phi\right] \tag{13}$$

can be written in an explicit form as

$$
\begin{aligned}
&\frac{1}{e^{\nu(r,t)/2}}\frac{1}{e^{\Lambda(r,t)/2}}\left\{-\left(\frac{\partial\mu(r,t)}{\partial t}\right)\left[A_{4,r}-\frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial r}\right)A_4\right]\right\}+\\
&+\frac{1}{e^{\nu(r,t)/2}}\frac{1}{e^{\Lambda(r,t)/2}}\left[-A_{4,rt}-\frac{1}{2}\left(\frac{\partial^2\nu(r,t)}{\partial t\partial r}\right)A_4-\frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial r}\right)A_{4,t}\right]+\\
&+\frac{1}{e^{\nu(r,t)/2}}\frac{1}{e^{\Lambda(r,t)/2}}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)\left[\frac{1}{4}\left(\frac{\partial\nu(r,t)}{\partial r}\right)A_4+\frac{1}{2}A_{4,r}\right]+\\
&+\frac{1}{e^{\nu(r,t)}}\left[A_{1,tt}+\left(\frac{\partial\mu(r,t)}{\partial t}\right)A_{1,t}+\frac{1}{2}\left(\frac{\partial^2\Lambda(r,t)}{\partial t^2}\right)A_1+\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)A_{1,t}\right]+\\
&+\frac{1}{e^{\nu(r,t)}}\left\{\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)A_1\left[-\frac{1}{4}\left(\frac{\partial\nu(r,t)}{\partial r}\right)+\frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\right]-\frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial t}\right)A_{1,t}\right\}=\\
&=-iee\frac{1}{e^{\Lambda(r,t)/2}}\left(\overline{\Phi}\Phi_{,r}-\overline{\Phi}_{,r}\Phi\right)-2e^2\overline{\Phi}\Phi A_1
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&\frac{1}{e^{\Lambda(r,t)}}\left[A_{4,rr}+\frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial r}\right)A_{4,r}-\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial r}\right)A_{4,r}\right]+\\
&+\frac{1}{e^{\Lambda(r,t)}}\left(\frac{\partial\nu(r,t)}{\partial r}\right)A_4\left[-\frac{1}{4}\left(\frac{\partial\Lambda(r,t)}{\partial r}\right)+\frac{1}{2}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\right]+\\
&+\frac{1}{e^{\Lambda(r,t)}}\left(\frac{\partial\mu(r,t)}{\partial t}\right)\left[\left(\frac{\partial\mu(r,t)}{\partial t}\right)A_{4,r}+\frac{1}{2}\left(\frac{\partial^2\nu(r,t)}{\partial t^2}\right)A_4\right]+\\
&+\frac{1}{e^{\Lambda(r,t)/2}}\left(\frac{\partial\mu(r,t)}{\partial r}\right)\left[A_{4,r}+\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)A_1\right]+\\
&+\frac{1}{e^{\nu(r,t)/2}}\frac{1}{e^{\Lambda(r,t)/2}}\left[A_{1,rt}-\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)A_{1,r}-\frac{1}{2}\left(\frac{\partial^2\Lambda(r,t)}{\partial t\partial r}\right)A_1\right]+\\
&+\frac{1}{e^{\nu(r,t)/2}}\frac{1}{e^{\Lambda(r,t)/2}}\left(\frac{\partial\nu(r,t)}{\partial r}\right)\left[\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)A_1+\frac{1}{2}A_{1,t}\right]=\\
&=ie\frac{1}{e^{\nu(r,t)/2}}\left(\overline{\Phi}\Phi_{,t}-\overline{\Phi}_{,t}\Phi\right)+2e^2\overline{\Phi}\Phi A_4
\end{aligned}
\tag{15}
$$

In this configuration, the necessary Lorentz condition is

$$
\begin{aligned}
&\frac{1}{e^{\nu(r,t)/2}}A_4\left[-\left(\frac{\partial\mu(r,t)}{\partial t}\right)-\frac{1}{2}\left(\frac{\partial\Lambda(r,t)}{\partial t}\right)\right]-\frac{1}{e^{\nu(r,t)/2}}A_{4,t}+\\
&+\frac{1}{e^{\Lambda(r,t)/2}}A_1\left[\frac{1}{2}\left(\frac{\partial\nu(r,t)}{\partial r}\right)+\left(\frac{\partial\mu(r,t)}{\partial r}\right)\right]+\frac{1}{e^{\Lambda(r,t)/2}}A_{1,r}=0
\end{aligned}
\tag{16}
$$

Building up the energy-momentum tensor

$$
T_{ab}=\overline{\Phi}_{:a}\Phi_{:b}+\overline{\Phi}_{:b}\Phi_{:a}+F_{ac}F_b{}^c-\eta_{ab}L\,. \tag{17}
$$

can be derived the Einstein equation $G_{ab} = kT_{ab}$, where the tensor G_{ab} have the explicit form (3)-(7).

For solving we start with the physically assumptions that the charged scalar field is the main source of both the electromagnetic and gravitational fields. Neglecting the feedback of gravity in the first order approximation, can write the equation for potential Φ:

$$\Phi_{,rr} + \frac{2}{r}\Phi_{,r} - \Phi_{,tt} - m_0{}^2\Phi = 0 \tag{18}$$

and it's hermitic conjugated. The considered solutions has the form [4, 8]

$$\Phi = \frac{N}{r}e^{i(\omega t - kr)}, \text{ and } \overline{\Phi} = \frac{N}{r}e^{-i(\omega t - kr)} \tag{19}$$

Using the same conditions, from Maxwell equations it can be read, from (14) and (15)

$$A_{1,rr} + \frac{2}{r}A_{1,r} - \frac{2}{r^2}A_1 - A_{1,tt} = -2ek\frac{|N|^2}{r^2} + 2e^2\frac{|N|^2}{r^2}A_1 \tag{20}$$

with $\omega = \sqrt{m_0^2 + k^2}$, and respectively

$$A_{4,rr} + \frac{2}{r}A_{4,r} - A_{4,tt} = 2e\omega\frac{|N|^2}{r^2} + 2e^2\frac{|N|^2}{r^2}A_4 \tag{21}$$

Considering for the gauge fields A_1 and A_4 the particular solutions of the form

$$A_1 = ek|N|^2 \text{ and } A_4(r,t) = 2e\omega|N|^2\log\left(\frac{r}{r_o}\right) + 2ek\frac{|N|^2}{r}t \tag{22}$$

from the Einstein's system, can be derived solutions for metric tensor functions

$$\mu(r,t) = F_1(r) + F_2(t) \tag{23}$$

$$F_1(r) = -2\int\frac{1}{r^2}\left(r + \sqrt{r^2 + \kappa|N|^2 - \left(\frac{1}{3}k^2 - 1\right)r^4 + 2\kappa|N|^2k^2r^2}\right)dr \tag{24}$$

$$F_2(t) = \frac{2}{3}\ln\left[\frac{9}{4k^2}\left(C_1\sin kt - C_2\cos kt\right)^2\right] \tag{25}$$

Using these results, from (3)-(7), can be derived an equation for the metric tensor function $\Lambda(r,t)$ of the form

$$\begin{aligned}&\frac{1}{r^2}\left[1 + \frac{1}{2}\frac{\left(2r + 4r^3\right)}{f(r)}\right] - \frac{1}{r^3}\left[r + f(r)\right] - \frac{1}{r^4}\left[r + f(r)\right]^2 + \frac{1}{4}\left(\frac{\partial\Lambda(r,t)}{\partial r}\right)^2 - \\ &-\frac{1}{2}\frac{1}{r^2}\left[r + f(r)\right]\left(\frac{\partial\Lambda(r,t)}{\partial r}\right) + \frac{1}{2}\frac{1}{r}\left(\frac{\partial\Lambda(r,t)}{\partial r}\right) + \frac{1}{2}\left(\frac{\partial^2\Lambda(r,t)}{\partial t^2}\right) = \kappa\frac{|N|^2}{r^4}\end{aligned} \tag{26}$$

where was used the definitions

$$f(r)=\sqrt{r^2+\kappa|N|^2+r4}\ \text{ and } \ h(r)=\sqrt{r^2\left(1+r^2\right)}.$$

Introducing these solutions, could be written as

$$\Lambda(r,t)=F(t)\int\left\{-\frac{4}{r+h(r)}-\frac{4r}{h(r)[r+h(r)]}-\frac{2r^3}{h(r)[r+h(r)]}-\frac{2r^2}{r+h(r)}\right\}dr \tag{27}$$

where

$$F(t)=\log\left(\frac{1}{2}C_1t+C_2\right) \tag{28}$$

3. Results and Discussions

Further, using these results, can be computed the fourth electric current component as

$$j_4=e\left[-2\frac{|N|^2}{r^3}\sqrt{k^2+m_0^2}+4e^2N^4\frac{\sqrt{k^2+m_0^2}}{r^2}\log\left(\frac{r}{r_0}\right)+4e^2N^4\frac{kt}{r^3}\right] \tag{29}$$

could be, further, evaluated the electric charge of the boson system. Defining

$$G(r)=\frac{\exp\left[2\alpha t\arctan\frac{1}{\sqrt{1+r^2}}-\alpha t\sqrt{1+r^2}\right]}{r^5\left(1-\frac{3}{\sqrt{9-3r^2k^2+9r^2}}\right)}\exp\left[-\frac{2}{3}\frac{\sqrt{9r^2-3r^4k^2+9r^4}}{r}\right] \tag{30}$$

The electric charge's magnitude is

$$Q=e\frac{|N|^2}{k^{4/3}}144^{2/3}\pi\omega\left[sin\left(\frac{1}{2}kt\right)+cos\left(\frac{1}{2}kt\right)\right]^2\int_0^\infty\left[\frac{G(r)^2}{9-3r^2k^2+9r^2}+3G(r)+1\right]dr \tag{31}$$

These values for the electric charge current's components are computed in order to get a comparative expression of the previous results [4 and 7]. Despite of MAPLE analytical software algorithm used in succeeding for these results, the magnitude for the boson system's electric charge in large space-time kinds, has to be evaluated numerically.

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